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Costly Financial Intermediation in Neoclassical Growth Theory*

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ABSTRACT

The neoclassical growth model is extended to include costly intermediated borrowing and lending between households. This is an important extension because substantial resources are used in intermediating the large amount of borrowing and lending. In 2007, in the United States, the amount intermediated was 1.7 times GNP, and the resources used in this intermediation amounted to at least 3.4 percent of GNP. The theory implies that financial intermediation services are an intermediate good and that the spread between borrowing and lending rates measures the efficiency of the financial sector.

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1. Introduction

A rich class of models studies savings for retirement. But these models abstract from the large costs of financial intermediation, even though most savings are intermediated. This paper extends the neoclassical growth model by incorporating an intermediation sector. It does so in such a way that it matches *both* the amount of borrowing and lending between households *and* the resources used in intermediation. Furthermore, the remaining appealing characteristics of the standard neoclassical growth model remain unaltered. In addition, the model provides a suitable framework to evaluate not only the efficiency gains from innovations in the financial sector but also the impact of demographic changes on intermediation and saving behavior.

In 2007, for the U.S economy, intermediation was large, around 1.7 times the gross national product (GNP).[1] The resources used in this process were not inconsequential, amounting to at least 3.4 percent of GNP. These two figures together imply that the average household borrowing rate is at least 2 percent higher than the average household lending rate. Relative to the level of the observed average rates of return on debt and equity securities, this spread is far from being insignificant.

Since our model abstracts from aggregate risk, by construction there is no premium for bearing aggregate risk. As explained later, the household borrowing rate is equal to the return on equity. The government can borrow at a lower rate than households—as empirically observed. Consequently, there is a difference in the return on equity and the interest rate on government debt. For our calibrated economy this difference is 2 percent, and abstracting from it may be inappropriate when computing statistics that report the

[1] About half of this was intermediated lending and the other half mutual organizations.

spread between different rates of return in the economy. We discuss this in more detail in Section 8.

Since in equilibrium the total amount borrowed by households is equal to the total amount of intermediated lending by households, a natural question that arises is, who are the borrowers and lenders? In our model, where the only reason for households to save is to finance retirement over an uncertain lifetime, one set of households chooses to save by accumulating capital and a second set by purchasing annuities. Since capital accumulation is partially financed by owners' equity and the remainder by borrowing, *capital owners are the borrowers*. In addition, since purchasing annuities is isomorphic to lending, *annuity holders are the lenders.*

We caution the reader regarding two issues. First, the model counterpart of annuities is not limited to commercial annuities but includes, more importantly, defined benefit pension plans and annuity-like promises of the government, such as Social Security and Medicare. We think of these plans as mandatory purchases of annuities. As pointed out by Abel (1987), Social Security and Medicare are implicit government liabilities and can be regarded as annuity-like promises of the government. Therefore, the empirical counterpart of the (model) government debt is the explicit government debt *plus* the present value of all the implicit government promises. In Section 8, where we examine some implications of our theory, we will include these annuity-like promises as part of annuity-like assets held by households.[2]

Second, we follow the tradition in macroeconomics that assumes that households own all the capital in the economy and rent it to businesses. Thus, we treat the capital

[2] We reemphasize that when we use the annuity construct in this paper, it includes all annuity-like payments, including Social Security, Medicare, and defined benefit pension plans as well as a small amount of commercial annuities.

owned by businesses as capital owned by the owners of these businesses, and therefore, all debt of nonfinancial businesses is debt of our household sector.

The output of the intermediary sector is an intermediary good. The value added by intermediation services is equal to the amount of borrowing times its price minus the amount of lending times its price. In equilibrium, the amount borrowed is equal to the amount lent. Hence, the price of this service is equal to the spread between the average borrowing and lending rates. Improvements in the financial system that reduce this spread are efficiency gains.

Table 1 details the aggregate balance sheet data for U.S households in 2007. Household assets include privately held capital stock (K), government debt (D^G),[3] and intermediated household lending (D^H). The latter is equal to the amount of household borrowing (D^H).

Table 1

Aggregate Balance Sheet of U.S. Households, 2007

Assets	**Liabilities**
$K = 3.4\ GDP$	$D^H = 1.7\ GDP$
$D^H = 1.7\ GDP$	
$D^G = 4.6\ GDP$	$Net\ Worth = 8.0\ GDP$

In 2007, about half the U.S. capital stock, the value of which was 3.4 times GNP, was financed by borrowing and half by owners' equity. This borrowing is done to finance owner-occupied housing, by proprietorships and partnerships to finance unincorporated

[3] Government debt includes explicit government debt (0.5 GDP) and implicit government debt (4.1 GDP)

businesses, and by shared ownership corporations to finance businesses. Households that own capital finance it partially by borrowing (D^H) and partially by equity. Further, the Modigliani-Miller theorem holds for our economy because for a given firm, the debt-equity financing decision does not matter. In the aggregate, total equity and private debt are determined.

Reason for household borrowing

We begin our study by examining household savings decisions. In practice, most household savings are for retirement. However, some savings is held in highly liquid financial instruments as a substitute for costly insurance against idiosyncratic risk such as a job loss.[4] Abstracting from these factors has little consequence for aggregate lending. In our model, households choose between two savings strategies. One strategy is to invest in equity and earn a real return of r_e percent. The other strategy is to purchase a lifetime annuity, which is actuarially fair at $r < r_e$ percent. Since the lifetime remaining after retirement is uncertain, households that choose the annuity option are in effect buying insurance against outliving their savings.

But why do some households choose to save by lending to financial intermediaries (with a low return) while others invest in equities (with a high return)? In this study, this disparity is attributable to household heterogeneity in the form of differences in the strength of preferences for bequests. That is, we assume that people are identical in all aspects other than the intensity of their bequest motive. The only source of uncertainty is the duration of the lifetime after retirement. Hence, an important difference

[4] In this study, we do not make a distinction between these two types of savings. For issues other than the ones we address in this paper, this may be a crucial element of reality that would have to be incorporated into the abstraction.

between both strategies is that the strategy of buying equities generates bequests upon death equal to net worth at the time of death, whereas buying annuities does not.[5] For our calibrated economy, people with a low, say nil, bequest motive will prefer the annuity strategy, whereas agents with even a modest bequest motive will prefer equities.[6] The strength of the bequest motive has little consequence for aggregate bequests with bequests being largely accidental.

To summarize, in equilibrium, those with even a modest preference for bequests accumulate capital assets and borrow during their working lives, and upon retirement, use capital income for consumption and interest payment on debt. Upon their death, they bequeath all their net worth. Households with little or no bequest motive buy annuities during their working years and use annuity benefits to finance their consumption over their retirement years.

As mention earlier, we abstract from the small amount of direct borrowing and lending between households and assume that all borrowing and lending between households is intermediated through financial institutions. Furthermore, in light of the finding that the premium for bearing nondiversifiable aggregate risk is small in models consistent with growth and business cycle facts, our analysis abstracts from *aggregate risk.*[7]

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[5] We permit an annuity payment upon death. The amount will be positive if the bequest preference parameter is not zero for anyone choosing the annuity strategy.

[6] As explain later, there is an additional requirement about the size of the spread.

[7] Using a model with no capital accumulation, Mehra and Prescott (1985) find a small equity premium. McGrattan and Prescott (2000) find that the equity premium is small in the growth model if it is restricted to be consistent with growth and business cycle facts. Lettau and Uhlig (2000) introduce habit formation into the standard growth model and find that the equity premium is small if the model parameters are restricted to be consistent with the business cycle facts. Many others using the growth model restricted to be consistent with the macro economic growth and business cycle facts have found the same thing.

The intermediation technology is constant returns to scale with intermediation costs being proportional to the amount intermediated. To calibrate the constant of proportionality, we use Flow of Funds Account statistics and data from National Income and Product Accounts (NIPA). The calibrated value of this parameter equals the net interest income of financial intermediaries, divided by the quantity of intermediated debt, and is approximately 2 percent.[8]

In the absence of aggregate uncertainty, the return on equity and the borrowing rate are identical, since the households who borrow are also marginal in equity markets. In our framework, government debt is intermediated at zero cost, and thus its return is equal to the household lending rate. An important feature is that the government can borrow at a lower rate than can households—a feature that mirrors reality.

In our model, all households in a cohort have identical labor income at every point in their working life. As a consequence, there is little difference in cross-sectional consumption at a point in time. However, sizable differences in net worth develop within a cohort over their working years. One implication is that preferences for bequests cannot be ignored when studying net worth distributions.

The paper is organized as follows. The economy is specified in Section 2. In Section 3, we discuss the decision problem of the households. Section 4 deals with the aggregation of individual behavior, Section 5 with the relevant balance sheets, and Section 6 characterizes the balanced growth equilibrium. We calibrate the economy in Section 7. In Section 8, we present and discuss our results. Section 9 concludes the paper.

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[8] See Section 7 (calibration) for details.

2. The Economy

In order to build a model that captures the large amount of observed borrowing and lending, as well as the large amount of resources used in this process, we introduce three key features of reality. The first feature is differences in bequest preferences, the second is an uncertain length of retirement, and the third is costly intermediation of borrowing and lending between households. This leads some households to buy costly annuities that make payments throughout the retirement years. Since buying an annuity is isomorphic to lending, households choosing the annuity option are the lenders in our model. Households with high bequest utility save by increasing their net worth, which is their holding of productive capital less their debt.

We model an overlapping-generations economy and consider its balanced growth path equilibrium. All households born at a given date are identical in all respects except for their bequest preference parameter α . They all have identical preferences with respect to consumptions over their lifetime, so the only dimension over which they differ is α . Those with a even a modest preference for bequests borrow and own capital; others with no or weak preferences for bequest lend by acquiring annuities.

What motivates bequests? Although a casual consideration of bequests naturally assumes that they exist because of parents' altruistic concern for the economic well-being of their offspring, results in Menchik and David (1983), Hurd (1989), Wilhelm (1996), Laitner and Juster (1996), Altonji, Hayashi, and Kotlikoff (1997), Laitner and Ohlsson (2001), Kopczuk and Lupton (2007), and Fuster, Imrohoroglu, and Imrohoroglu (2008) suggest otherwise: households with children do not, in general, exhibit behavior in greater accord with a bequest motive than do childless households. This finding leads us to conclude that the existing literature supports our assumption that some people have preferences for making bequests. These empirical results lead us to eschew the perspective of Barro (1974) and Becker and Barro (1988), who postulate that each generation receives utility from the consumption of the generations to follow, and simply model bequests as being motivated by a well-defined "joy of giving,"[9] as in Abel and Warshawsky (1988) and Constantinides, Donaldson, and Mehra (2007).

