Quantitative Economics 2 (2011), 1–36 1759-7331/20110001

Costly financial intermediation in neoclassical growth theory

Rajnish Mehra
Department of Economics and Finance, Arizona State University, and NBER

Facundo Piguillem
Department of Economics, Einaudi Institute for Economics and Finance

Edward C. Prescott
Department of Economics, Arizona State University, and Federal Reserve Bank of Minneapolis

The neoclassical growth model is extended to include costly intermediated borrowing and lending between households. This is an important extension as substantial resources are used to intermediate the large amount of borrowing and lending between households. In 2007, in the United States, the amount intermediated was 1.7 times gross national product (GNP), and the resources used in this intermediation amounted to at least 3.4 percent of GNP. The theory implies that financial intermediation services are an intermediate good, and that the spread between borrowing and lending rates measures the efficiency of the financial sector.

Keywords. Aggregate intermediation, borrowing, lending, equity premium, government debt, life cycle savings, retirement.

JEL classification. D31, E2, E21, E44, G1, G11, G12, G23, H0, H62.

Rajnish Mehra: rajnish.mehra@asu.edu
Facundo Piguillem: facundo.piguillem@gmail.com
Edward C. Prescott: edward.prescott@asu.edu
This paper has circulated under the title "Intermediated Quantities and Returns." We thank the editor, the two referees, Andy Abel, Costas Azariadis, Sudipto Bhattacharya, Bruce Lehmann, John Cochrane, George Constantinides, Cristina De Nardi, Douglas Diamond, John Donaldson, John Heaton, Jack Favilukis, Francisco Gomes, Fumio Hayashi, Daniel Lawver, Anil Kashyap, Juhani Linnainmaa, Robert Lucas, Ellen McGrattan, Krishna Ramaswamy, Jesper Rangvid, Kent Smetters, Michael Woodford, Dimitri Vayanos, Amir Yaron, Stephen Zeldes, the seminar participants at the Arizona State University, Bank of Korea, University of Calgary, UCLA, UCSD, Charles University, University of Chicago, Columbia University, Duke University, Federal Reserve Bank of Chicago, ITAM, London Business School, London School of Economics, University of Mannheim, University of Minnesota, University of New South Wales, Peking University, Reykjavik University, Rice University, University of Tokyo, University of Virginia, Wharton, Yale University, Yonsei University, the Economic Theory conference in Kos, the conference on Money, Banking and Asset Markets at the University of Wisconsin, and ESSFM in Gerzensee for helpful comments. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

DOI: 10.3982/QE40

1. Introduction

There is a rich class of models that study savings for retirement, but these models abstract from the large costs of financial intermediation, despite the fact that most savings are intermediated. This paper extends the neoclassical growth model by incorporating an intermediation sector. It does so in such a way that it matches *both* the amount of borrowing and lending between households *and* the resources used in intermediation. Furthermore, all the appealing characteristics of the standard neoclassical growth model remain unaltered. In addition, the model provides a suitable framework to evaluate not only efficiency gains from innovations in the financial sector, but also the impact of demographic changes on intermediation and saving behavior.

Our paper presents a model that is consistent with the economic growth facts, documented by Kaldor (1961) and used by Solow (2000), *and* provides a prototype framework that allows us to address the amount of borrowing and lending between households *and* the resources used in intermediation. To the best of our knowledge, this is the first such extension. One interpretation of our model would be a theory of growth with financial intermediation. Given the large amount of resources used in intermediation, we consider this to be an important extension of the existing growth models.

In 2007, for the U.S. economy, intermediation was large—around 1.7 times the annual gross national product (GNP).[1] The resources used in this process were not inconsequential, amounting to at least 3.4 percent of GNP. These two figures together imply that the average household borrowing rate is at least 2 percent higher than the average household lending rate. Relative to the level of the observed average rates of return on debt and equity securities, this spread is far from being insignificant.

Since our model abstracts from aggregate risk, by construction there is no premium for bearing aggregate risk. As explained later, the household borrowing rate is equal to the return on equity. The government can borrow at a lower rate than households—as empirically observed. Consequently, there is a difference in the return on equity and the interest rate on government debt. For our calibrated economy this difference is 2 percent, and abstracting from it may be inappropriate when computing statistics that report the spread between different rates of return in the economy. We discuss this in Section 8.

Since in equilibrium the total amount borrowed by households is equal to the total amount of intermediated lending by households, a natural question that arises is who are the borrowers and lenders? In our model, where the only reason for households to save is to finance retirement over an uncertain lifetime, one set of households chooses to save by accumulating capital and a second set by purchasing annuities. Since capital accumulation is partially financed by owners' equity and the remainder by borrowing, *capital owners are the borrowers*. In addition, since purchasing annuities is isomorphic to lending, *annuity holders are the lenders*.

We caution the reader regarding two issues. First, the model counterpart of annuities is not limited to commercial annuities, but includes, more importantly, defined benefit

[1] About half of this is intermediated lending by commercial banks. The other half is lending by other financial intermediaries such as mutual organizations.

pension plans and, even more importantly, annuity-like promises of the government, such as Social Security and Medicare. We think of these plans as mandatory purchases of annuities. As pointed out by Abel (1986), Social Security and Medicare are implicit government liabilities and can be regarded as annuity-like promises of the government. When we examine some implications of our theory, we will include these annuity-like promises as part of annuity-like assets held by households.

Payments for these "annuities" are made throughout the working life of households, and our model tries to capture this. Empirically, commercially available annuities, purchased at or near retirement, account for a very small fraction of savings for retirement due to well known adverse selection issues. Consequently, our paper abstracts from these annuities. The biggest annuities are in the form of Social Security retirement benefits and Medicare, which are mandatory purchases of annuities during a household's working life. In addition, there are defined benefit retirement plans, which are essentially annuities that people effectively purchase during their working life.

An integral part of our analysis is that households *endogenously* borrow and lend. Some households lend to financial intermediaries, while others borrow from these intermediaries to partially finance capital investment in the businesses they own. While there are a myriad of reasons why households borrow and lend, in our model, for simplicity, we motivate this by only one such reason (the intensity for bequests). This keeps the analysis simple and tractable. The reasons matter little for the inference we draw.

Later in Section 8, when we examine some implications of our theory, we will include these annuity-like promises as part of annuity assets held by model households.[2]

We follow the tradition in macroeconomics by assuming that households own all the capital in the economy and rent it to businesses. Thus, we treat the capital owned by businesses as capital owned by the owners of these businesses, and, therefore, all debt of nonfinancial businesses is debt of the household sector.

The output of the intermediary sector is an intermediary good. The value added by intermediation services is equal to the amount of borrowing times its price minus the amount of lending times its price. In equilibrium, the amount borrowed is equal to the amount lent. Hence, the price of this service is equal to the spread between the average borrowing and lending rates. Improvements in the financial system which reduce this spread are efficiency gains.

In 2007, about half of the U.S. capital stock, the value of which was 3.4 times GNP, was financed by borrowing and half by owners' equity. This borrowing is done to finance owner-occupied housing, by proprietorships and partnerships to finance unincorporated businesses, and by shared ownership corporations to finance businesses. Households that own capital finance it partially by borrowing and partially by equity. Further, the Modigliani–Miller theorem holds for our economy because, for a given firm, the debt–equity financing decision does not matter. In the aggregate, total equity and private debt are determined.

[2]We reemphasize that when we use the annuity construct in this paper, it includes all annuity-like payments, including Social Security, Medicare, defined benefit pension plans, and the small amount of commercial annuities.

Reason for household borrowing

We begin our study by examining household saving decisions. In practice, most household savings are for retirement. However, some of it is held in highly liquid financial instruments as a substitute for costly insurance against idiosyncratic risk such as a job loss.[3] Abstracting from these factors has little consequence for aggregate lending. In our model, households choose between two savings strategies. One strategy is to invest in equity and earn a real return of r_e percent. The other strategy is to purchase a lifetime annuity, which is actuarially fair at $r < r_e$ percent. Since the lifetime remaining after retirement is uncertain, households that choose the annuity option are in effect buying insurance against outliving their savings.

But why do some households choose to save by lending to financial intermediaries (with a low return) while others invest in equities (with a high return)? In this study, this is due to household heterogeneity in the form of differences in the strength of preferences for bequests. That is, we assume that people are identical in all aspects other than the intensity of their bequest motive. The only source of uncertainty is the duration of the lifetime after retirement. Hence, an important difference between the two strategies is that the buying equities strategy generates bequests upon death equal to net worth at the time of death, while buying annuities does not.[4] For our calibrated economy, people with a low bequest motive will prefer the annuity strategy, while agents with even a modest bequest motive will prefer equities.[5] The strength of the bequest motive has little consequence for aggregate bequests, as they are largely accidental.

To summarize, in equilibrium, those who have even a modest preference for bequests accumulate capital assets and borrow during their working lives, and upon retirement, use capital income for consumption and interest payment on debt. Upon their deaths they bequeath all their net worth. Households with little or no bequest motive buy annuities during their working years and use annuity benefits to finance their consumption over their retirement years.

As mentioned earlier, we abstract from the small amount of direct borrowing and lending between households, and assume that all borrowing and lending between households is intermediated through financial institutions. Furthermore, in light of the finding that the premium for bearing nondiversifiable aggregate risk is small in models consistent with growth and business cycle facts, our analysis abstracts from *aggregate risk*.[6]

[3]In this study we do not make a distinction between these two types of saving. For issues other than those we address in this paper, this may be a crucial element of reality that would have to be incorporated into the abstraction.

[4]We permit an annuity payment upon death. It will be positive if the bequest preference parameter is not zero for anyone choosing the annuity strategy.

[5]As explained later, there is an additional requirement about the size of the spread.

[6]Using a model with no capital accumulation, Mehra and Prescott (1985) found a small equity premium. McGrattan and Prescott (2000) found that the equity premium is small in the growth model if it is restricted to be consistent with growth and business cycle facts. Lettau and Uhlig (2000) introduced habit formation into the standard growth model and found that the equity premium is small if the model parameters are restricted to be consistent with the business cycle facts. Many others using the growth model restricted to be consistent with the macroeconomic growth and business cycle facts have found the same thing.

The intermediation technology is constant returns to scale with intermediation costs being proportional to the amount intermediated. To calibrate the constant of proportionality, we use Flow of Funds Account statistics and data from National Income and Product Accounts. The calibrated value of this parameter equals the net interest income of financial intermediaries divided by the quantity of intermediated debt, and is approximately 2 percent.[7]

In the absence of aggregate uncertainty, the return on equity and the borrowing rate are identical, since the households who borrow are also marginal in equity markets. In our framework, government debt is intermediated at zero cost, and thus its return is equal to the household lending rate. An important feature is that the government can borrow at a lower rate than can households, which mirrors reality.