Households

Any systematic consideration of bequests mandates that the analysis be undertaken in the context of an overlapping-generations model. Accordingly, we analyze an overlapping-generations economy and determine its balanced growth behavior. Each period, a set of individuals of measure 1 enter the economy. Two types enter at each date: type-A, who derive no utility from leaving a bequest, and type-B, whose utility is an increasing function of the amount they bequeath.[10] The measure of type $i \in \{A, B\}$ is μ_i. The total measure of people born at each date is 1, so $\mu_A + \mu_B = 1$.

Individuals have finite expected lives. They enter the labor force at age 22, work for T years, and then retire.[11] Model age j is 0 when a person begins his or her working life. The first year of retirement is model age $j = T$.

All workers receive an identical wage income. Wage income grows at the economy's balanced growth rate γ. At retirement, individuals face idiosyncratic

[9] See also Hurd and Mundaca (1989), De Nardi, Imrohoroglu, and Sargent (1999), De Nardi (2004), and Hansen and Imrohoroglu (2006).

[10] The "no utility from a bequest" assumption is a simplifying one and is not necessary for the analysis. All that is needed is that the utility from bequest be sufficiently small that the type-A choose to acquire annuities.

[11] We implicitly assume that parents finance the consumption of their children under the age of 22; in other words, children's consumption is a part of their parents' consumption.

uncertainty about the length of their remaining lifetime. Their retirement lifetimes are exponentially distributed. Once individuals retire, the probability of surviving to the next period is $\sigma = (1-\delta)$, where δ is the probability of death. Expected life is $T + 1/\delta$. We emphasize that there is no aggregate uncertainty.[12]

Individuals of type α, born at time *t,* order their preferences over age-contingent consumption and bequests by[13]

(2.1) $$\sum_{j=0}^{T} \beta^j \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \beta^j \sigma^{j-T} \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \alpha\delta\beta^j \sigma^{j-T-1} \log b_{t+j,j} .$$

Here, $\beta < 1$ is the discount factor and α is the strength of bequest parameter. Variable $c_{t+j,j}$ is the period consumption of a *j*-year-old born at time *t,*[14] conditional on being alive at time $t + j$. An individual who is born at time *t* and dies at age $j-1$ consumes nothing at time $t + j$ and bequeaths $b_{t,t+j}$ units of the period $t + j$ consumption good and consumes nothing subsequently. Each generation supplies one unit of labor inelastically for $j = 0,1,\dots,T-1$. Thus, aggregate labor supply is $L = T$ given that the measure of each generation is 1.

We only need to analyze the decision problems of an individual of a type-α born at time $t = 0$. The solution to the problem for a type α born at any other time *t* can be found using the fact that along a balanced growth path

(2.2) $$c_{t,j} = (1+\gamma)^t c_{0,j} .$$

[12] The Blanchard (1985) model has individuals with exponential life. The Díaz-Giménez et al. (1992) model has individuals with both an exponential working life and an exponential retirement life.
[13] Our model has no factor giving rise to life cycle consumption patterns over the working life as in Fernández-Villaverde and Krueger (2002).
[14] In this paper, the first subscript represents calendar time and the second subscript represents the age at that time.

Further, to simplify the notation, we use c_j to denote the consumption of a *j*-year-old at time *j* rather than $c_{j,j}$. An analogous change of notation applies to the other variables.

Production Technology

The aggregate production function is

(2.3) $$Y_t = F(K_t, z_t L_t) = K_t^{\theta}(z_t L_t)^{1-\theta}$$

(2.4) $$z_{t+1} = (1+\gamma)z_t ,$$

where K_t is capital, L_t is labor, and z_t is the labor-augmenting technological change parameter, which grows at rate γ. The parameter z_0 is chosen so that $Y_0 = 1$.

Output is produced competitively, so

(2.5) $$\delta_k + r_e = F_K(K_t, z_t L_t)$$

(2.6) $$e_t = z_t F_L(K_t, z_t L_t) ,$$

where δ_k is the depreciation rate, r_e is both the household borrowing rate and the return on equity, and e_t is the wage rate.

Income is received as either wage income E_t or gross capital income R_t. Thus,

(2.7) $$Y_t = E_t + R_t ,$$

where $E_t = L_t e_t = (1-\theta)Y_t$ and $R_t = (\delta_k + r_e)K_t = \theta Y_t$. Components of output are consumption C_t, investment X_t, and intermediation services I_t; thus,

(2.8) $$Y_t = C_t + X_t + I_t .$$

Along a balanced growth path, investment $X_t = (\delta_k + \gamma)K_t$ and $K_{t+1} = (1+\gamma)K_t$.

Financial Intermediation Technology

The intermediation technology displays constant returns to scale, with the intermediation cost in units of the composite output good being proportional to the amount of borrowing and lending intermediated. The cost is ϕ times the amount of borrowing and lending between households.[15] The intermediary also intermediates between households lending to the government. There are no costs associated with this intermediation. The intermediary receives interest rate r_e on its lending to households and effectively pays interest rate r on its borrowing from households. Given the technology, equilibrium interest rates satisfy

$$r_e - r = \phi .$$

The lending contract between households and intermediaries is not the standard one but rather an annuity contract. A household can enter into an annuity contract at age 0. An annuity contract specifies an age-contingent premium payment path during working life, a benefit path contingent on being alive subsequent to retirement, and a payment upon death. The amount being lent by an individual who has chosen the annuity contract is the value of pension fund reserves for that contract at that point in time. These reserves are equal to the expected present value of future payments less the expected present value of future premium payments, if any. The present value is calculated using r as the lending rate at which households can lend to intermediaries. Competitive intermediaries will offer any annuity contract with the property that the expected present value of benefits is equal to the present value of the premiums using r in the present value calculations.

[15] Miller and Upton (1974) pioneered in having a financial sector in their dynamic general equilibrium model. They had no intermediation costs.

The alternative to entering into an annuity contract to save for retirement is to accumulate capital and to borrow to partially finance that capital. Our model has three sectors: a household sector, a government sector, and a financial sector. The nonfinancial business sector is consolidated with household sector.

Government Policy

Government policy is characterized by a tax rate τ on labor income, an interest rate r on government debt, and the path of government debt $\{D_t^G = D_0^G(1+\gamma)^t\}$. The feasible government policy parameters are constrained to a one-dimensional manifold. Theoretically, it does not matter which of the three policy parameters is picked. We chose r because it simplified finding the equilibrium and we have a wealth of observations as to a reasonable value for its choice. The government finances interest payments on its debt by issuing new debt and by taxing labor income. The government's period t budget constraint is

(2.9) $$(1+r)D_t^G = \tau E_t + D_{t+1}^G.$$

Since $D_{t+1}^G = (1+\gamma)D_t^G$ in balanced growth,

(2.10) $$(r-\gamma)D_t^G = \tau(1-\theta)Y_t.$$

The government pursues a tax rate policy that pegs[16] r, which equals the interest rate on government debt. This being a balanced growth analysis, government debt grows at rate $\gamma > 0$, which means that the government deficits are positive and grow at rate γ as well.

[16] In this paper, we fix this rate at 3 percent. This is discussed further in Section 7 on calibration.

The intermediary holds all the government debt, and there are no intermediation costs associated with holding this asset on the part of the intermediary.

Bequests

Aggregate bequests at date t are

(2.11) $$B_t = B_0(1+\gamma)^t \ .$$

We let $\bar{b} = B_{30}$. The inheritance of a type-B born at $t=0$ is

(2.12) $$\bar{b}^B = \bar{b}$$

and is received at date $t = 30$. The inheritance of a type-A born at $t=0$ is

(2.13) $$\bar{b}^A = \bar{b}(1+r)/(1+r_e) \ .$$

A type-A's inheritance is slightly smaller than that of a type-B because their inheritances are intermediated and intermediation is costly.

3. Optimal Individual Decisions

We consider the optimal individual decision problem, taking as given (i) the size of the inheritance the individual will receive at model age 30 (chronological age 52), (ii) the wages at each date of the individual's working life, (iii) the labor income tax rate τ, and (iv) the borrowing and lending rates r_e and r. The first problem facing an individual is whether to choose the annuity strategy A or the no annuity strategy B. The parameters of the calibrated economy are such that a type-A will choose the annuity strategy, whereas a type-B will choose the no annuity strategy. The second problem is to determine the optimal lifetime consumption and savings decisions conditional on the strategy chosen. We determine, given α, the optimal consumption/saving behavior for each strategy and the resulting lifetime utility, and then determine which of the two strategies is best for that individual type.

A convention followed is that a bar over a variable denotes a constant. In the case where the constant depends upon a person's type, that is, on α, this functional dependence is indicated. This distinction is necessary because the best strategy will differ across household types.

The Best No Annuity Strategy

Finding the best no annuity strategy can be split into two subproblems. The first problem is the one after retirement, which is stationary and is solved using recursive techniques. The state variable is net worth, which is in units of the *current period consumption good*. The value of a unit of *k* is $(1+r_e)k$ to a household choosing the no annuity strategy. The second problem is to determine consumptions and savings over the working life.

The problem becomes stationary and recursive at retirement age *T*, with net worth *w* being the state variable. The value function $f(w)$ is the maximal obtainable expected current and future utility flows if a retiree is alive and has net worth *w*. The optimality equation is

(3.1)
$$f(w) = \max_{c,w'} \{\log c + \sigma\beta f(w') + \delta\beta\alpha \log w'\}$$
$$\text{s.t.} \quad c + \frac{w'}{(1+r_e)} \leq w .$$

The solution to this optimality equation has the form

(3.2)
$$f(w) = \overline{f}_1(\alpha) + \overline{f}_2(\alpha)\log w,$$

where

(3.3) $$\overline{f}_2(\alpha) = \frac{1+\alpha\ \beta\delta}{1-\sigma\beta}.$$

The optimal consumption/saving policy for retirees is

(3.4) $$\begin{aligned} &c = w / \overline{f}_2(\alpha) \\ &w' = (1+r_e)(w-c). \end{aligned}$$

The bequests, conditional on $j - 1$ being the person's last year of life, is

(3.5) $$b_j = w_j\,.$$

The problem facing an individual at birth who follows the no annuity strategy (which we call strategy B because it is the one chosen by those with a sufficiently strong preference for making a bequest) is

(3.6) $$\begin{aligned} &U^B(\alpha) = \max_{\{c_j\}_{j=0}^{T-1}, w_T} \{\sum_{j=0}^{T-1} \beta^j \log c_j + \beta^T [\overline{f_1}(\alpha) + \overline{f_2}(\alpha) \log w_T]\} \\ &\text{s.t.} \\ &\sum_{j=0}^{T-1} \frac{c_j}{(1+r_e)^j} + \frac{w_T}{(1+r_e)^T} \le v_0^B = \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\overline{b}^B}{(1+r_e)^{30}}. \end{aligned}$$

Here, v_0^B is the present value of wages and inheritance of an individual born at $t = 0$. The solution (see Appendix 2 for more details) is

(3.7) $$\begin{aligned} &c_j^B = \overline{c}(\alpha)\beta^j(1+r_e)^j v_0^B \qquad\qquad j < T \\ &w_T^B = (1 - \sum_{j=0}^{T-1} \overline{c}(\alpha)\beta^j)(1+r_e)^T\ v_0^B, \end{aligned}$$

where $$\overline{c}(\alpha) = \frac{(1-\beta)}{1-\beta^T + (1-\beta)\beta^T f_2(\alpha)}\ .$$

The preretirement age j net worth of an individual following this strategy satisfies

$$
\begin{aligned}
& w_0^B = 0 \\
(3.8) \quad & w_j^B = (1+r_e)(w_{j-1}^B - c_{j-1}^B + (1-\tau)e_0(1+\gamma)^{j-1}) \qquad \text{for } 1 \le j < T, j \ne 30 \\
& w_{30}^B = (1+r_e)(w_{29}^B - c_{29}^B + (1-\tau)e_0(1+\gamma)^{29}) + \overline{b}^B.
\end{aligned}
$$

The Best Annuity Strategy

The best annuity strategy for a type α is the solution to the following:

$$
U^A(\alpha) = \max_{\{b_j, c_j\}} \{ \sum_{j=0}^{T} \beta^j \log c_j + \sum_{j=T+1}^{\infty} \beta^j \sigma^{j-T} \log c_j + \sum_{T+1}^{\infty} \beta^t \sigma^{j-T-1} \delta \alpha \log b_j \}
$$

(3.9) s.t.

$$
\sum_{j=0}^{T} \frac{c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\sigma^{j-T} c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\sigma^{j-T-1} \delta b_j}{(1+r)^j} \le v_0^A,
$$

where r is the lending rate and

$$
(3.10) \qquad v_0^A = \sum_{t=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^t}{(1+r)^t} + \frac{\overline{b}^A}{(1+r)^{30}}.
$$

The constant v_0^A is the present value of future wage income and inheritances using the lending rate r of a person born at $t = 0$. The superscript A denotes the annuity strategy and not an individual type. In equilibrium, type-A will choose strategy A.

There are other constraints, specifically, that the worker choosing this strategy does not borrow. For the economies considered in this study, these constraints are not binding and can therefore be ignored. If, however, the economy were such that the no-borrowing constraint were binding for some j, then the solution below would not be the solution to the problem formulated above.

The nature of the annuity contract is that the payment to a retiree who is alive at age $j \ge T$ is c_j. If the individual dies at age j, payment b_j is made to that person's estate. The solution to this program is

(3.11) $$c_j^A = \bar{c}(\alpha)\beta^j(1+r)^j\, v_0^A \qquad j \geq 0$$

(3.12) $$b_j^A = \alpha\bar{c}(\alpha)(1+r)^j \beta^j v_0^A \qquad j \geq T+1.$$

The net worth of an individual choosing this strategy is the pension fund reserves associated with that individual's annuity contract. Pension fund reserves (from the point of view of the intermediary) for a given annuity contract for an individual born at $t=0$ at age *j* in equilibrium equals the expected present value at time $t=j$ of payments that will be made less the value (at time $t=j$ as well) of premiums that will be received.

For workers, they can be determined as the present value of past premiums. Thus, pension fund reserves for individuals' annuity holders born at $t=0$ at age *j* satisfy

(3.13) $$\begin{aligned} w_0^A &= 0 \\ w_j^A &= \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1}\right)(1+r) && \text{for } 1 \leq j < T,\, j \neq 30 \\ w_j^A &= \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1}\right)(1+r) + \bar{b}^A && \text{for } j = 30\ . \end{aligned}$$

For retirees, conditional on being alive, pension fund reserves for individuals born at $t=0$ at age *j* are equal to the expected present value of the future payments:

(3.14) $$w_j^A = \sum_{t=0}^{\infty}(1-\delta)^t \frac{c_{j+t}^A}{(1+r)^t} + \sum_{t=0}^{\infty}\delta(1-\delta)^{t-1}\frac{b_{j+t}^A}{(1+r)^t} \qquad j > T$$

The Best Strategy

In general, there will be a $\alpha^*(\phi)$ such that a household chooses strategy B if its α exceeds $\alpha^*(\phi)$ and the annuity strategy otherwise. Proposition 1 is used to establish this result under a restriction that is satisfied for the calibrated model economy.

Proposition 1: If $\frac{1+r+\phi}{1+r} > \beta\left[\frac{1-(1-\delta)\beta}{\beta\delta}\right]$, then $\frac{\partial U^B(\alpha)}{\partial\alpha} - \frac{\partial U^A(\alpha)}{\partial\alpha} > 0$.

Proof: See Appendix 1.□

The value of $\phi > 0$ affects the relative attractiveness of the two strategies. Proposition 2 establishes that an α household will choose the annuity strategy if ϕ is sufficiently small and the no annuity strategy if ϕ is sufficiently large.

Proposition 2: For sufficiently small ϕ, $U^B(0) < U^A(0)$. For sufficiently large α, $U^B(0) > U^A(0)$.

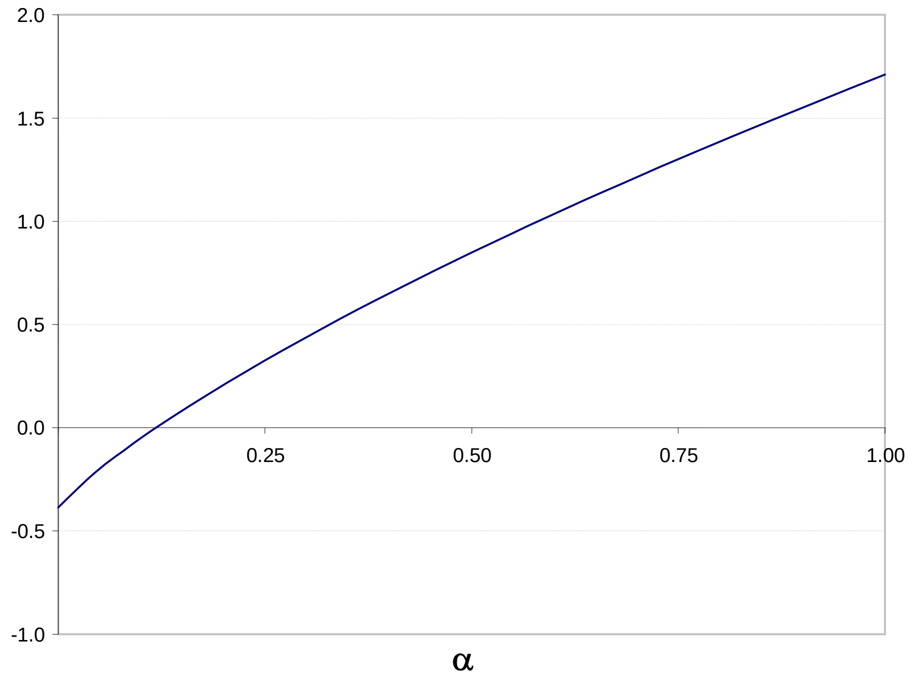
Proof outline: For small non-negative ϕ, the value of insurance associated with strategy A exceeds the value of the higher return associated with strategy B. This is why strategy A dominates for small ϕ. For large ϕ, the cost of the annuity is large and the higher return associated with the no annuity strategy dominates. This is why strategy B dominates for large ϕ.□

Figure 1 plots the difference in utilities for the two strategies, as a function of α, for the prices, tax rate, and bequest for our calibrated economy. We see that individuals with bequest preference parameter $\alpha < 0.12$ choose to annuitize.

Figure 1

Utility Difference between the Best No Annuity and Best Annuity Strategy:

$U^B(\alpha) - U^A(\alpha)$



4. Aggregate Behavior of the Household Sector

Aggregate Consumption

Aggregate consumption depends upon the labor tax rate τ and inheritance $\bar{b}$ as well as the prices $\{e, r, r_e\}$. Equilibrium prices do not depend upon the household side, and can be determined from the policy choice of r and profit-maximizing conditions. Having formulated the optimal consumption strategies for the two types of individuals, we characterize the aggregate consumption, asset holdings, and bequest at time $t = 0$ by individual type given $\bar{b}$ and τ and the equilibrium prices. Two aggregate equilibrium relations must be solved for the variables $\bar{b}$ and τ.

There are two types of households $i \in \{A, B\}$. The type-A has $\alpha_A = 0$ and will in equilibrium choose the annuity strategy A given the model economy. The type-B has α_B, which is sufficiently large that the equilibrium is such that they chose not annuitize. The measure of type-*i* of age *j* at $t = 0$ is

$$\mu_j^i = \begin{cases} \mu_0^i & j \le T \\ \sigma^{j-T} \mu_0^i & j > T. \end{cases} \tag{4.1}$$

The aggregate consumption of the type-*i* households at time 0 is C^i:

$$C^i(\bar{b}, \tau) = \mu^i \sum_{j=0}^{T-1} c_j^i (1+\gamma)^{-j} + \mu^i \sum_{j=T}^{\infty} \sigma^{j-T} c_j^i (1+\gamma)^{-j} . \tag{4.2}$$

Here, we have used the fact that each subsequent generation has a consumption-age profile that is higher by a factor of $(1+\gamma)^j$ in balanced growth.

Aggregate consumption is

$$C(\bar{b}, \tau) = C^A(\bar{b}, \tau) + C^B(\bar{b}, \tau) . \tag{4.3}$$

Aggregate Asset Holdings

The aggregate net worth at time 0 of a type-$i \in \{A, B\}$ is

$$W(\bar{b}, \tau) = \mu_0^i \sum_{j=0}^{T} w_j^i (1+\gamma)^{-j} + \mu_0^i \sum_{j=T+1}^{\infty} \sigma^{j-T} w_j^i (1+\gamma)^{-j} . \tag{4.4}$$

Net worth is prior to consumption and receipt of wage income and includes net interest income and dividend income. In the case of the intermediary, net worth includes intermediation cost liabilities. Net worth is prior to consumption and is denominated in units of the current period consumption good.