In our model, all households in a cohort have identical labor income at every point in their working life. As a consequence of this, there is little difference in cross-sectional consumption at a point in time. However, sizable differences in net worth develop within a cohort over their working years. One implication is that preferences for bequests cannot be ignored when studying net worth distributions.

The paper is organized as follows. The economy is specified in Section 2. In Section 3, we discuss the decision problem of the households. Section 4 deals with the aggregation of individual behavior, Section 5 deals with the relevant balance sheets, and Section 6 characterizes the balanced growth equilibrium. We calibrate the economy in Section 7. In Section 8, we present and discuss our results. Section 9 concludes the paper.

2. The economy

To build a model that captures the large amount of observed borrowing and lending, as well as the large amount of resources used in this process, we introduce three key features of reality: differences in bequest preferences, an uncertain length of retirement, and costly intermediation of borrowing and lending between households. This leads some households to buy *costly* annuities that make payments throughout their retirement years. Since buying an annuity is isomorphic to lending, households choosing the annuity option are the lenders in our model. Households with high bequest utility save by increasing their net worth, which is their holding of productive capital less their debt.

We model an overlapping generations economy and consider its balanced growth path equilibrium. All households born at a given date are identical in all respects except for their bequest preference parameter α. They all have identical preferences with respect to consumptions over their lifetime, so the only dimension over which they differ is α. Those who have a not small α (type-B) borrow and own capital; others with $\alpha = 0$ or weak preferences for bequest (type-A) lend by acquiring annuities.

What motivates bequests? While a casual consideration of bequests naturally assumes that they exist because of parents' altruistic concern for the economic well-being of their offspring, results in Menchik and David (1983), Hurd (1989), Wilhelm (1996), Laitner and Juster (1996), Altonji, Hayashi, and Kotlikoff (1997), Laitner and Ohlsson

[7] See Section 7 (calibration) for details.

(2001), Kopczuk and Lupton (2007), and Fuster, Imrohoroglu, and Imrohoroglu (2008) suggest otherwise: households with children do not, in general, exhibit behavior in greater accord with a bequest motive than do childless households. This, we think, leads us to conclude that the existing literature supports our assumption that some people have preferences for making bequests. These empirical results lead us to eschew the perspective of Barro (1974) and Becker and Barro (1988), who postulated that each generation receives utility from the consumption of the generations to follow, and simply to model bequests as being motivated by a well defined "joy of giving,"[8] as in Abel and Warshawsky (1988) and Constantinides, Donaldson, and Mehra (2007).

Households

Any systematic consideration of bequests mandates that the analysis be undertaken in the context of an overlapping generations model. Accordingly, we analyze an overlapping generations economy and determine its balanced growth behavior. Each period, a set of individuals of measure 1 enters the economy. Two types enter at each date: type A, who derive no utility from leaving a bequest, and type B, whose utility is an increasing function of the amount they bequeath.[9] The measure of type $i \in \{A, B\}$ is μ_i. The total measure of people born at each date is 1, so $\mu_A + \mu_B = 1$.

Individuals have finite expected lives. They enter the labor force at age 22, work for T years, and then retire.[10] Model age j is 0 when a person begins his or her working life. The first year of retirement is model age $j = T$.

All workers receive an identical wage income. Wage income grows at the economy's balanced growth rate γ. At retirement, individuals face idiosyncratic uncertainty about the length of their remaining lifetimes. Their retirement lifetimes are exponentially distributed. Once individuals retire, the probability of surviving to the next period is $\sigma = (1 - \delta)$, where δ is the probability of death. Expected life is $T + 1/\delta$. We emphasize that there is no aggregate uncertainty.[11]

Individuals of type α, born at time t, order their preferences over age-contingent consumption and bequests by[12]

$$\sum_{j=0}^{T} \beta^j \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \beta^j \sigma^{j-T} \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \alpha\delta\beta^j \sigma^{j-T-1} \log b_{t+j,j}. \tag{2.1}$$

[8] See also Hurd and Mandcada (1989), De Nardi, Imrohoroglu, and Sargent (1999), De Nardi (2004), and Hansen and Imrohoroglu (2008).

[9] The "no utility from a bequest" assumption is a simplifying one and is not necessary for the analysis. All that is needed is that the utility from a bequest be sufficiently small that the type A choose to acquire annuities.

[10] We implicitly assume that parents finance the consumption of their children under the age of 22; in other words, children's consumption is a part of their parents' consumption.

[11] The Blanchard (1985) model has individuals with exponential life. The Díaz-Giménez, Prescott, Fitzgerald, and Alvarez (1992) model has individuals with both an exponential working life and an exponential retirement life.

[12] Our model has no factor giving rise to life cycle consumption patterns over the working life as in Fernández-Villaverde and Krueger (2002).

Here $\beta < 1$ is the discount factor and α is the strength of bequest parameter. Variable $c_{t+j,j}$ is the period consumption of a j-year-old born at time t,[13] conditional on being alive at time $t+j$. An individual who is born at time t and dies at age $j-1$ consumes nothing at time $t+j$, bequeaths $b_{t,t+j}$ units of the period $t+j$ consumption good, and consumes nothing subsequently. Each generation supplies one unit of labor inelastically for $j = 0, 1, \ldots, T-1$. Thus, aggregate labor supply is $L = T$ given that the measure of each generation is 1.

We only need to analyze the decision problems of a type α individual born at time $t = 0$. The solution to the problem for a type α born at any other time t can be found using the fact that along a balanced growth path,

$$c_{t,j} = (1+\gamma)^t c_{0,j}. \tag{2.2}$$

Further, to simplify the notation, we use c_j to denote the consumption of a j-year-old at time j rather than $c_{j,j}$. An analogous change of notation applies to the other variables.

Production technology

The aggregate production function is

$$Y_t = F(K_t, z_t L_t) = K_t^{\theta}(z_t L_t)^{1-\theta}, \tag{2.3}$$

$$z_{t+1} = (1+\gamma) z_t, \tag{2.4}$$

where K_t is capital, L_t is labor, and z_t is the labor-augmenting technological change parameter, which grows at a rate γ. The parameter z_0 is chosen so that $Y_0 = 1$.

Output is produced competitively, so

$$\delta_k + r_e = F_K(K_t, z_t L_t), \tag{2.5}$$

$$e_t = z_t F_L(K_t, z_t L_t), \tag{2.6}$$

where δ_k is the depreciation rate, r_e is both the household borrowing rate and the return on equity, and e_t is the wage rate.

Income is received as either wage income E_t or gross capital income R_t. Thus,

$$Y_t = E_t + R_t, \tag{2.7}$$

where $E_t = L_t e_t = (1-\theta) Y_t$ and $R_t = (\delta_k + r_e) K_t = \theta Y_t$. Components of output are consumption C_t, investment X_t, and intermediation services I_t; thus,

$$Y_t = C_t + X_t + I_t. \tag{2.8}$$

Along a balanced growth path, investment $X_t = (\delta_k + \gamma) K_t$ and $K_{t+1} = (1+\gamma) K_t$.

[13]In this paper, the first subscript represents calendar time and the second subscript represents the age at that time.

Financial intermediation technology

The intermediation technology displays constant returns to scale, with the intermediation cost in units of the composite output good being proportional to the amount of borrowing and lending intermediated. The cost is ϕ times the amount of borrowing and lending between households.[14] The intermediary also intermediates between households lending to the government. There are no costs associated with this intermediation. The intermediary receives interest rate r_e on its lending to households and effectively pays interest rate r on its borrowing from households. Given the technology, equilibrium interest rates satisfy

$$r_e - r = \phi.$$

The lending contract between households and intermediaries is not the standard one, but rather an annuity contract. A household can enter into an annuity contract at age 0. An annuity contract specifies an age-contingent premium payment path during working life, a benefit path contingent on being alive subsequent to retirement, and a payment upon death. The amount being lent by an individual who has chosen the annuity contract is the value of pension fund reserves for that contract at that point in time. These reserves are equal to the expected present value of future payments less the expected present value of future premium payments, if any. The present value is calculated using r, the rate at which households can lend to intermediaries. Competitive intermediaries will offer any annuity contract with the property that the expected present value of benefits is equal to the present value of the premiums using r in the present value calculations.

The alternative to entering into an annuity contract to save for retirement is to accumulate capital and to borrow to partially finance that capital. Our model has three sectors: a household sector, a government sector, and a financial sector. The nonfinancial business sector is consolidated with the household sector.

Government policy

Government policy is characterized by a tax rate τ on labor income, an interest rate r on government debt, and the path of government debt $\{D_t^G = D_0^G(1+\gamma)^t\}$. The feasible government policy parameters are constrained to a one dimensional manifold. Theoretically it does not matter which of the three policy parameters is picked. We chose r because it simplifies finding the equilibrium and there is a wealth of observations as to a reasonable value for its choice. The government finances interest payments on its debt by issuing new debt and by taxing labor income. The government's period t budget constraint is

$$(1+r)D_t^G = \tau E_t + D_{t+1}^G. \tag{2.9}$$

[14] Miller and Upton (1974) pioneered having a financial sector in their dynamic general equilibrium model. They had no intermediation costs.

Since $D^G_{t+1} = (1+\gamma)D^G_t$ in balanced growth,

$$(r-\gamma)D^G_t = \tau(1-\theta)Y_t. \tag{2.10}$$

The government pursues a tax rate policy that pegs[15] r, which equals the interest rate on government debt. This being a balanced growth analysis, government debt grows at rate $\gamma > 0$, which means that government deficits are positive and grow at rate γ as well.

The intermediary holds all the government debt, and there are no intermediation costs associated with holding this asset.

Bequests

Aggregate bequests at date t are

$$B_t = B_0(1+\gamma)^t. \tag{2.11}$$

We let $\bar{b} = B_{30}$. The inheritance of a type B born at $t = 0$ is

$$\bar{b}^B = \bar{b} \tag{2.12}$$

and is received at date $t = 30$. The inheritance of a type A born at $t = 0$ is

$$\bar{b}^A = \bar{b}(1+r)/(1+r_e). \tag{2.13}$$

The reason that a type A's inheritance is slightly smaller than that of a type B is that their inheritances are intermediated and intermediation is costly.