Aggregate Inheritance

At time 0 the measure of the people age $j > T$ who *die* and leave a bequest is $\mu_0^B \delta\sigma^{j-T-1}$; thus, the total bequests given by these households is

$$B_j = \mu_0^B \delta\,\sigma^{j-T-1} w_j^B \quad j > T\ .$$

Hence, the *aggregate bequests* at time 0 are

(4.5) $$B_0 = \sum_{j=T+1}^{\infty} B_{0j}(1+\gamma)^{-j}\ .$$

Aggregate Private Debt

The aggregate indebtedness of a type-B satisfies

(4.6) $$D^B(\overline{b},\tau) = K - W^B(\overline{b},\tau)/(1+r_e)\,,$$

because the price of existing capital in terms of the consumption good is $(1+r_e)$ and the household is obligated to make a payment of $(1+r_e)D^B(\overline{b},\tau)$.

5. Balance Sheets

Assets and liabilities are beginning of period numbers and are in units of the consumption good. We consider only economies for which there is intermediated borrowing and lending in equilibrium. Given there is a large amount of intermediated borrowing and lending, these economies are the ones of empirical interest.

Type-A Sector: The assets of the type-A consist of pension fund reserves. They have no liabilities. The value of these pension reserves (in terms of the consumption good) is: Pension fund reserves = $(1+r)D^B(\overline{b},\tau)+(1+r)D^G(\overline{b},\tau)$. Their balance sheet is as follows:

Balance Sheet of Type-A Households

Assets	**Liabilities**
Pension fund reserves	0 Net worth

Hence, their net worth satisfies

$$W^A(\bar{b},\tau) = (1+r)D^B(\bar{b},\tau) + (1+r)D^G(\bar{b},\tau).$$

Type-B Sector: Those following the no annuity strategy have aggregate debt $D^B(\bar{b},\tau)$ and hold all the economy's capital, K. Their balance sheet is as follows:

Balance Sheet of Type-B Households

Assets	**Liabilities**
$(1+r_e)K$	$(1+r_e)D^B(\bar{b},\tau)$ Net worth

Here, we have adjusted the assets and liabilities by a factor $(1+r_e)$ to get the net worth in units of the consumption good. Their net worth is

$$W^B(\bar{b},\tau) = (1+r_e)K - (1+r_e)D^B(\bar{b},\tau).$$

Financial Intermediary Sector: The assets of the financial intermediary are the liabilities of the government and the type-B households, whereas its liabilities are the pension assets of type-A households and the amount payable for intermediation services. The net worth of the financial intermediaries is zero.

Balance Sheet of the Intermediaries

Assets	**Liabilities**
Government debt = $(1+r)D^G(\bar{b},\tau)$	Pension promises = $(1+r)[D^B(\bar{b},\tau)+D^G(\bar{b},\tau)]$
Private debt = $(1+r_e)D^B(\bar{b},\tau)$	Amounts payable for intermediation services = $D^B(\bar{b},\tau)(r_e-r)$
	Net worth = 0

Government: The assets of the government are the present value of the tax receipts on labor income, whereas its liabilities are the debt it has outstanding.

Balance Sheet of the Government

Assets	**Liabilities**
$\dfrac{\tau(1-\theta)Y}{r-\gamma}$	$D^G(\bar{b},\tau)$
	Net worth = 0

Since labor is supplied inelastically and taxed at a rate τ, the government effectively owns a fraction τ of an individual's time endowment (now and in all future periods). In our model economy, the net worth of the government is zero and government debt is an asset for debt holders in our model.

6. Equilibrium Relations

We normalize Y to 1 and determine the value of a set of balanced growth variables at $t = 0$. All variables grow at rate γ except aggregate labor supply, which is constant and equal to 40, r_e, financial intermediation, and aggregate consumption. Given that Y has been normalized to 1 at time 0, the cost share relationships determine time 0 capital stock K and wage e:

(6.1) $$(r_e + \delta_k)K = \theta Y$$

(6.2) $$e\,L = (1-\theta)Y$$

From the intermediary's problem, the lending rate satisfies

(6.3) $$r_e = r + \phi\,.$$

Three Equilibrium Conditions

Prices $\{e, r, r_e\}$ are determined from policy and technology. Therefore, only $\bar{b}$ and τ are needed to completely specify the household budget constraints. Conditional on these variables, aggregate consumption, $C(\bar{b},\tau)$, and aggregate intermediation, $I(\bar{b},\tau)$, will be determined by aggregating individual household variables. Aggregation, given the individual decisions conditional on $\bar{b}$ and τ, is specified in Appendix 2.

One aggregate equilibrium condition is the aggregate resource constraint,

(6.4) $$C(\bar{b},\tau) + X + \phi I(\bar{b},\tau) = K^{\alpha} L^{1-\alpha}\,,$$

where $X = (\delta_k + \gamma)K$ is investment. Intermediation services satisfy

(6.5) $$I(\bar{b},\tau) = K - \frac{W^B(\bar{b},\tau)}{(1+r_e)}\,.$$

We assume that type-B households hold all the capital and the intermediaries none. This assumption resolves an unimportant indeterminacy. Increasing the amount of capital held by a type-B and that type-B's indebtedness by the same amount does not affect that type-B's net worth, which is what is relevant. This portfolio shift by a type-B household is offset by portfolio shifts by other type-B households. The aggregate indebtedness of a type-B is denoted by $D^B(\bar{b},\tau)$ and is equal to $I(\bar{b},\tau)$.

The second equilibrium condition is that the inheritance of households at a point in time equals aggregate bequests at that point in time. We consider $t=0$ and let $B(\bar{b},\tau)$ be the aggregate bequest at that time. The second equilibrium condition is

(6.6) $$\bar{b} = B(\bar{b},\tau)(1+\gamma)^{30}.$$

There is a third equilibrium condition, namely, the government's budget constraint. This constraint $(1+r)D_t^G = \tau E_t + D_{t+1}^G$ equates payments to receipts. Given $D_{t+1}^G = (1+\gamma)D_t^G$, $E_0 = (1-\theta)Y_0$, and the normalization $Y_0 = 1.0$, the time 0 government budget constraint is

(6.7) $$(r-\gamma)D^G(\bar{b},\tau) = \tau(1-\theta).$$

Equilibrium

The first two equilibrium conditions are linear in $(\bar{b},\tau)$, so solving for a candidate solution is straightforward. This solution is the equilibrium only if, in addition, (i) the best strategy for type-B households is the no annuity strategy, (ii) the best strategy for type-A households is the annuity strategy, (iii) type-B borrows and does not lend, and (iv) type-A lends and does not borrow. The reason for the last constraint is that these equilibrium conditions hold provided that the no-borrowing constraint on annuity holders is not binding, and it will not be binding if (iv) holds.

7. Calibration

The parameters that need to be calibrated are those related to the households $\{\alpha^A, \alpha^B, \beta, \mu^A, \mu^B, T, \delta\}$; the intermediation technology parameter $\{\phi\}$; the production good technology parameters $\{\theta, \delta_k, \gamma\}$; and the policy parameter r. The other two policy parameters $\{\tau, D^G\}$ are endogenous. As mentioned before, the choice r as a parameter and τ as an endogenous variable is only for convenience; reversing their roles will not affect the results described in Section 8.

Many of these parameters are well documented in the literature; others are not. We proceed by listing the parameters with the selected values and a brief motivation.

Parameters Associated with Individuals

$\beta = 0.99$ (Annuity holders' c grows at almost 2 percent over their lifetimes)

$\delta = 0.05$ (Implies a postretirement life expectancy of 20 years)

$\alpha^A = 0$ (Assumption: Type-A individuals have low bequest intensity)

$\alpha^B = 1$ (Assumption: Type-B individuals have high bequest intensity)

$T = 40$ (Workers retire at chronological age 63)

$\mu^B = 0.162$ (Specified so that the amount intermediated matches U.S. data)

$\mu^A = 1 - \mu^B = 0.838$

Intermediation parameters

$\phi = .02$ (Consistent with the average difference in borrowing and lending rates)

Policy parameters

$r = 0.03$ (Assumption about government fiscal policy)

The motivation for this policy is that this has been the approximate return on lending by households (see McGrattan and Prescott, 2003).

Goods production parameters

$\theta = 0.3$ (Capital income share)

$\gamma = 0.02$ (Average growth rate of U.S. per capita output)

$\delta_k = 0.0382$ (Consistent with capital output ratio = 3.4, given $r_e = .05$.

In calibrating ϕ we proceed as follows. Our model economy has household, government, and financial intermediary sectors. All nonfinancial business borrowing is consolidated with the household sector. We start with the net interest income of the financial intermediation sector. Fees are a small part of this sector's product and most of them are for transaction services, which is not intermediation in the sense used in this study. Using data from NIPA[17] for year 2007, the interest received amounted to 0.165 times GNP,[18] and interest paid amounted to 0.110 times GNP. To estimate the services associated with intermediating borrowing and lending, we first subtracted intermediation services furnished without payment to households, since we did not want to include implicit purchases of transaction services by the household. We also subtracted part of bad debt, viewing it as interest not received by the intermediary, to obtain an estimate of the cost of intermediating borrowing and lending between households of 3.4 percent of GNP in 2007 (see Table 2).

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[17] Source: NIPA (U.S. Department of Commerce, 2007) Tables 7.11 and 2.4.5.
[18] Source: NIPA Table 1.7.5.

Using data from the Flow of Funds, we found that the debt outstanding of in our household sector, which includes nonfinancial businesses, equals 1.72 times GNP.[19] The implied intermediation spread is thus 2.0 percent, and in turn the calibrated $\phi = 0.02$. This number results in the after-tax returns being close to their historical averages (see McGrattan and Prescott 2003, 2005).

Table 2

Financial Intermediary Sector Accounts Relative to GNP Year 2007

Interest received	0.165	Table 7.11 NIPA line 28
Less interest paid	0.110	Table 7.11 NIPA line 4
Equals net interest income	0.055	
Less services furnished without payment	0.016	Table 2.4.5 NIPA line 89
Less bad debt expenses	0.005	Table 7.16 NIPA line 12*
Equals services for intermediating household borrowing and lending	0.034	
Amount intermediated between households	1.721	Table D.3 Flow of Funds (total amount in column 1 less state, local, and federal government)

*This datum is for 2005, the latest for which this datum is currently available. We assumed that half the total bad debt was in that of financial intermediaries.