3. Optimal individual decisions

We consider the optimal individual decision problem, taking as given (i) the size of the inheritance the individual will receive at model age 30 (chronological age 52), (ii) wages at each date of the individual's working life, (iii) the labor income tax rate τ, and (iv) the borrowing and lending rates r_e and r. The first problem facing an individual is whether to choose the annuity strategy A or the no annuity strategy B. The parameters of the calibrated economy are such that a type A will choose the annuity strategy, while a type B will choose the no annuity strategy. The second problem is to determine the optimal lifetime consumption and savings decisions conditional on the strategy chosen. We determine, given α, the optimal consumption/saving behavior for each strategy and the resulting lifetime utility, and then determine which of the two strategies is best for that individual type.

A convention followed is that a bar over a variable denotes a constant. In the case where the constant depends on a person's type, that is, on α, this functional dependence is indicated. This is necessary because the best strategy will differ across household types.

[15] In this paper, we fix this at 3 percent. This is discussed further in Section 7 on calibration.

The best no annuity strategy

This problem can be split into two subproblems. The first problem is the one after retirement, which is stationary and is solved using recursive techniques. The state variable is net worth, which is in units of the *current period consumption good*. The value of a unit of k is $(1+r_e)k$ to a household choosing the no annuity strategy. The second problem is to determine consumptions and savings over the working life.

The problem becomes stationary and recursive at retirement age T, with net worth w being the state variable. The value function $f(w)$ is the maximal obtainable expected current and future utility flows if a retiree is alive and has net worth w. The optimality equation is

$$\begin{aligned} f(w) &= \max_{c,w'}\{\log c + \sigma\beta f(w') + \delta\beta\alpha \log w'\} \\ \text{s.t.} \quad & c + \frac{w'}{(1+r_e)} \leq w. \end{aligned} \tag{3.1}$$

The solution to this optimality equation has the form

$$f(w) = \bar{f}_1(\alpha) + \bar{f}_2(\alpha)\log w, \tag{3.2}$$

where

$$\bar{f}_2(\alpha) = \frac{1+\alpha\beta\delta}{1-\sigma\beta}. \tag{3.3}$$

The optimal consumption/saving policy for retirees is

$$\begin{aligned} c &= w/\bar{f}_2(\alpha), \\ w' &= (1+r_e)(w-c). \end{aligned} \tag{3.4}$$

The bequests, conditional on $j-1$ being the person's last year of life, is

$$b_j = w_j. \tag{3.5}$$

The problem facing an individual at birth who follows the no annuity strategy (which we call strategy B because it is the one chosen by those with a sufficiently strong preference for making a bequest) is

$$\begin{aligned} U^B(\alpha) &= \max_{\{c_j\}_{j=0}^{T-1}, w_T} \left\{\sum_{j=0}^{T-1} \beta^j \log c_j + \beta^T[\bar{f}_1(\alpha) + \bar{f}_2(\alpha)\log w_T]\right\} \\ \text{s.t.} \quad & \sum_{j=0}^{T-1} \frac{c_j}{(1+r_e)^j} + \frac{w_T}{(1+r_e)^T} \\ & \leq v_0^B = \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\bar{b}^B}{(1+r_e)^{30}}. \end{aligned} \tag{3.6}$$

Here v_0^B is the present value of wages and inheritance of an individual born at $t=0$. The solution (see Appendix B for more details) is

$$
\begin{aligned}
c_j^B &= \bar{c}(\alpha)\beta^j(1+r_e)^j v_0^B, \quad j<T, \\
w_T^B &= \left(1-\sum_{j=0}^{T-1}\bar{c}(\alpha)\beta^j\right)(1+r_e)^T v_0^B,
\end{aligned}
\tag{3.7}
$$

where

$$
\bar{c}(\alpha) = \frac{(1-\beta)}{1-\beta^T+(1-\beta)\beta^T f_2(\alpha)}.
$$

The preretirement age j net worth of an individual following this strategy satisfies

$$
\begin{aligned}
w_0^B &= 0, \\
w_j^B &= (1+r_e)(w_{j-1}^B - c_{j-1}^B + (1-\tau)e_0(1+\gamma)^{j-1}), \quad \text{for } 1\le j<T, j\neq 30, \\
w_{30}^B &= (1+r_e)(w_{29}^B - c_{29}^B + (1-\tau)e_0(1+\gamma)^{29}) + \bar{b}^B.
\end{aligned}
\tag{3.8}
$$

The best annuity strategy

The best annuity strategy for a type α is the solution to

$$
\begin{aligned}
U^A(\alpha) &= \max_{\{b_j,c_j\}}\left\{\sum_{j=0}^{T}\beta^j\log c_j + \sum_{j=T+1}^{\infty}\beta^j\sigma^{j-T}\log c_j + \sum_{T+1}^{\infty}\beta^t\sigma^{j-T-1}\delta\alpha\log b_j\right\} \\
\text{s.t.}\quad & \sum_{j=0}^{T}\frac{c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty}\frac{\sigma^{j-T}c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty}\frac{\sigma^{j-T-1}\delta b_j}{(1+r)^j} \le v_0^A,
\end{aligned}
\tag{3.9}
$$

where r is the lending rate and

$$
v_0^A = \sum_{t=0}^{T-1}\frac{(1-\tau)e_0(1+\gamma)^t}{(1+r)^t} + \frac{\bar{b}^A}{(1+r)^{30}}.
\tag{3.10}
$$

The constant v_0^A is the present value of future wage income and inheritances using the lending rate r of a person born at $t=0$. The superscript A denotes the annuity strategy and not an individual type. In equilibrium, type A will choose strategy A.

There are other constraints, specifically, that the worker choosing this strategy does not borrow. For the economies considered in this study, these constraints are not binding and can therefore be ignored. If, however, the economy were such that the no-borrowing constraint were binding for some j, then the solution below would not be the solution to the problem formulated above.

The nature of the annuity contract is that the payment to a retiree who is alive at age $j\ge T$ is c_j. If the individual dies at age j, payment b_j is made to that person's estate. The solution to this program is

$$c_j^A = \bar{c}(\alpha)\beta^j(1+r)^j v_0^A, \quad j \geq 0, \tag{3.11}$$

$$b_j^A = \alpha\bar{c}(\alpha)(1+r)^j\beta^j v_0^A, \quad j \geq T+1. \tag{3.12}$$

The net worth of an individual choosing this strategy is the pension fund reserves associated with that individual's annuity contract. Pension fund reserves (from the point of view of the intermediary) for a given annuity contract for an individual born at $t=0$ at age j in equilibrium equals the expected present value at time $t=j$ of payments that will be made less the value (at time $t=j$ as well) of premiums that will be received.

For workers, they can be determined as the present value of past premiums. Thus, pension fund reserves for individual annuity holders born at $t=0$ at age j satisfy

$$\begin{aligned} w_0^A &= 0, \\ w_j^A &= (w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1})(1+r), \quad \text{for } 1 \leq j < T, j \neq 30, \\ w_j^A &= (w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1})(1+r) + \bar{b}^A, \quad \text{for } j = 30. \end{aligned} \tag{3.13}$$

For retirees, conditional on being alive, pension fund reserves for individuals born at $t=0$ at age j are equal to the expected present value of the future payments:

$$w_j^A = \sum_{t=0}^{\infty}(1-\delta)^t \frac{c_{j+t}^A}{(1+r)^t} + \sum_{t=0}^{\infty}\delta(1-\delta)^{t-1}\frac{b_{j+t}^A}{(1+r)^t}, \quad j > T. \tag{3.14}$$

The best strategy

In general there will be an $\alpha^*(\phi)$ such that a household chooses strategy B if its α exceeds $\alpha^*(\phi)$ and chooses the annuity strategy otherwise. Proposition 1 is used to establish this result under a restriction that is satisfied for the calibrated model economy.

Proposition 1. *If*

$$\frac{1+r+\phi}{1+r} > \beta\left[\frac{1-\beta\delta}{\beta\delta}\right],$$

then

$$\frac{\partial U^B(\alpha)}{\partial\alpha} - \frac{\partial U^A(\alpha)}{\partial\alpha} > 0.$$

The proof is given in Appendix A.

The value of $\phi > 0$ affects the relative attractiveness of the two strategies. Proposition 2 establishes that an α household will choose the annuity strategy if ϕ is sufficiently small and the no annuity strategy if ϕ is sufficiently large.

Proposition 2. *For sufficiently small* ϕ, $U^B(0) < U^A(0)$. *For sufficiently large* α, $U^B(0) > U^A(0)$.

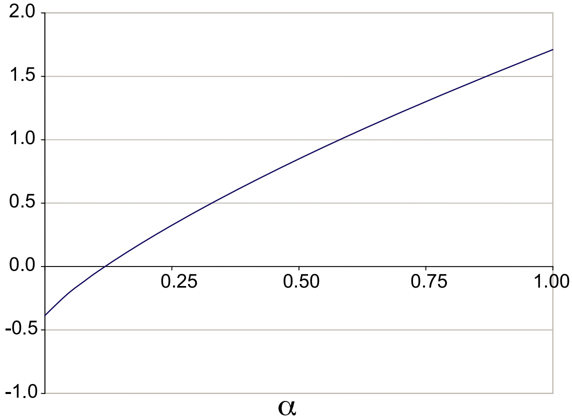

FIGURE 1. Utility difference between the best no annuity and the best annuity strategy: $U^B(\alpha) - U^A(\alpha)$.

PROOF OUTLINE. For small nonnegative ϕ, the value of insurance associated with strategy A exceeds the value of the higher return associated with strategy B. This is why strategy A dominates for small ϕ. For large ϕ, the cost of the annuity is large and the higher return associated with the no annuity strategy dominates. This is why strategy B dominates for large ϕ. □

Figure 1 plots the difference in utilities for the two strategies, as a function of α, for the prices, tax rate, and bequest for our calibrated economy. We see that individuals who have bequest preference parameter $\alpha < 0.12$ choose to annuitize. This is consistent with the results in Yaari (1965).

4. AGGREGATE BEHAVIOR OF THE HOUSEHOLD SECTOR

Aggregate consumption

Aggregate consumption depends on the labor tax rate τ and inheritance $\bar{b}$ as well as the prices $\{e, r, r_e\}$. Equilibrium prices do not depend on the household side, and can be determined from the policy choice of r and profit-maximizing conditions. Having formulated the optimal consumption strategies for the two types of individuals, we characterize the aggregate consumption, asset holdings, and bequest at time $t = 0$ by individual type given $\bar{b}$, τ, and the equilibrium prices. Two aggregate equilibrium relations must be solved for the variables $\bar{b}$ and τ.