Transaction costs incurred by households associated with buying and selling assets are not part of intermediation costs. The assets in our model are capital *K*, government debt, Type-B household debt, and pension fund reserves. With regard to *K* transactions—say, the brokerage fees associated with transferring ownership of an owner-occupied house—NIPA treats these costs as an investment and justifies this as

[19] Source: Flow of Funds (Board of Governors, 2007) Table D.3. See Table 2 in this paper for further details.

putting the house to more productive use. With government debt, transfer of ownership costs is zero in our model and virtually zero in fact. Pension fund reserves are not traded between households, and therefore there are almost no costs associated with transferring ownership. The total costs of buying and selling household debt between financial intermediaries are small and are part of intermediation costs. Households incur brokerage fees associated with transferring ownership of financial securities between households. These fees are not payment for intermediating debt between households and therefore are not part of the cost of intermediated borrowing and lending between households. Brokerage fees paid by intermediaries are part of the costs of intermediating borrowing and lending between households.

8. Results

We considered four values for α^B, a parameter for which we have little information. For each value of α^B, we search for the μ^B for which the intermediated borrowing and lending between households is 1.72 times GNP. The results are summarized in Table 3, which shows results that are not sensitive to the size of the bequest preference parameter α^B. Given that the aggregate results are insensitive to α^B, subsequently we deal only with the case $\alpha^B = 1$.[20]

[20] As in Cagetti and De Nardi (2006), there is little consequence of inheritance for the net worth distribution.

Table 3

Summary of Aggregate Results

Economy	$\alpha^B = 1/3$	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
μ^A	0.833	0.838	0.851	0.867
μ^B	0.167	0.162	0.149	0.133
National Accounts				
C_A	0.636	0.639	0.651	0.663
C_B	0.132	0.128	0.117	0.104
X	0.198	0.198	0.198	0.198
I	0.034	0.034	0.034	0.034
Y	1.000	1.000	1.000	1.000
Depreciation	0.13	0.13	0.13	0.13
Compensation	0.70	0.70	0.70	0.70
Profits	0.17	0.17	0.17	0.17
Net Worth				
Type-A	6.29	6.33	6.42	6.53
Type-B	1.66	1.66	1.66	1.66
Government Debt/Y	4.55	4.59	4.68	4.79
Bequest/Y	0.0341	0.0347	0.0365	0.0390
Tax rate	0.0650	0.0655	0.0668	0.0684

Aggregate U.S Households Statistics

Based on the aggregate balance sheet of U.S. households presented in the introduction (see Table 1), a snapshot of the U.S economy for the year 2007 is as follows:

K	=	3.4014 GDP	Capital owned by households
D^{EG}	=	0.5 GDP	Explicit government debt
D^{IG}	=	4.1 GDP	Implicit government debt
D^{G}	=	4.6 GDP	Total government debt
D^{H}	=	1.7 GDP	Intermediated lending by households
D^{H}	=	1.7 GDP	Borrowing by households

Household net worth $= K + D_G = 8.0$ GDP

We have calibrated μ^B to match the borrowing and lending by households. The strength of bequest parameter for type-B has little consequence, provided it is not so small that this type-B chooses the annuity strategy.

One test of our model is how well it replicates some other statistics for the U.S. economy, such as government debt, bequests, and inheritances. Next, we examine each statistic in turn.

Government Debt

Government debt in our model, which is 4.6 times GNP, may at first sight appear large relative to U.S. federal, state, and local government, which was only 0.5 GNP in 2007. However, as argued earlier, there are huge implicit liabilities of the U.S. government. The present value of the implicit Social Security Retirement and Medicare promises are over four times GNP by most estimates. The empirical counterpart of model government debt is explicit government debt *plus* the present value of implicit government promises, which most estimate to be about 4 times GNP. In light of this fact, the stock of government debt in our model is reasonable.

An additional point is that if no one had a bequest motive, the steady-state capital stock would be the same, namely, 3.4 times GNP, and government debt in our model would be slightly larger. Policy and not the nature of bequest preferences is what determines the capital-output ratio.

Bequests

Total annual bequests in our model, as seen in Table 3, are 0.035 times GNP for $\alpha^B = 1$. The aggregate value of estates in 2007 that exceeded $675,000 was 0.00123 times GNP.[21] Some of these estates are inter-spousal and should not be included. This is more than offset by bequests that were under the limit for which estate tax returns had to be filed. Adding these and inter vivos transfers and adjusting for underreporting of gifts associated with the transfer of family businesses to the younger generation would result in aggregate bequests being close to model aggregate bequests.

Modigliani's (1988) estimate of bequest flows is close to the flow in our model. He reports bequests of 0.02 times GNP. He adds life insurance, death benefits, and newly established trusts to conclude that bequests are at least 0.027 times GNP.

Another measure of the size of bequests is the amount an individual inherits expressed in units of the individual's annual wage at the time of inheritance. Each individual receives at chronological age 52 an amount equal to 1.98 times his or her annual wage at that time. Menchick and David (1983) estimate the average inheritance received by all males to be $20,000 (in 1967 dollars). We estimate the average gross annual wage for that year as $8,840, arriving at a ratio of inheritance received to annual

[21] U.S. Department of the Treasury (2007), Historic Table 17, p. 203.

wage equal to 2.26.[22] However, if we correct for inter-spousal transfers, the inheritance received could well reduce the average inheritance to $13,220, which results in a ratio of inheritance received to annual wage equal to 1.5. These considerations suggest that inheritances are consistent with the predictions of our model.[23]

Inheritances

Another variable of interest is the fraction of wealth that is inherited. A significant component of wealth is human capital, which is the present value of wages in our model world where labor is supplied inelastically. The other part is the present value of inheritance. As shown in Table 4, human capital is about 95.5 percent of wealth at entry into the workforce and would be higher if there were population growth. These results are for type-A households, which discount using a 3 percent rate. The share is a little lower for type-B households, which use a 5 percent discount rate. Anything that reduces the ratio of bequests to GNP reduces this number, so for the model with a 1 percent population growth rate, as in the United States, this ratio is nearly 97 percent.

Table 4

Inheritance as Fraction of Wealth at Entry into Workforce

	$\alpha^B = 1/3$	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
Type-A	0.044	0.045	0.047	0.050
Type-B	0.035	0.036	0.038	0.040

[22] Nominal GDP in 1967 was $833 billion. Assuming that 70 percent of GDP is labor income (consistent with our model economy), we obtain an estimate of total wage income of $583 billion in 1967. Then, since the total employment in that year was 65.9 million, the average gross annual wage income is $8,840.

[23] We examined the consequences of population growth and found that they were small. Bequests fall to 0.03 times GNP as the population growth increases to the point at which the growth rate of the economy equals the interest rate.

The issues as to the importance of bequests for the size of the capital stock are mute in our model, since policy determines the capital stock and not the nature of preferences for bequests. However, a statistic of interest is the one estimated by Kotlikoff and Summers (1981). This statistic is the present value of inheritances people alive have received, using a 3 percent interest rate. Their estimate of this number is 0.80 times the total household net worth. Modigliani's (1988) estimate of this number is much smaller: 0.20. Modigliani (1988, Table 1, p. 19) presents a number of other estimates, all of which range between 0.10 and 0.20. This ratio number for our model economy is 0.18, which is in line with these estimates.

In our model economy, 93 percent of bequests are accidental. We came up with this number as follows. Setting $\alpha = 0$ for type-B households and requiring type-B households to follow the no annuity strategy results in this number. Treating these accidental bequests as savings for retirement along with all type-A savings implies that 99 percent of savings is for retirement purposes and 1 percent is for bequests.

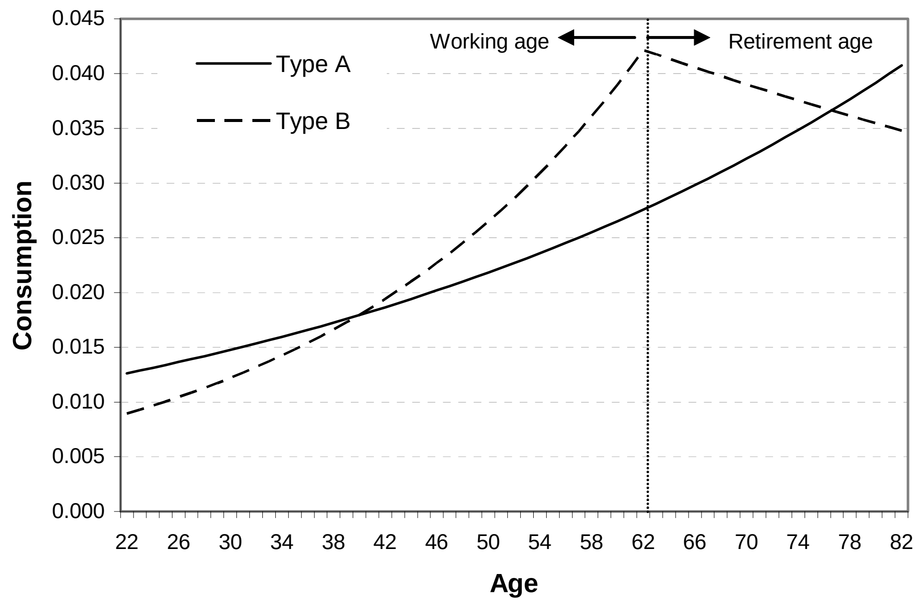
Some Micro Findings

Our abstraction has implications for micro observations as well. Unlike the macro findings, the model's micro findings are not a quantitative theory of the consequence of the bequest motive for the distributions of consumption, net worth, and equity holdings and consequently must be interpreted with care. They do, however, show that the bequest motive, or for that matter any factor that leads people to partially finance their capital acquisitions with debt, is quantitatively important for these statistics. With this caveat, the micro distributional relations for our model economy are as follows.

Figure 2 plots the lifetime consumption patterns of the two types of households. Type-A's consumption grows at a constant annual rate of 1.97 percent throughout their lifetime. Type-B's starts out lower and grows more rapidly during their working life, with this growth rate being 3.95 percent. Upon retirement, the consumption growth rate turns negative, falling to −0.95 percent. At retirement, a type-B retiree's consumption is higher than that of an equal age type-A retiree.