There are two types of households $i \in \{A, B\}$. The type A has $\alpha_A = 0$ and will in equilibrium choose the annuity strategy A given the model economy. The type B has α_B, which is sufficiently large that the equilibrium is such that they chose not to annuitize. The measure of type i of age j at $t = 0$ is

$$\mu_j^i = \begin{cases} \mu_0^i, & j \le T, \\ \sigma^{j-T}\mu_0^i, & j > T. \end{cases} \tag{4.1}$$

The aggregate consumption of the type-i households at time 0 is C^i:

$$C^i(\bar{b}, \tau) = \mu^i \sum_{j=0}^{T-1} c^i_j (1+\gamma)^{-j} + \mu^i \sum_{j=T}^{\infty} \sigma^{j-T} c^i_j (1+\gamma)^{-j}. \tag{4.2}$$

Here we have used the fact that each subsequent generation has a consumption–age profile that is higher by a factor of $(1+\gamma)^j$ in balanced growth.

Aggregate consumption is

$$C(\bar{b}, \tau) = C^A(\bar{b}, \tau) + C^B(\bar{b}, \tau). \tag{4.3}$$

Aggregate asset holdings

The aggregate net worth at time 0 of a type $i \in \{A, B\}$ is

$$W(\bar{b}, \tau) = \mu^i_0 \sum_{j=0}^{T} w^i_j (1+\gamma)^{-j} + \mu^i_0 \sum_{j=T+1}^{\infty} \sigma^{j-T} w^i_j (1+\gamma)^{-j}. \tag{4.4}$$

Net worth is prior to consumption and receipt of wage income, and includes net interest income and dividend income. In the case of the intermediary, net worth includes intermediation cost liabilities. Net worth is prior to consumption and is denominated in units of the current period consumption good.

Aggregate inheritance

At time 0, the measure of the people aged $j > T$ who *die* and leave a bequest is $\mu^B_0 \delta \sigma^{j-T-1}$; thus, the total bequests given by these households is

$$B_j = \mu^B_0 \delta \sigma^{j-T-1} w^B_j, \quad j > T.$$

Hence, the *aggregate bequests* at time 0 are

$$B_0 = \sum_{j=T+1}^{\infty} B_{0j} (1+\gamma)^{-j}. \tag{4.5}$$

Aggregate private debt

The aggregate indebtedness of a type B satisfies

$$D^B(\bar{b}, \tau) = K - W^B(\bar{b}, \tau)/(1+r_e), \tag{4.6}$$

because the price of existing capital in terms of the consumption good is $(1+r_e)$ and the household is obligated to make a payment of $(1+r_e)D^B(\bar{b}, \tau)$.

5. Balance sheets

Assets and liabilities are beginning of period numbers and are in units of the consumption good. We consider only economies for which there is intermediated borrowing and lending in equilibrium. Given there is a large amount of intermediated borrowing and lending, these economies are the ones of empirical interest.

Type-A sector

The assets of the type A consist of pension fund reserves. They have no liabilities. The value of these pension reserves (in terms of the consumption good) is pension fund reserves $= (1+r)D^B(\bar{b},\tau) + (1+r)D^G(\bar{b},\tau)$. The balance sheet of type-A households is

Assets	Liabilities
Pension fund reserves	0 Net worth

Hence, their net worth satisfies

$$W^A(\bar{b},\tau) = (1+r)D^B(\bar{b},\tau) + (1+r)D^G(\bar{b},\tau).$$

Type-B sector

Those following the no annuity strategy have aggregate debt $D^B(\bar{b},\tau)$ and hold all the economy's capital. The balance sheet of type-B households is

Assets	Liabilities
$(1+r_e)K$	$(1+r_e)D^B(\bar{b},\tau)$ Net worth

Here we have adjusted the assets and liabilities by a factor of $(1+r_e)$ to get the net worth in units of the consumption good. Their net worth is

$$W^B(\bar{b},\tau) = (1+r_e)K - (1+r_e)D^B(\bar{b},\tau).$$

Financial intermediary sector

The assets of the financial intermediary are the liabilities of the government and the type-B households, while its liabilities are the pension assets of type-A households and the amount payable for intermediation services. The net worth of the financial intermediaries is zero, as their balance sheet indicates:

Assets	Liabilities
Government debt $= (1+r)D^G(\bar{b},\tau)$ Private debt $= (1+r_e)D^B(\bar{b},\tau)$	Pension promises $= (1+r)[D^B(\bar{b},\tau) + D^G(\bar{b},\tau)]$ Amounts payable for intermediation services $= D^B(\bar{b},\tau)(r_e - r)$ Net worth $= 0$

Government

The assets of the government are the present value of the tax receipts on labor income, while its liabilities are the debt it has outstanding. The balance sheet of the government is

Assets	Liabilities
$\dfrac{\tau(1-\theta)Y}{r-\gamma}$	$D^G(\bar{b}, \tau)$
	Net worth $= 0$

Since labor is supplied inelastically and taxed at a rate τ, the government effectively owns a fraction τ of an individual's time endowment (now and in all future periods). In our model economy, the net worth of the government is zero and government debt is an asset for debt holders.

6. Equilibrium relations

We normalize Y to 1 and determine the value of a set of balanced growth variables at $t = 0$. All aggregate variables grow at rate γ except aggregate labor supply and the interest rates, which are constant. Given that Y has been normalized to 1 at time 0, the cost share relationships determine time 0 capital stock K and wage e:

$$(r_e + \delta_k)K = \theta Y, \tag{6.1}$$

$$eL = (1-\theta)Y. \tag{6.2}$$

From the intermediary's problem, the lending rate satisfies

$$r_e = r + \phi. \tag{6.3}$$

Three equilibrium conditions

Prices $\{e, r, r_e\}$ are determined from policy and technology. Therefore, only $\bar{b}$ and τ are needed to completely specify the household budget constraints. Conditional on these variables, aggregate consumption, $C(\bar{b}, \tau)$, and aggregate intermediation, $I(\bar{b}, \tau)$, will be determined by aggregating individual household variables. Aggregation, given the individual decisions conditional on $\bar{b}$ and τ, is specified in Appendix B.

One aggregate equilibrium condition is the aggregate resource constraint

$$C(\bar{b}, \tau) + X + \phi I(\bar{b}, \tau) = K^\alpha L^{1-\alpha}, \tag{6.4}$$

where $X = (\delta_k + \gamma)K$ is investment. Intermediation services satisfy

$$I(\bar{b}, \tau) = K - \frac{W^B(\bar{b}, \tau)}{(1 + r_e)}. \tag{6.5}$$

We assume that type-B households hold all the capital and the intermediaries hold none. This is done to resolve an unimportant indeterminacy. Increasing the amount of capital held by a type B and that type B's indebtedness by the same amount does not affect that type B's net worth, which is what is relevant. This portfolio shift by a type-B household is offset by portfolio shifts by other type-B households. The aggregate indebtedness of a type B is denoted by $D^B(\bar{b}, \tau)$ and is equal to $I(\bar{b}, \tau)$.

The second equilibrium condition is that the inheritance of households at a point in time equals aggregate bequests at that point in time. We consider $t = 0$ and let $B(\bar{b}, \tau)$ be the aggregate bequest at that time. The second equilibrium condition is

$$\bar{b} = B(\bar{b}, \tau)(1 + \gamma)^{30}. \tag{6.6}$$

There is a third equilibrium condition, namely, the government's budget constraint. This constraint $(1+r)D_t^G = \tau E_t + D_{t+1}^G$ equates payments to receipts. Given $D_{t+1}^G = (1+\gamma)D_t^G$, $E_0 = (1-\theta)Y_0$, and the normalization $Y_0 = 1.0$, the time 0 government budget constraint is

$$(r - \gamma)D^G(\bar{b}, \tau) = \tau(1 - \theta). \tag{6.7}$$

Equilibrium

The first two equilibrium conditions are linear in $(\bar{b}, \tau)$, so solving for a candidate solution is straightforward. This solution is the equilibrium only if, in addition, (i) the best strategy for type-B households is the no annuity strategy, (ii) the best strategy for type-A households is the annuity strategy, (iii) type B borrows and does not lend, and (iv) type A lends and does not borrow. The reason for the last constraint is that these equilibrium conditions hold provided that the no-borrowing constraint on annuity holders is not binding and it will not be binding if (iv) holds.

7. Calibration

The parameters that need to be calibrated are those related to the households $\{\alpha^A, \alpha^B, \beta, \mu^A, \mu^B, T, \delta\}$, the intermediation technology parameter $\{\phi\}$, the production good technology parameters $\{\theta, \delta_k, \gamma\}$, and the policy parameter r. The other two policy parameters $\{\tau, D^G\}$ are endogenous. As mentioned before, the choice of r as a parameter and τ as an endogenous variable is only for convenience; reversing their roles will not affect the results described in Section 8.

Many of these parameters are well documented in the literature; others are not. We proceed by listing the parameters with the selected values and a brief motivation.

Parameters associated with individuals

$\beta = 0.99$ (annuity holders' c grows at almost 2 percent over their lifetimes),

$\delta = 0.05$ (implies a post-retirement life expectancy of 20 years),

$\alpha^A = 0$ (assumption: type-A individuals have low bequest intensity),

$\alpha^B = 1$ (assumption: type-B individuals have high bequest intensity),

$T = 40$ (workers retire at chronological age 63),

$\mu^B = 0.162$ (specified so that the amount intermediated matches U.S. data),

$\mu^A = 1 - \mu^B = 0.838.$

Intermediation parameter

$\phi = 0.02$ (consistent with the average difference in borrowing and lending rates).

Policy parameter

$r = 0.03$ (assumption about government fiscal policy).

The motivation for this policy is that this has been the approximate return on lending by households (see McGrattan and Prescott (2003)).

Goods production parameters

$\theta = 0.3$ (capital income share),

$\gamma = 0.02$ (average growth rate of U.S. per capita output),

$\delta_k = 0.0382$ (consistent with capital–output ratio $= 3.4$, given $r_e = 0.05$).

In calibrating ϕ, we proceed as follows. Our model economy has household, government, and financial intermediary sectors. All nonfinancial business borrowing is consolidated with the household sector. We start with the net interest income of the financial intermediation sector. Fees are a small part of this sector's product and most of them are for transaction services, which is not intermediation in the sense used in this study. Using data from National Income and Product Accounts (NIPA)[16] for year 2007, the interest received amounted to 0.165 times gross national product (GNP)[17] and interest paid amounted to 0.110 times GNP. To estimate the services associated with intermediating borrowing and lending, we first subtracted intermediation services furnished without payment to households because we did not want to include implicit purchases of transaction services by the household. We also subtracted part of bad debt, viewing it as interest not received by the intermediary, to obtain an estimate of the cost of intermediating borrowing and lending between households of 3.4 percent of GNP in 2007. See Table 1.