Figure 2

Lifetime Consumption Pattern

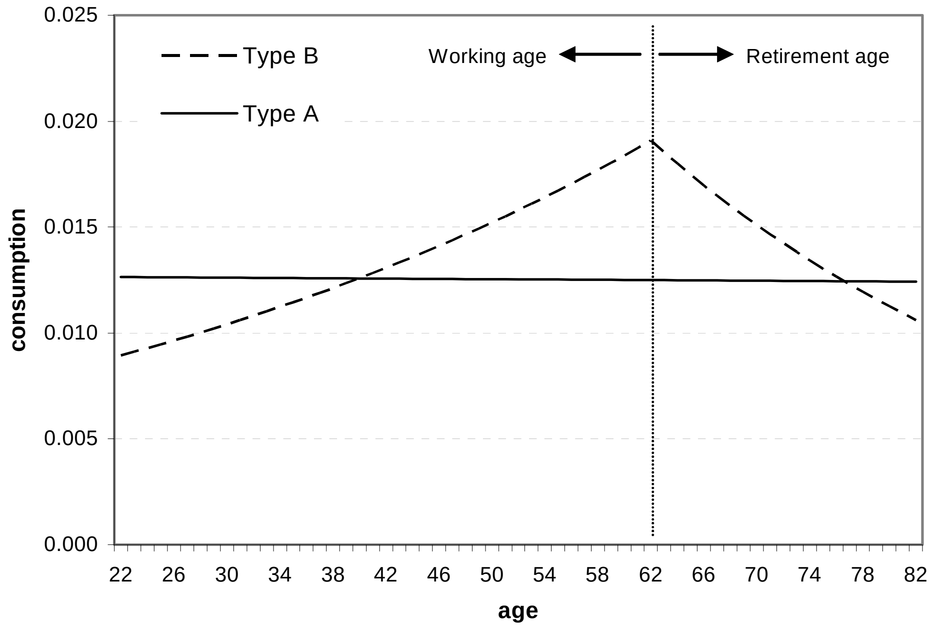


Cross-Sectional Consumption

Figure 3 plots cross-sectional consumption by age for the two types. All type-A individuals that are alive have virtually the same consumption. Young type-B workers have lower consumption, and older workers have higher consumption. For the type-B retirees, consumption level declines with age.

Figure 3

Cross-Sectional Consumption by Age

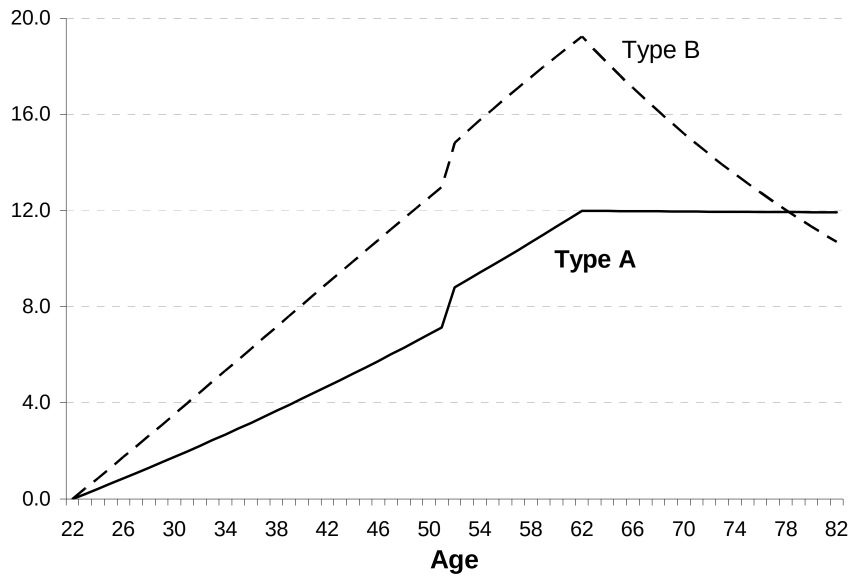


Net Worth by Age

In Figure 4 we plot net worth relative to current annual wage income, which has a stationary distribution. At retirement, the net worth of a type-A household is 12 times the annual wage, and that of a type-B is 19 times the annual wage. The disparity in net worth (corrected for age) is modest, being a maximum of about 1.6 at retirement age. After retirement, disparity falls until age 78 and then starts growing, with the type-A household becoming the one with the greater net worth. The jump in net worth at chronological age 52 is due to inheritance.

Figure 4

Net Worth as a Function of Age in Units of Annual Wage Income



Lorenz Curves

Figure 5 plots the Lorenz curves for consumption, net worth, and capital or equity holdings. In the case of capital, we assume all type-B households have the same ratio of debt liabilities to capital in their portfolios in order to resolve the portfolio indeterminacy at the individual level. We truncate the distribution at age 112, so the curves are not exact, but they are very good approximations given the small fraction of population over this age.

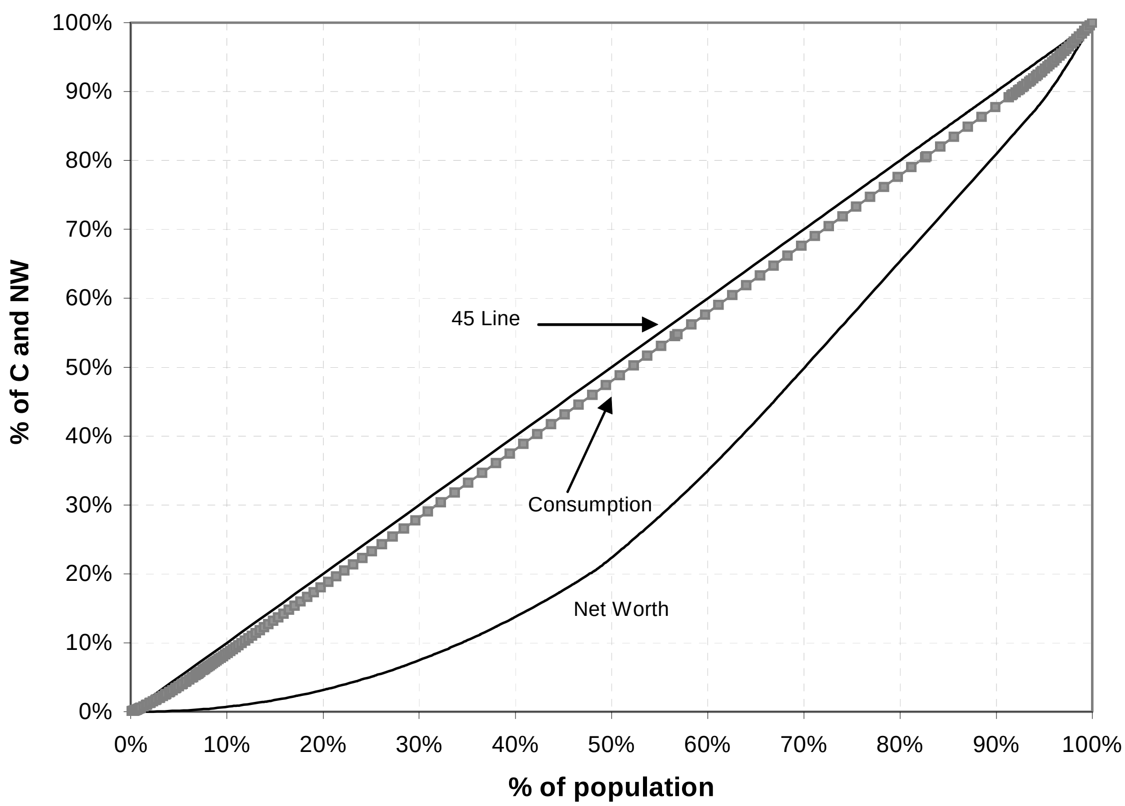
Our principal findings are that there is almost no disparity in consumption levels and sizable disparities in net worth levels. This shows that the dispersion in net worth is a bad proxy for dispersion in consumption.[24]

[24] The Gini coefficients for the consumption and net worth Lorenz curves are 0.038 and 0.35, respectively.

In our model economy, all individuals have the same human capital endowments. If the model were modified to have people earn proportionally different wages, to a first approximation, an individual's allocation is proportional to that individual's wage.[25] Thus, introducing wage disparity would add disparity in consumption and net worth. Introducing entrepreneurs (Cagetti and De Nardi 2006) and idiosyncratic risk (Castãneda, Díaz-Giménez, and Ríos-Rull 2003; Chatterjee et al. 2007) would increase disparity as well.

Figure 5

Lorenz Curve for Consumption, Net Worth, and Capital



[25] If bequests were distributed proportional to the human capital factor, the scaling result would hold exactly.

Cost of Financial Market Constraints

What are the gains to a household of having access to the equity market at no intermediation cost? Table 5 reports the cost of not having this access—which was the case for most Americans prior to the development of low-cost indexed mutual funds—as being about 4.0 percent of wealth at time of entry into the workforce. This wealth is the present value of labor income and inheritance.

Table 5

Cost to a Type-A of Not Having Access to the Annuity Market in Units of Wealth at Entry into Workforce

α^B	Change in v_0^A
1/3	0.77%
1	0.79%
3	0.84%
6	0.90%

Table 6

Cost to a Type-B of Not Being Permitted to Hold Equity Directly in Units of Wealth at Entry into Workforce

α^B	Change in v_0^B
1/3	1.24%
1	4.00%
3	9.74%
6	15.77%

Tables 5 and 6 show the percentage increase in either v_0^k, that is, wealth at time of entry into the workforce, which is necessary to compensate an $i \in \{A, B\}$ in wealth equivalents if forced to switch to a system other than their preferred choice. Since both consumption and bequest are linear functions of initial wealth, the percentage changes in both consumptions and bequests are the same as the percentage change in initial wealth.

What are the costs to a type-A if for some reason, such as adverse selection problems or legal constraints, they do not have access to annuity markets and must use the equity option for saving? The cost is small, being approximately 0.8 percent of lifetime consumption.

Implications for the Equity Premium

In our framework, there is no equity premium because there is no aggregate uncertainty. The return on equity and the borrowing rate are both equal to 5 percent. This is a no arbitrage condition. The return on government debt is 3 percent. If we use the conventional definition of the equity premium—the return on a broad equity index less the return on government debt—we would erroneously conclude that in our model the equity premium was 2 percent. The difference in the government borrowing rate and the return on equity is *not* an equity premium; it arises because of costly financial intermediation. Analogously, if in the U.S. economy borrowing and lending rates for equity investors differ (and they do), the equity premium should be measured relative to the investor borrowing rate rather than the government's borrowing rate (the investor lending rate). Measuring the premium relative to the government's borrowing overstates the premium for bearing aggregate risk by the difference between the investor's borrowing and lending rates.[26] If such a correction were made to the results reported in Mehra and Prescott (1985), the equity premium would be 4 percent rather than the reported 6 percent.

[26] For a detailed exposition of this and related issues, the reader is referred to Mehra and Prescott (2008).

9. Concluding Comments

In this paper, we develop a heterogeneous household economy where households differ along only one dimension: their preferences for bequest. In equilibrium, households with a low desire to bequeath lend and hold annuities, whereas those with a high desire to bequeath borrow and own capital. This distinction is important because the total amount of borrowing by households and the government must equal the amount lent by households. Our simple framework mimics reality with respect to *both* the amount of intermediated borrowing and lending between households *and* the average spread in borrowing and lending rates resulting from intermediation costs.

We view this as a first step in what we think will prove to be a productive research program. Possible extensions include building in differential survival rates and addressing the issues of adverse selection and moral hazard when pricing annuities. This extension might justify our requirement that people choose between the annuity and the no annuity strategies early in their careers. This research program, if successful, will require extension of the theory of household lifetime consumption behavior because the bequest motive is not the only salient factor that differentiates people. Differences in preferences with respect to consumption today versus consumption in the future and differences in preferences that give rise to differences in lifetime labor supply are likely to be important as well.

Another possible extension is to model non-steady-state behavior as in Geanakoplos, Magill, and Quinzii (2004), who consider the importance of demographic waves for stock market valuation, or as in Braun, Ikeda, and Joines (2007) for saving behavior within the overlapping-generations framework.

Appendix 1: Proof of Proposition 1

The prices (r, r_e, e_0) , tax rate τ , and inheritance implied by $\overline{b_0}$ are given to an individual. Note $0 < r < r_e$. Let $U_A(\alpha)$ and $U_B(\alpha)$ represent the maximum attainable utility of an agent of measure zero in this economy who follows strategy A (annuity) or B (bequest), respectively, as a function of $\alpha \in \mathbb{R}_+$. Define $\Delta(\alpha) = U_B(\alpha) - U_A(\alpha)$.

Proposition 1: If $\frac{1+r_e}{1+r} > \beta\left[\frac{1-\sigma\beta}{\beta\delta}\right]^{1-\beta\sigma}$ then

$$\frac{\partial \Delta U(\alpha)}{\partial \alpha} > 0 \quad \forall \alpha \in \mathbb{R}_+ .$$

Proof: The maximum utility as a function of α attainable by an agent who follows an annuity strategy (A), taking as given the parameters of the economy, can be expressed as

$$U_A(\alpha) = \sum_{j=0}^{T-1} \beta^j \log(c_j^A) + \beta^T (\phi_A(\alpha) + \theta_A(\alpha) log(w_T^A)),$$

where

$$\theta_A(\alpha) = \frac{1+\beta\alpha\delta}{1-\beta(1-\delta)}$$

$$\phi_A(\alpha) = \frac{(\theta_A(\alpha)-1)\log[(1+r)\beta] - \log[\theta_A(\alpha)] + \beta\alpha\delta[\log(\alpha) - \log[\theta_A(\alpha)]]}{1-\beta(1-\delta)}$$

$$c_j^A = \overline{c}(\alpha)\beta^j (1+r)^j v_0^A \qquad j < T$$

$$w_T^A = \theta_A(\alpha)\overline{c}(\alpha)\beta^T (1+r)^T v_0^A$$

($\overline{c}(\alpha)$ and v_0^A are defined in Section 3).

Similarly, the maximum utility as a type α who follows an annuity strategy (B) is

$$U_B(\alpha) = \sum_{j=0}^{T-1} \beta^j \log(c_j^B) + \beta^T (\phi_B(\alpha) + \theta_B(\alpha) log(w_T^B)),$$

where

$$\theta_B(\alpha) = \frac{1+\beta\alpha\delta}{1-\beta(1-\delta)}$$

$$\phi_B(\alpha) = \frac{(\theta_B(\alpha)-1)\log(1+r_e) + (\theta_B(\alpha)-1)\log[(\theta_B(\alpha)-1)] - \theta_B(\alpha)\log[(\theta_B(\alpha)]}{1-\beta(1-\delta)}$$

$$c_j^B = \bar{c}(\alpha)\beta^j(1+r_e)^j v_0^B \qquad j<T$$
$$w_T^B = \theta_B(\alpha)\bar{c}(\alpha)\beta^T(1+r_e)^T v_0^B$$

($\bar{c}(\alpha)$ and v_0^B are defined in Section 3).

Using the properties of the logarithm function and defining $\theta(\alpha)=\theta_A(\alpha)=\theta_B(\alpha)$,

(A1.1) $$\Delta(\alpha) = \sum_{j=0}^{T-1}\beta^j \log\left(\frac{(1+r_e)^j v_0^B}{(1+r)^j v_0^A}\right) + \beta^T(\phi_B(\alpha)-\phi_A(\alpha)+\theta(\alpha)log(\frac{w_T^B}{w_T^A}))$$

Since the first term is independent of α, it follows that

(A1.2) $$\frac{\partial\Delta(\alpha)}{\partial\alpha} = \beta^T\frac{\partial[\phi_B(\alpha)-\phi_A(\alpha)]}{\partial\alpha} + \beta^T\theta'(\alpha)log\left(\frac{w_T^B}{w_T^A}\right),$$

where $\theta'(\alpha) = \dfrac{\beta\delta}{1-\beta\sigma} > 0$, which does not depend on α.
Further,

$$\frac{w_T^B}{w_T^A} = \frac{v_0^B(1+r_e)^T}{v_0^A(1+r)^T} = \frac{\sum_{j=0}^{T-1}\frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^{j-T}} + \frac{\bar{b}}{(1+r_e)^{30-T}}}{\sum_{j=0}^{T-1}\frac{(1-\tau)e_0(1+\gamma)^j}{(1+r)^{j-T}} + \frac{\bar{b}}{(1+r)^{30-T}}} > 1 \text{ since } r_e > r,\ j<T \text{ and } 30<T.$$

This implies that the second term in (A1.2) is positive, i.e., $\beta^T\theta'(\alpha)log\left(\frac{w_T^B}{w_T^A}\right) > 0$.

To prove our assertion that $\dfrac{\partial\Delta U(\alpha)}{\partial\alpha} > 0$ is positive, we proceed in three steps:

a. We show that $\lim_{\alpha\to 0}\dfrac{\partial\Delta(\alpha)}{\partial\alpha} > 0$;
b. We show that $\dfrac{\partial^2\Delta(\alpha)}{\partial\alpha^2} < 0$; and that

c. $\lim_{\alpha\to\infty}\frac{\partial\Delta(\alpha)}{\partial\alpha}>0$.

Some straightforward algebra yields

(A1.3) $$\frac{\partial[\phi_B(\alpha)-\phi_A(\alpha)]}{\partial\alpha}=\frac{\theta'(\alpha)}{1-\beta\sigma}\left(\log\left(\frac{1+r_e}{(1+r)\beta}\right)+\log\left(\frac{\theta(\alpha)-1}{\alpha}\right)-\beta\sigma\log\left(\frac{\theta(\alpha)}{\alpha}\right)\right)$$

From (A1.3) it is readily seen that $\lim_{\alpha\to 0}\frac{\partial[\phi_B(\alpha)-\phi_A(\alpha)]}{\partial\alpha}\to+\infty$. This follows since the last term tends to $+\infty$ and all the other terms are bounded. This coupled with the fact that $\beta^T\theta'(\alpha)log\left(\frac{w_T^B}{w_T^A}\right)>0$ proves that $\lim_{\alpha\to 0}\frac{\partial\Delta(\alpha)}{\partial\alpha}>0$.

The second derivative $\frac{\partial^2\Delta(\alpha)}{\partial\alpha^2}$ is negative by direct differentiation,
$$\frac{\partial^2\Delta(\alpha)}{\partial\alpha^2}=\frac{-\beta^{T+1}\delta(1-\delta)}{\alpha(1-\beta(1-\delta))((1+(\alpha-1)\delta)(1+\alpha\beta\delta)}<0,$$

since the denominator is always positive and the numerator is negative.

Finally, it can be shown that $\lim_{\alpha\to\infty}\frac{\partial\Delta(\alpha)}{\partial\alpha}>0$ under the condition stated in the theorem. Notice that (taking the limit of A1.3) when $\alpha\to\infty$, equation (A1.2) is positive if and only if

$$\frac{1}{1-\beta\sigma}\log\left(\frac{(1+r_e)}{(1+r)\beta}\right)+\log\left(\theta'(\alpha)\right)+\log\left(\frac{v_0^B(1+r_e)^T}{v_0^A(1+r)^T}\right)>0.$$

The last term in the previous expression has already been shown to be positive. Thus, a sufficient condition for this inequality is

$$\frac{1}{1-\beta\sigma}\log\left(\frac{(1+r_e)}{(1+r)\beta}\right)+\log\left(\theta'(\alpha)\right)>0 \ .$$

This inequality can be written as $\frac{1+r_e}{1+r}>\beta\left[\frac{1-\sigma\beta}{\beta\delta}\right]^{1-\beta\sigma}$.