Using data from the Flow of Funds, we found the debt outstanding of our household sector, which includes nonfinancial businesses, equals 1.72 times GNP.[18] The implied intermediation spread is thus 2.0 percent and in turn the calibrated $\phi = 0.02$. This number results in the after-tax returns being close to their historical averages (see McGrattan and Prescott (2003, 2005)).

In dealing with transaction costs associated with buying and selling assets, and fees such as those paid by investors to, say, a trust company, we follow the convention used

[16] Source: U.S. Department of Commerce (2008, Tables 7.11 and 2.4.5).

[17] Source: NIPA Table 1.7.5.

[18] Source: Flow of Funds (Board of Governors of the Federal Reserve System (2008, Table D.3)). See Table 2 herein for further details.

TABLE 1. Financial intermediary sector accounts relative to GNP year 2007.

Interest received	0.165	NIPA, Table 7.11, line 28
Less interest paid	0.110	NIPA, Table 7.11, line 4
Equals net interest income	0.055	
Less services furnished without payment	0.016	NIPA, Table 2.4.5, line 89
Less bad debt expenses	0.005	NIPA, Table 7.16, line 12[a]
Equals services for intermediating household borrowing and lending	0.034	
Amount intermediated between households	1.721	Flow of Funds, Table D.3 (total amount in column 1 less state, local, and federal government)

[a]This datum is for 2005, the latest for which it is currently available. We assumed half of the total bad debt was in that of financial intermediaries.

by U.S. national accounts and do not include them as a part of intermediation costs. The assets in our model are capital K, government debt, type-B household debt, and pension fund reserves. With regard to K transactions, say, the brokerage fees associated with transferring ownership of an owner-occupied house, NIPA treats these costs as an investment and justifies this as putting the house to more productive use. With government debt, transfer of ownership costs are zero in our model and virtually zero in fact. Pension fund reserves are not traded between households and, therefore, there are almost no costs associated with transferring ownership. The total costs of buying and selling of household debt between financial intermediaries are small and are part of intermediation costs. Households incur brokerage fees associated with transferring ownership of financial securities between households. These fees are not payment for intermediating debt between households and, therefore, are not part of the cost of intermediated borrowing and lending between households. Brokerage fees paid by intermediaries are part of the costs of intermediating borrowing and lending between households.

8. RESULTS

We considered four values for α^B, a parameter for which we have little information. For each value of α^B, we search for the μ^B for which the intermediated borrowing and lending between households is 1.72 times GNP. The results are summarized in Table 2, which shows results are not sensitive to the size of the bequest preference parameter α^B. Given that the aggregate results are insensitive to α^B, subsequently we deal only with the case $\alpha^B = 1$.[19]

Balance sheet of households

Table 3 details the aggregate balance sheet data for U.S. households implied by our model. Our model is calibrated so that both the privately held capital stock (K) and

[19]Like Cagetti and De Nardi (2006), there is little consequence of inheritance for the net worth distribution.

TABLE 2. Summary of aggregate results.

Economy	$\alpha^B = 1/3$	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
μ^A	0.833	0.838	0.851	0.867
μ^B	0.167	0.162	0.149	0.133
National accounts				
C_A	0.636	0.639	0.651	0.663
C_B	0.132	0.128	0.117	0.104
X	0.198	0.198	0.198	0.198
I	0.034	0.034	0.034	0.034
Y	1.000	1.000	1.000	1.000
Depreciation	0.13	0.13	0.13	0.13
Compensation	0.70	0.70	0.70	0.70
Profits	0.17	0.17	0.17	0.17
Net worth				
Type A	6.29	6.33	6.42	6.53
Type B	1.66	1.66	1.66	1.66
Government debt/Y	4.55	4.59	4.68	4.79
Bequest/Y	0.0341	0.0347	0.0365	0.0390
Tax rate	0.0650	0.0655	0.0668	0.0684

the intermediated household borrowing and lending (D^H) match U.S. statistics; government debt (D^G) is endogenously determined. One test of our model is how well it replicates this and other statistics, such as bequests and inheritances, for the U.S. economy. We examine each in turn.

Government debt

Government debt in our model, which is 4.6 times GNP, may at first sight appear large relative to U.S. federal, state, and local government debt, which was only 0.5 GNP in 2007. However, there are huge implicit annuity-like liabilities of the U.S. government, such as Social Security retirement and Medicare benefits. Households value the expected present value of these annuity-like net benefits and consider them as assets that contribute to their net worth. Hence, in the aggregate balance sheet of our model economy, the empirical counterpart of model government debt is explicit government debt *plus* the expected present value of these net benefits. Careful studies by Gokhale and

TABLE 3. Balance sheet of households.

Assets	Liabilities
$K = 3.4$ GDP	$D^H = 1.7$ GDP
$D^H = 1.7$ GDP	
$D^G = 4.6$ GDP	Net worth $= 8.0$ GDP

Smetters (2003, 2006) estimated the present value of these net benefits as between 4.2 and 5 GNP.[20] In light of this, the stock of government debt in our model is reasonable.

An additional point is that if no one had a bequest motive, the steady-state capital stock would be the same, namely, 3.4 times GNP, and government debt in our model would be slightly larger. Policy and not the nature of bequest preferences is what determines the capital–output ratio.

Bequests

A surprising finding is that the model's prediction regarding the magnitude of the bequests is insensitive to the strength of the bequest motive. We believe this insensitivity is due to the fact that bequest expenditures in the intertemporal budget constraint are small relative to the sum of all event contingent total expenditures, coupled with the fact that the measure of agents who leave a bequest (type B) is a small fraction of the total population.

Total annual bequests in our model, as seen in Table 3, are 0.035 times GNP for $\alpha^B = 1$. The aggregate value of estates in 2007 that exceeded \$675,000 was 0.00123 times GNP.[21] Some of these estates are interspousal and should not be included. This is more than offset by bequests that were under the limit for which estate tax returns had to be filed. Adding these and inter vivo transfers, and adjusting for underreporting of gifts associated with the transfer of family businesses to the younger generation would result in aggregate bequests being close to model aggregate bequests.

Modigliani's (1988) estimate of bequest flows is close to the flow in our model. He reported bequests of 0.02 times GNP. He added life insurance, death benefits, and newly established trusts to conclude that bequests are at least 0.027 times GNP.

Another measure of the size of bequests is the amount an individual inherits expressed in units of the individual's annual wage at time of inheritance. Each individual receives at chronological age 52 an amount equal to 1.98 times his or her annual wage at that time. Menchick and David (1983) estimated the average the inheritance received by all males to be \$20,000 (in 1967 dollars). We estimate the average gross annual wage for that year as \$8840, arriving at a ratio of inheritance received to annual wage equal to 2.26.[22] However, correcting for interspousal transfers might account for the difference. These considerations suggest that inheritances are consistent with the predictions of our model.[23]

[20] Their estimates were \$44 trillion in 2002 and \$63 trillion in 2005.

[21] U.S. Department of the Treasury (2007, Historic Table 17, p. 203).

[22] Nominal GDP in 1967 was \$833 billion. Assuming that 70 percent of GDP is labor income (consistent with our model economy), we obtain an estimate of total wage income of \$583 billion in 1967. Then, since the total employment in that year was 65.9 million, the average gross annual wage income is \$8840.

[23] We examined the consequences of population growth and found that they were small. Bequests fall to 0.03 times GNP as the population growth increases to the point at which the growth rate of the economy equals the interest rate.

Table 4. Inheritance as fraction of wealth at entry into workforce.

	$\alpha^B = 1/3$	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
Type A	0.044	0.045	0.047	0.050
Type B	0.035	0.036	0.038	0.040

Inheritance

Another variable of interest is the fraction of wealth that is inherited. A significant component of wealth is human capital, which is the present value of wages in our model world where labor is supplied inelastically. The other part is the present value of inheritance. As shown in Table 4, human capital is about 95.5 percent of wealth at entry into the workforce and would be higher if there were population growth. These results are for type-A households that discount using a 3 percent rate. The share is a little lower for type-B households that use a 5 percent discount rate. Anything that reduces the ratio of bequests to GNP reduces this number, so for the model with a 1 percent population growth rate, as in the United States, this ratio is near 97 percent.

The issues as to the importance of bequests for the size of the capital stock are mute in our model, because policy determines the capital stock and not the nature of preferences for bequests. However, a statistic of interest is the one estimated by Kotlikoff and Summers (1981). This statistic is the present value of inheritances people alive have received, using a 3 percent interest rate. Their estimate of this number is 0.80 times the total household net worth. Modigliani's (1988) estimate of this number is much smaller: 0.20. Modigliani (1988, Table 1, p. 19) presented a number of other estimates, all of which range between 0.10 and 0.20. This ratio number for our model economy is 0.18, which is in line with these estimates.

In our model economy, 93 percent of bequests are accidental. We came up with this number as follows. Setting $\alpha = 0$ for type-B households and requiring type-B households to follow the no annuity strategy results in this number. Treating these accidental bequests as savings for retirement along with all type-A savings implies that 99 percent of savings is for retirement purposes and 1 percent is for bequests.

Testable implications

Although our model was not developed to match both the explicit and implicit liabilities of the government, the aggregate savings predicted by our theory are approximately equal to that observed. The total government debt and bequests/GDP implied by our model is in line with the U.S. historical experience. This, we believe, is an important testable implication.

Some microfindings

Our abstraction has implications for micro-observations as well. Unlike the macrofindings, the model's microfindings are not a quantitative theory of the consequences of the

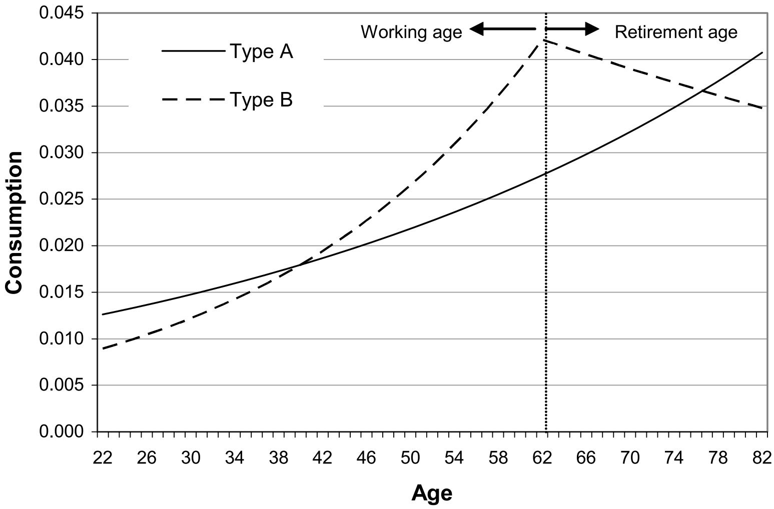

FIGURE 2. Lifetime consumption pattern.

bequest motive for the distributions of consumption, net worth, and equity holdings, and consequently must be interpreted with care. They do, however, show that the bequest motive or, for that matter, any factor that leads people to partially finance their capital acquisitions with debt, is quantitatively important for these statistics. With this caveat, the microdistributional relations for our model economy are as follows.

Figure 2 plots the lifetime consumption patterns of the two types of households. Type-A's consumption grows at a constant annual rate of 1.97 percent throughout their lifetimes. Type-B's consumption starts out lower and grows more rapidly during their working lives, with this growth rate being 3.95 percent. Upon retirement, the consumption growth rate turns negative, falling to −0.95 percent. At retirement, a type-B retiree's consumption is higher than that of an equal age type-A retiree.[24]

Cross-sectional consumption

Figure 3 plots cross-sectional consumption by age for the two types. All type-A's who are alive have virtually the same consumption. Young type-B workers have lower consumption and older workers have higher consumption. For the type-B retirees, consumption level declines with age.

Net worth by age

In Figure 4, we plot net worth relative to current annual wage income, which has a stationary distribution. At retirement, the net worth of a type-A household is 12 times the

[24]There is a rich literature on the life cycle consumption patterns, including the works of Attanasio, Banks, Meghir, and Weber (1999) and Hansen and Imrohoroglu (2008), among others. This is not the concern of this paper, but the fact that life cycle patterns differ between those choosing to annuitize and those choosing not to annuitize has implications for the empirical pattern of life cycle consumption.

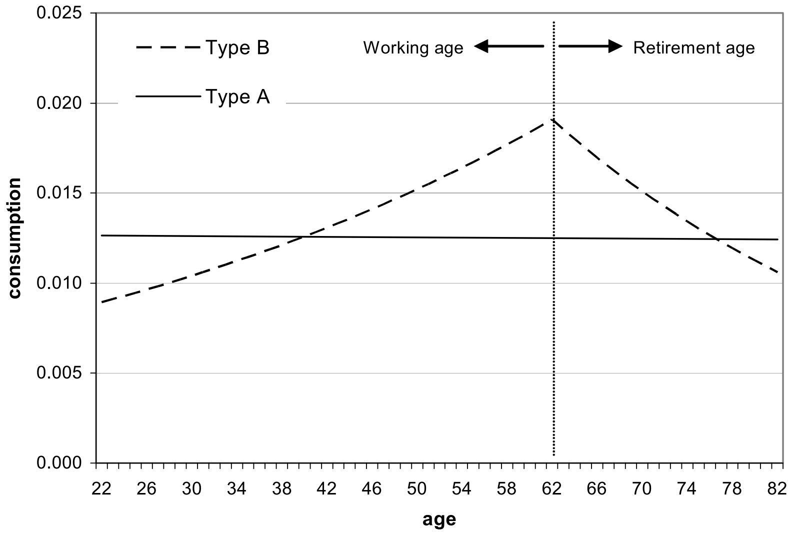

FIGURE 3. Cross-sectional consumption by age.

annual wage, and that of a type-B household is 19 times the annual wage. The disparity in net worth (corrected for age) is modest, being a maximum of about 1.6 at retirement age. After retirement, disparity falls until age 78 and then starts to grow, with the type-A household becoming the one with the greater net worth. The jump in net worth at chronological age 52 is due to inheritance.

Lorenz curves

Figure 5 plots the Lorenz curves for consumption, net worth, and capital or equity holdings. In the case of capital, we assume all type-B households have the same ratio of debt

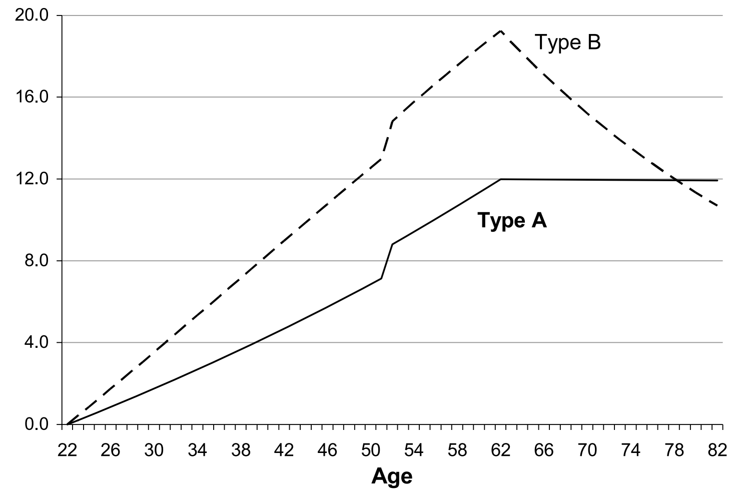

FIGURE 4. Net worth as a function of age in units of annual wage income.

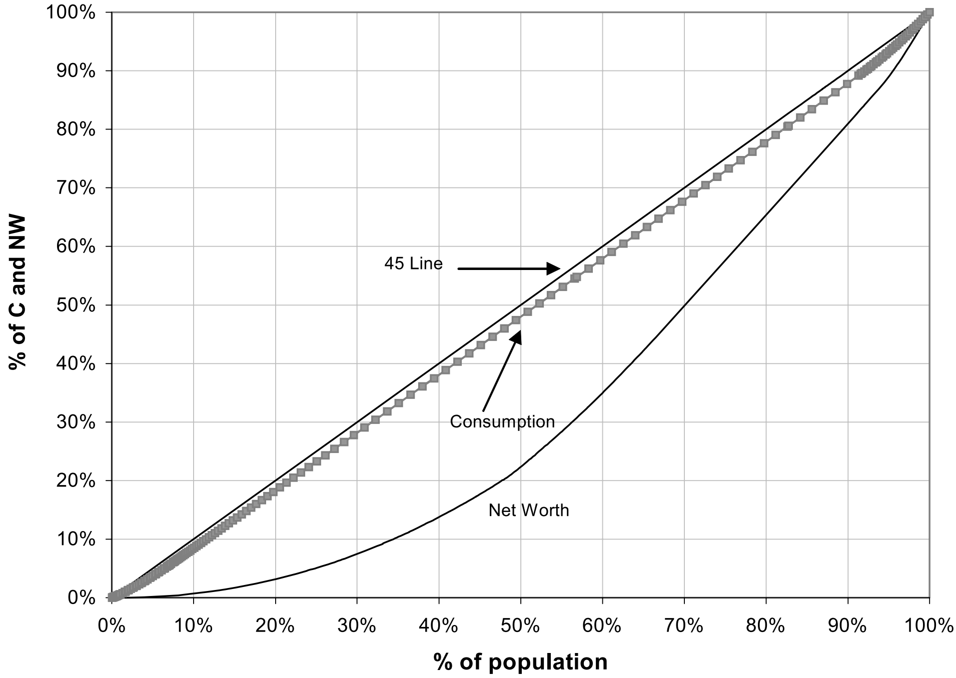

FIGURE 5. Lorenz curve for consumption, net worth, and capital.

liabilities to capital in their portfolios so as to resolve the portfolio indeterminacy at the individual level. We truncate the distribution at age 112, so the curves are not exact, but are very good approximations given the small fraction of the population over this age.

Our model is not designed to address issues about wealth distribution, as we have abstracted from any heterogeneity in human capital. All agents have the same earnings stream. Our principal findings are that there is almost no disparity in consumption levels and sizable disparities in net worth levels. This shows that the dispersion in net worth may be a bad proxy for dispersion in consumption.[25]

In our model economy, all individuals have the same human capital endowments. If the model were modified to have people earn proportionally different wages, to a first approximation an individual's allocation is proportional to that individual's wage.[26] Thus, introducing wage disparity would add disparity in consumption and net worth. Introducing entrepreneurs (Cagetti and De Nardi (2006)) and idiosyncratic risk (Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007)) would increase disparity as well.

Cost of financial market constraints

What are the gains to a household of having access to the equity market at no intermediation cost? Table 5 reports the cost of not having this access, which was the case for most

[25]The Gini coefficients for the consumption and net worth Lorenz curves are 0.038 and 0.35, respectively.

[26]If bequests were distributed proportional to the human capital factor, the scaling result would hold exactly.

TABLE 5. Cost to a type A of not having access to the annuity market in units of wealth at entry into the workforce.

α^B	Change in v_0^A
1/3	0.77%
1	0.79%
3	0.84%
6	0.90%

TABLE 6. Cost to a type B of not being permitted to hold equity directly in units of wealth at entry into the workforce.

α^B	Change in v_0^B
1/3	1.24%
1	4.00%
3	9.74%
6	15.77%

Americans prior to the development of low-cost indexed mutual funds, as being about 4.0 percent of wealth at time of entry into the workforce. This wealth is the present value of labor income and inheritance.

Tables 5 and 6 show the percentage increase in either v_0^k, that is, wealth at time of entry into the workforce, which is necessary to compensate an $i \in \{A, B\}$ in wealth equivalents if forced to switch to a system other than their preferred choice. Since both consumption and bequest are linear functions of initial wealth, the percentage changes in both consumption and bequests are the same as the percentage change in initial wealth.

What are the costs to a type A if for some reason, such as adverse selection problems or legal constraints, they do not have access to annuity markets and must use the equity option for saving? The cost is small, being approximately 0.8 percent of lifetime consumption.

Implications for the equity premium

In our framework, there is no equity premium because there is no aggregate uncertainty. The return on equity and the borrowing rate are both equal to 5 percent. This is a no arbitrage condition. The return on government debt is 3 percent. If we use the conventional definition of the equity premium—the return on a broad equity index less the return on government debt—we would erroneously conclude that in our model the equity premium was 2 percent. The difference in the government borrowing rate and the return on equity is *not* an equity premium; it arises because of the wedge between borrowing and lending rates. Analogously, if in the U.S. economy, borrowing and lending rates for equity investors differ (and they do), the equity premium should be measured relative to the investor borrowing rate rather than the government's borrowing rate (the investor lending rate). Measuring the premium relative to the government's borrowing rate artificially increases the premium for bearing aggregate risk by the difference between the investor's borrowing and lending rates.[27] If such a correction were made to the results reported in Mehra and Prescott (1985), the equity premium would be 4 percent rather than the reported 6 percent.