Since a), b), and c) are satisfied, it follows that $\frac{\partial \Delta U(\alpha)}{\partial \alpha} > 0 \quad \forall \alpha \in \mathbb{R}_{+}$. **QED**

Appendix 2: Aggregation

General Formulas

There are two types $i \in \{A, B\}$. The A-type has $\alpha^{A} = 0$ and in equilibrium they choose the annuity strategy given the model economy. The measure of type *i* of age *j* at $t = 0$ is

(A2.1)
$$\mu_j^i = \begin{cases} \mu_0^i & j \le T \\ (1-\delta)^{j-T} \mu_0^i & j > T \end{cases}$$

The aggregate quantity for variable *Z* of type $i \in \{A, B\}$ agents at $t = 0$ is Z_0^k,

(A2.2)
$$Z_0^i = \mu_0^i \sum_{j=0}^{T-1} z_j^i (1+\gamma)^{-j} + \mu_0^i \sum_{j=T}^{\infty} (1-\delta)^{j-T} z_j^i (1+\gamma)^{-j} ,$$

where z_j^i is the individual allocation of type-*i* at age *j* born at $t = 0$. Notice that we have used the fact that each subsequent generation has a consumption-age profile that is higher by a factor of $(1+\gamma)^j$ under balanced growth. Aggregate quantity of *Z* at time 0, Z_0 is

$$Z_0 = Z_0^A + Z_0^B .$$

Agent Type-B

Aggregate assets of agent type-B and aggregate bequest

The aggregate assets for B-type agents are computed using the law of motion of net worth. From the individual problem,

$$w_0^B = 0$$
$$w_j^B = \left(w_{j-1}^B - c_{j-1}^B + (1-\tau)e_0(1+\gamma)^{j-1} \right)(1+r_e) \qquad \text{for } j \le T \,\&\, j \ne 30$$
$$w_j^B = \left(w_{j-1}^B - c_{j-1}^B + (1-\tau)e_0(1+\gamma)^{j-1} \right)(1+r_e) + \bar{b} \quad \text{for } j = 30$$
$$w_j^B = \left(w_{j-1}^B - c_{j-1}^B \right)(1+r_e) \qquad \text{for } j > T$$

From equations (3.4) and (3.7), the consumption for type-B is given by

(A2.3) $$c_j^B = \begin{cases} [\beta(1+r_e)]^j \overline{c}^B(\alpha^B)v_0^B & j < T \\ \dfrac{w_j^B}{f_2(\alpha^B)} & j \geq T \end{cases},$$

where

(A2.4) $$\overline{c}^B(\alpha) = \frac{(1-\beta)}{1-\beta^T + (1-\beta)\beta^T f_2(\alpha)}$$

$$v_0^B = \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\overline{b}}{(1+r_e)^{30}}$$

and $$\overline{f}_2(\alpha) = \frac{1+\alpha\ \beta\delta}{1-\sigma\beta}.$$

Using (A2.2) aggregate net worth is

$$W^B(\overline{b},\tau) = \mu_0^B \sum_{j=0}^{T-1} w_j^B (1+\gamma)^{-j} + \mu_0^B \sum_{j=T}^{\infty} \sigma^{j-T} w_j^B (1+\gamma)^{-j}.$$

The summation over $j=0,\ldots,T-1$ is performed numerically, whereas the total net worth of the retirees is

(A2.5) $$\mu_0^B \sum_{j=T}^{\infty} \sigma^{j-T} w_j^B (1+\gamma)^{-j} = \frac{w_T^B \mu_0^B (1+\gamma) f_2(\alpha^B)}{(1+\gamma)^T [(1+\gamma) f_2(\alpha^B) - \sigma(1+r_e)(f_2(\alpha^B)-1)]},$$

where, from the individual problem,

$$w_T^B = f_2(\alpha^B)[\beta(1+r_e)]^T \overline{c}^B(\alpha^B) v_0^B.$$

Since $\alpha^A = 0$, all bequests are coming from the type-B, and as shown in Section 3.1 is given by

$$b_j^B = w_j^B \qquad j \geq T+1$$

if a type-B dies prior to the end of the previous period subsequent to consuming, and zero otherwise.

Since the measure of agents dying at each age $j \geq T+1$ is $\mu_0^B \delta \sigma^{j-T-1} = \delta \mu_{j-1}^B$, the aggregate bequest is

$$B_0(\bar{b},\tau) = \sum_{j=T+1}^{\infty} \delta \frac{\mu_{j-1}^B b_j^B}{(1+\gamma)^j} = \sum_{j=T+1}^{\infty} \delta \frac{\mu_{j-1}^B w_j^B}{(1+\gamma)^j} = \frac{\delta}{\sigma} \sum_{j=T+1}^{\infty} \frac{\mu_j^B w_j^B}{(1+\gamma)^j}.$$

Using (A2.5), it is straightforward to find that

$$B_0(\bar{b},\tau) = \frac{\delta w_T^B \mu_0^B}{\sigma(1+\gamma)^T} \left[\frac{(1+\gamma) f_2(\alpha^B)}{[(1+\gamma) f_2(\alpha^B) - \sigma(1+r_e)(f_2(\alpha^B)-1)]} - 1 \right]$$

or

(A2.6) $$B_0(\bar{b},\tau) = \frac{\delta w_T^B \mu_0^B}{(1+\gamma)^T} \left[\frac{(f_2(\alpha^B)-1)(1+r_e)}{[(1+\gamma) f_2(\alpha^B) - \sigma(1+r_e)(f_2(\alpha^B)-1)]} \right].$$

Aggregate consumption type-B

Similarly, using (A2.2) and (A2.3), the aggregate consumption of type-B agents at time 0 can be expressed as

(A2.7) $$C_0^B = \Phi_1^B v_0^B,$$

where

$$\Phi_1^B = \bar{c}(\alpha^B) \left[\sum_{j=0}^{T-1} \left(\frac{\beta(1+r_e)}{1+\gamma} \right)^j + \beta^T \sum_{j=T}^{\infty} \left(\frac{(1+r_e)}{1+\gamma} \right)^j \left(\frac{f_2(\alpha^B)-1}{f_2(\alpha^B)} \right)^{j-T} \sigma^{j-T} \right] \mu_0^B$$

or

$$\Phi_1^B = (1+\gamma) \bar{c}^B(\alpha) \left[\frac{1 - \left[\frac{\beta(1+r_e)}{1+\gamma} \right]^T}{(1+\gamma) - \beta(1+r_e)} + \frac{(1+\alpha^B \beta \delta) \left[\frac{\beta(1+r_e)}{1+\gamma} \right]^T}{(1+\gamma) - \beta\sigma^2 + \alpha^B \beta\delta(\gamma+\delta) - \beta\sigma(1-\delta(1-\alpha^B)) r_e} \right] \mu_0^B$$

Agent Type-A

Aggregate assets of agent type-A

The aggregate bequest is measured in units of agent type-B assets, and therefore the inheritance received by agent type-A measured in her assets' units is $\bar{b}^A = \bar{b}(1+r)/(1+r_e)$. The aggregate assets for agents type-A are computed using the law of motion of net worth. From the individual problem,

$$\begin{aligned} &w_0^A = 0 \\ &w_j^A = \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1} \right)(1+r) && \text{for } j \le T \ \& \ j \ne 30 \\ \text{(A2.8)} \qquad &w_j^A = \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1} \right)(1+r) + \bar{b}^A && \text{for } j = 30 \\ &w_j^A = \sum_{t=0}^{\infty}(1-\delta)^t \frac{c_{j+t}^A}{(1+r)^t} + \sum_{t=0}^{\infty}\delta(1-\delta)^{t-1}\frac{b_{j+t}^A}{(1+r)^t} && j > T \end{aligned}$$

Using (A2.2) aggregate net worth is calculated as

$$W^A(\bar{b},\tau) = \mu_0^A \sum_{j=0}^{T-1} w_j^A (1+\gamma)^{-j} + \mu_0^A \sum_{j=T}^{\infty} \sigma^{j-T} w_j^A (1+\gamma)^{-j} .$$

As for type-B, the summation for *j=0,…,T* is performed numerically. Since in the calibration, $\alpha^A = 0$. From equation (3.11), consumption for type-A agents, born at period zero when they reach age *j* (at time *j*), is

$$c_j^A = \bar{c}(\alpha^A)(1+r)^j \beta^j \, v_0^A \qquad j \ge 0 .$$

Then, agents alive at time 0 of age *j* consume

$$\text{(A2.9)} \qquad c_{0,j}^A = \bar{c}(\alpha^A) v_0^A \left[\frac{\beta(1+r)}{(1+\gamma)} \right]^j \qquad j \ge 0 .$$

Using (A2.8) and (A2.9), the net worth for retired agents can be written as

$$w_j^A = \frac{c_{j,0}^A}{1\text{-}\beta\sigma} \qquad j > T \quad .$$

Then,

$$\mu_0^A \sum_{j=T+1}^{\infty} \sigma^{j-T} \frac{w_j^A}{(1+\gamma)^{-j}} = \frac{\mu_0^A \bar{c}(\alpha^A) v_0^A}{1\text{-}\beta\sigma} \left[\frac{(1+r)\beta}{1+\gamma}\right]^{T+1} \left[\frac{1+\gamma}{1+\gamma-\beta(1+r)\sigma}\right].$$

Aggregate consumption type-A

Again, using (A2.2) and (A2.9), the aggregate consumption of type-A agents at time 0 can be expressed as

(A2.10) $$C_0^A = \Phi_1^A v_0^A ,$$

where

$$\Phi_1^A = \bar{c}(\alpha^A) \left[\sum_{j=0}^{T-1} \left(\frac{\beta(1+r)}{1+\gamma} \right)^j + \sum_{j=T}^{\infty} \left(\frac{\beta(1+r)}{1+\gamma} \right)^j \sigma^{j-T} \right] \mu_0^A$$

or

$$\Phi_1^A = (1+\gamma)\bar{c}(\alpha^A) \left[\frac{1-\left[\frac{\beta(1+r)}{1+\gamma}\right]^T}{(1+\gamma)-\beta(1+r)} + \frac{\left[\frac{\beta(1+r)}{1+\gamma}\right]^T}{(1+\gamma)-\beta(1+r)\sigma} \right] \mu_0^A ,$$

where

$$v_0^A = \sum_{t=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r)^j} + \frac{\bar{b}^A}{(1+r)^{30}}.$$

Balance Sheets

Type B: $(1+r_e)K = (1+r_e)D^B(\bar{b},\tau) + W^B(\bar{b},\tau)$

Type A: $(1+r)A^A(\bar{b},\tau) = W^A(\bar{b},\tau)$

Intermediary: $(1+r_e-\phi)D^B(\bar{b},\tau) + (1+r)G^G(\bar{b},\tau) = (1+r)A^A(\bar{b},\tau)$

Notice that the net worth of both the intermediary and the government is 0.

Equilibrium Conditions

Three equilibrium conditions can potentially be used to solve the model:

1) Feasibility: $$\text{Y}= C_0(\bar{b},\tau) + \text{X} + \phi\left[K - \frac{W^B(\bar{b},\tau)}{1+r_e}\right],$$

where $C_0(\bar{b},\tau) = C_0^A(\bar{b},\tau) + C_0^B(\bar{b},\tau)$

2) Bequest=inheritance: $\bar{b} = B_0(\bar{b},\tau)(1+\gamma)^{30}$

3) Assets Markets $\frac{W^B(\bar{b},\tau)}{1+r_e} + \frac{W^A(\bar{b},\tau)}{1+r} = G^G(\bar{b},\tau) + \mathrm{K}$

Since this is a linear system in $(\bar{b},\tau)$, one equation is redundant and the solution is straightforward. We chose to use the first two equilibrium conditions, and then we check that the third one is satisfied as well.

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