9. Concluding comments

In this paper, we develop a heterogeneous household economy where households differ along only one dimension: their preferences for bequest. In equilibrium, households that have a low desire to bequeath lend and hold annuities, while those that have a high desire to bequeath borrow and own capital. This is important because the total amount of borrowing by households and the government must equal the amount lent by households. Our simple framework mimics reality with respect to *both* the amount of intermediated borrowing and lending between households, *and* the average spread in borrowing and lending rates resulting from intermediation costs. In addition, the amount of aggregate savings predicted by the theory is approximately equal to the observed amount of aggregate savings. This is an important test of our theory, as it was not developed to match both the explicit and implicit liabilities of the government.[28]

We view this as a first step in what we think will prove to be a productive research program. Possible extensions include incorporating differential survival rates and addressing the issues of adverse selection and moral hazard when pricing annuities. This extension might justify our requirement that people choose between the annuity and the no annuity strategies early in their careers. This research program, if successful, will require extension of the theory of household lifetime consumption behavior because the bequest motive is not the only salient factor that differentiates people. Differences in preferences with respect to consumption today versus consumption in the future, and differences in preferences that give rise to differences in lifetime labor supply are likely to be important as well.

Another possible extension is to model non-steady-state behavior as in Geanakoplos, Magill, and Quinzii (2004), who considered the importance of demographic waves for stock market valuation, or as in Braun, Ikeda, and Joines (2007), for saving behavior within the overlapping generations framework.

Appendix A: Proof of Proposition 1

The prices (r, r_e, e_0), tax rate τ, and inheritance implied by $\bar{b}_0$ are given to an individual. Note $0 < r < r_e$. Let $U_A(\alpha)$ and $U_B(\alpha)$ represent the maximum attainable utility of an agent of measure zero in this economy who follows strategy A (annuity) or B (bequest), respectively, as a function of $\alpha \in \mathbb{R}_+$. Define $\Delta(\alpha) = U_B(\alpha) - U_A(\alpha)$.

[27]For a detailed exposition of this and related issues, the reader is referred to Mehra and Prescott (2008).

[28]We thank one of the referees for bringing this to our attention.

Proposition 1. *If*

$$\frac{1+r_e}{1+r} > \beta\left[\frac{1-\sigma\beta}{\beta\delta}\right]^{1-\beta\sigma},$$

then

$$\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0 \quad \forall\alpha\in\mathbb{R}_+.$$

Proof. The maximum utility as a function of α attainable by an agent who follows an annuity strategy (A), taking as given the parameters of the economy, can be expressed as

$$U_A(\alpha) = \sum_{j=0}^{T-1}\beta^j\log(c_j^A) + \beta^T(\phi_A(\alpha) + \theta_A(\alpha)\log(w_T^A)),$$

where

$$\theta_A(\alpha) = \frac{1+\beta\alpha\delta}{1-\beta(1-\delta)},$$

$$\phi_A(\alpha) = \frac{(\theta_A(\alpha)-1)\log[(1+r)\beta] - \log[\theta_A(\alpha)] + \beta\alpha\delta[\log(\alpha) - \log[\theta_A(\alpha)]]}{1-\beta(1-\delta)},$$

$$c_j^A = \bar{c}(\alpha)\beta^j(1+r)^j v_0^A, \quad j<T,$$

$$w_T^A = \theta_A(\alpha)\bar{c}(\alpha)\beta^T(1+r)^T v_0^A$$

($\bar{c}(\alpha)$ and v_0^A are defined in Section 3).

Similarly, the maximum utility as a type α who follows an annuity strategy (B) is

$$U_B(\alpha) = \sum_{j=0}^{T-1}\beta^j\log(c_j^B) + \beta^T(\phi_B(\alpha) + \theta_B(\alpha)\log(w_T^B)),$$

where

$$\theta_B(\alpha) = \frac{1+\beta\alpha\delta}{1-\beta(1-\delta)},$$

$$\phi_B(\alpha) = \frac{(\theta_B(\alpha)-1)\log(1+r_e) + (\theta_B(\alpha)-1)\log[(\theta_B(\alpha)-1)] - \theta_B(\alpha)\log[(\theta_B(\alpha)]}{1-\beta(1-\delta)},$$

$$c_j^B = \bar{c}(\alpha)\beta^j(1+r_e)^j v_0^B, \quad j<T,$$

$$w_T^B = \theta_B(\alpha)\bar{c}(\alpha)\beta^T(1+r_e)^T v_0^B$$

($\bar{c}(\alpha)$ and v_0^B are defined in Section 3).

Using the properties of the logarithm function and defining $\theta(\alpha)=\theta_A(\alpha)=\theta_B(\alpha)$,

$$\Delta(\alpha) = \sum_{j=0}^{T-1}\beta^j\log\left(\frac{(1+r_e)^j v_0^B}{(1+r)^j v_0^A}\right) + \beta^T\left(\phi_B(\alpha) - \phi_A(\alpha) + \theta(\alpha)\log\left(\frac{w_T^B}{w_T^A}\right)\right). \tag{A.1}$$

Since the first term is independent of α, and the ratio of w_T^B/w_T^A is independent of α as well, it follows that

$$\frac{\partial\Delta(\alpha)}{\partial\alpha} = \beta^T \frac{\partial[\phi_B(\alpha) - \phi_A(\alpha)]}{\partial\alpha} + \beta^T \theta'(\alpha)\log\left(\frac{w_T^B}{w_T^A}\right), \tag{A.2}$$

where $\theta'(\alpha) = \frac{\beta\delta}{1-\beta\sigma} > 0$, which does not depend on α, and

$$\frac{w_T^B}{w_T^A} = \frac{v_0^B(1+r_e)^T}{v_0^A(1+r)^T} = \frac{\displaystyle\sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^{j-T}} + \frac{\bar{b}}{(1+r_e)^{30-T}}}{\displaystyle\sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r)^{j-T}} + \frac{\bar{b}}{(1+r)^{30-T}}} > 1$$

since $r_e > r$, $j < T$, and $30 < T$. This implies that the second term in (A.2) is positive, that is, $\beta^T\theta'(\alpha)\log(\frac{w_T^B}{w_T^A}) > 0$.

To prove our assertion that $\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0$ is positive, we proceed in three steps:

(a) We show that $\lim_{\alpha\to 0}\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0$.

(b) We show that $\frac{\partial^2\Delta(\alpha)}{\partial\alpha^2} < 0$.

(c) We show that $\lim_{\alpha\to\infty}\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0$.

Some straightforward algebra yields

$$\begin{aligned} &\frac{\partial[\phi_B(\alpha) - \phi_A(\alpha)]}{\partial\alpha} \\ &\quad = \frac{\theta'(\alpha)}{1-\beta\sigma}\left(\log\left(\frac{1+r_e}{(1+r)\beta}\right) + \log\left(\frac{\theta(\alpha)-1}{\alpha}\right) - \beta\sigma\log\left(\frac{\theta(\alpha)}{\alpha}\right)\right). \end{aligned} \tag{A.3}$$

From (A.3), it is readily seen that $\lim_{\alpha\to 0}\frac{\partial[\phi_B(\alpha)-\phi_A(\alpha)]}{\partial\alpha} \to +\infty$. This follows since the last term tends to $+\infty$ and all the other terms are bounded. This coupled with the fact that $\beta^T\theta'(\alpha)\log(\frac{w_T^B}{w_T^A}) > 0$ proves that $\lim_{\alpha\to 0}\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0$.

The second derivative $\frac{\partial^2\Delta(\alpha)}{\partial\alpha^2}$ is negative by direct differentiation,

$$\frac{\partial^2\Delta(\alpha)}{\partial\alpha^2} = \frac{-\beta^{T+1}\delta(1-\delta)}{\alpha(1-\beta(1-\delta))(1+(\alpha-1)\delta)(1+\alpha\beta\delta)} < 0,$$

since the denominator is always positive and the numerator is negative.

Finally, it can be shown that $\lim_{\alpha\to\infty}\frac{\partial\Delta(\alpha)}{\partial\alpha} > 0$ under the condition stated in the theorem. Notice that (taking the limit of (A.3) when $\alpha\to\infty$) equation (A.2) is positive if and only if

$$\frac{1}{1-\beta\sigma}\log\left(\frac{(1+r_e)}{(1+r)\beta}\right) + \log(\theta'(\alpha)) + \log\left(\frac{v_0^B(1+r_e)^T}{v_0^A(1+r)^T}\right) > 0.$$

The last term in the above expression has already been shown to be positive. Thus a sufficient condition for this inequality is

$$\frac{1}{1-\beta\sigma}\log\left(\frac{(1+r_e)}{(1+r)\beta}\right)+\log(\theta'(\alpha))>0.$$

This inequality can be written as $\frac{1+r_e}{1+r}>\beta[\frac{1-\sigma\beta}{\beta\delta}]^{1-\beta\sigma}$.

Since (a), (b), and (c) are satisfied, it follows that $\frac{\partial\Delta(\alpha)}{\partial\alpha}>0\ \forall\alpha\in\mathbb{R}_+$. □

Appendix B: Aggregation

General formulas

There are two types $i\in\{A,B\}$. The A type has $\alpha^A=0$ and in equilibrium chooses the annuity strategy. The measure of type i of age j at $t=0$ is

$$\mu_j^i=\begin{cases}\mu_0^i, & j\le T,\\ (1-\delta)^{j-T}\mu_0^i, & j>T.\end{cases}\tag{B.1}$$

The aggregate quantity for variable Z of type $i\in\{A,B\}$ agents at $t=0$ is Z_0^i,

$$Z_0^i=\mu_0^i\sum_{j=0}^{T-1}z_j^i(1+\gamma)^{-j}+\mu_0^i\sum_{j=T}^{\infty}(1-\delta)^{j-T}z_j^i(1+\gamma)^{-j},\tag{B.2}$$

where z_j^i is the individual allocation of type i at age j born at $t=0$. Notice that we have used the fact that each subsequent generation has a consumption–age profile that is higher by a factor of $(1+\gamma)^j$ under balanced growth. The aggregate quantity of Z at time 0 is

$$Z_0=Z_0^A+Z_0^B.$$

Agent type B

Aggregate assets of agent type B and aggregate bequest The aggregate assets for B-type agents are computed using the law of motion of net worth. From the individual problem,

$$\begin{aligned}
&w_0^B=0,\\
&w_j^B=(w_{j-1}^B-c_{j-1}^B+(1-\tau)e_0(1+\gamma)^{j-1})(1+r_e), \quad \text{for } j\le T \text{ and } j\neq 30,\\
&w_j^B=(w_{j-1}^B-c_{j-1}^B+(1-\tau)e_0(1+\gamma)^{j-1})(1+r_e)+\bar{b}, \quad \text{for } j=30,\\
&w_j^B=(w_{j-1}^B-c_{j-1}^B)(1+r_e), \quad \text{for } j>T.
\end{aligned}$$

From equations (3.4) and (3.7), the consumption for type B is given by

$$c_j^B = \begin{cases} [\beta(1+r_e)]^j \bar{c}^B(\alpha^B) v_0^B, & j < T, \\ \dfrac{w_j^B}{f_2(\alpha^B)}, & j \geq T, \end{cases} \tag{B.3}$$

where

$$\begin{aligned} \bar{c}^B(\alpha) &= \frac{(1-\beta)}{1-\beta^T + (1-\beta)\beta^T f_2(\alpha)}, \\ v_0^B &= \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\bar{b}}{(1+r_e)^{30}}, \end{aligned} \tag{B.4}$$

and

$$f_2(\alpha) = \frac{1+\alpha\beta\delta}{1-\sigma\beta}.$$

Using (B.2), aggregate net worth is

$$W^B(\bar{b}, \tau) = \mu_0^B \sum_{j=0}^{T-1} w_j^B (1+\gamma)^{-j} + \mu_0^B \sum_{j=T}^{\infty} \sigma^{j-T} w_j^B (1+\gamma)^{-j}.$$

The summation over $j = 0, \dots, T-1$ is performed numerically, while for total net worth of the retirees is

$$\begin{aligned} &\mu_0^B \sum_{j=T}^{\infty} \sigma^{j-T} w_j^B (1+\gamma)^{-j} \\ &\quad = \frac{w_T^B \mu_0^B (1+\gamma) f_2(\alpha^B)}{(1+\gamma)^T [(1+\gamma) f_2(\alpha^B) - \sigma(1+r_e)(f_2(\alpha^B)-1)]}, \end{aligned} \tag{B.5}$$

where from the individual problem,

$$w_T^B = f_2(\alpha^B)[\beta(1+r_e)]^T \bar{c}^B(\alpha^B) v_0^B.$$

Since $\alpha^A = 0$, all bequests are coming from the type B and, as shown in Section 3, are given by

$$b_j^B = w_j^B, \quad j \geq T+1,$$

if a type B dies prior to the end of the previous period and are zero otherwise.

Since the measure of agents dying at each age $j \geq T+1$ is $\mu_0^B \delta \sigma^{j-T-1} = \delta \mu_{j-1}^B$, the aggregate bequest is

$$B_0(\bar{b}, \tau) = \sum_{j=T+1}^{\infty} \delta \frac{\mu_{j-1}^B b_j^B}{(1+\gamma)^j} = \sum_{j=T+1}^{\infty} \delta \frac{\mu_{j-1}^B w_j^B}{(1+\gamma)^j} = \frac{\delta}{\sigma} \sum_{j=T+1}^{\infty} \frac{\mu_j^B w_j^B}{(1+\gamma)^j}.$$

Using (B.5), it is straightforward to find that

$$B_0(\bar{b},\tau)=\frac{\delta w_T^B\mu_0^B}{\sigma(1+\gamma)^T}\left[\frac{(1+\gamma)f_2(\alpha^B)}{[(1+\gamma)f_2(\alpha^B)-\sigma(1+r_e)(f_2(\alpha^B)-1)]}-1\right]$$

or

$$B_0(\bar{b},\tau)=\frac{\delta w_T^B\mu_0^B}{(1+\gamma)^T}\left[\frac{(f_2(\alpha^B)-1)(1+r_e)}{[(1+\gamma)f_2(\alpha^B)-\sigma(1+r_e)(f_2(\alpha^B)-1)]}\right]. \tag{B.6}$$

Aggregate consumption type B Similarly, using (B.2) and (B.3), the aggregate consumption of type-B agents at time 0 can be expressed as

$$C_0^B=\Phi_1^B v_0^B, \tag{B.7}$$

where

$$\Phi_1^B=\bar{c}(\alpha^B)\left[\sum_{j=0}^{T-1}\left(\frac{\beta(1+r_e)}{1+\gamma}\right)^j+\beta^T\sum_{j=T}^{\infty}\left(\frac{(1+r_e)}{1+\gamma}\right)^j\left(\frac{f_2(\alpha^B)-1}{f_2(\alpha^B)}\right)^{j-T}\sigma^{j-T}\right]\mu_0^B$$

or

$$\begin{aligned}\Phi_1^B=(1+\gamma)\bar{c}^B(\alpha\beta)\Bigg[&\frac{1-\left[\frac{\beta(1+r_e)}{1+\gamma}\right]^T}{(1+\gamma)-\beta(1+r_e)}\\&+\frac{(1+\alpha^B\beta\delta)\left[\frac{\beta(1+r_e)}{1+\gamma}\right]^T}{(1+\gamma)-\beta\sigma^2+\alpha^B\beta\delta(\gamma+\delta)-\beta\sigma(1-\delta(1-\alpha^B))r_e}\Bigg]\mu_0^B.\end{aligned}$$

Agent type A

Aggregate assets of agent type A The aggregate bequest is measured in units of agent type-B assets; therefore, the inheritance received by agent type A measured in her assets' units is $\bar{b}^A=\bar{b}(1+r)/(1+r_e)$. The aggregate assets for agents type A are computed using the law of motion of net worth. From the individual problem,

$$\begin{aligned}&w_0^A=0,\\&w_j^A=(w_{j-1}^A-c_{j-1}^A+(1-\tau)e_0(1+\gamma)^{j-1})(1+r),\quad \text{for } j\le T \text{ and } j\neq 30,\\&w_j^A=(w_{j-1}^A-c_{j-1}^A+(1-\tau)e_0(1+\gamma)^{j-1})(1+r)+\bar{b}^{\mathrm{A}},\quad \text{for } j=30,\\&w_j^A=\sum_{t=0}^{\infty}(1-\delta)^t\frac{c_{j+t}^A}{(1+r)^t}+\sum_{t=0}^{\infty}\delta(1-\delta)^{t-1}\frac{b_{j+t}^A}{(1+r)^t},\quad j>T.\end{aligned} \tag{B.8}$$

Using (B.2), aggregate net worth is calculated as

$$W^A(\bar{b},\tau)=\mu_0^A\sum_{j=0}^{T-1}w_j^A(1+\gamma)^{-j}+\mu_0^A\sum_{j=T}^{\infty}\sigma^{j-T}w_j^A(1+\gamma)^{-j}.$$

As for type B, the summation for $j=0,\dots,T$ is performed numerically. From equation (3.11), consumption for type-A agents, born at period zero, when they reach age j (at time j) is

$$c_j^A=\bar{c}(\alpha^A)(1+r)^j\beta^j v_0^A,\quad j\geq 0.$$

Then agents alive at time 0 of age j consume

$$c_{0,j}^A=\bar{c}(\alpha^A)v_0^A\left[\frac{\beta(1+r)}{(1+\gamma)}\right]^j,\quad j\geq 0. \tag{B.9}$$

Using (B.8) and (B.9), net worth for retired agents can be written as

$$w_j^A=\frac{c_{j,0}^A}{1-\beta\sigma},\quad j>T.$$

Then

$$\mu_0^A\sum_{j=T+1}^{\infty}\sigma^{j-T}\frac{w_j^A}{(1+\gamma)^{-j}}=\frac{\mu_0^A\bar{c}(\alpha^A)v_0^A}{1-\beta\sigma}\left[\frac{(1+r)\beta}{1+\gamma}\right]^{T+1}\left[\frac{1+\gamma}{1+\gamma-\beta(1+r)\sigma}\right].$$

Aggregate consumption type A Again, using (B.2) and (B.9), the aggregate consumption of type-A agents at time 0 can be expressed as

$$C_0^A=\Phi_1^A v_0^A, \tag{B.10}$$

where

$$\Phi_1^A=\bar{c}(\alpha^A)\left[\sum_{j=0}^{T-1}\left(\frac{\beta(1+r)}{1+\gamma}\right)^j+\sum_{j=T}^{\infty}\left(\frac{\beta(1+r)}{1+\gamma}\right)^j\sigma^{j-T}\right]\mu_0^A$$

or

$$\Phi_1^A=(1+\gamma)\bar{c}(\alpha^A)\left[\frac{1-\left[\frac{\beta(1+r)}{1+\gamma}\right]^T}{(1+\gamma)-\beta(1+r)}+\frac{\left[\frac{\beta(1+r)}{1+\gamma}\right]^T}{(1+\gamma)-\beta(1+r)\sigma}\right]\mu_0^A,$$

where

$$v_0^A=\sum_{t=0}^{T-1}\frac{(1-\tau)e_0(1+\gamma)^j}{(1+r)^j}+\frac{\bar{b}^A}{(1+r)^{30}}.$$

Balance sheets

Type B: $(1 + r_e)K = (1 + r_e)D^B(\bar{b}, \tau) + W^B(\bar{b}, \tau)$.
Type A: $(1 + r)A^A(\bar{b}, \tau) = W^A(\bar{b}, \tau)$.
Intermediary: $(1 + r_e - \phi)D^B(\bar{b}, \tau) + (1 + r)G^G(\bar{b}, \tau) = (1 + r)A^A(\bar{b}, \tau)$.
Notice that the net worth of both the intermediary and the government is 0.

Equilibrium conditions

There are three equilibrium conditions that can potentially be used to solve the model:

- *Feasibility*: $Y = C_0(\bar{b}, \tau) + X + \phi[K - (W^B(\bar{b}, \tau))/(1 + r_e)]$, where $C_0(\bar{b}, \tau) = C_0^A(\bar{b}, \tau) + C_0^B(\bar{b}, \tau)$.
- *Bequest = inheritance*: $\bar{b} = B_0(\bar{b}, \tau)(1 + \gamma)^{30}$.
- *Assets markets*: $W^B(\bar{b}, \tau)/(1 + r_e) + W^A(\bar{b}, \tau)/(1 + r) = G^G(\bar{b}, \tau) + K$.

Since this is a linear system in $(\bar{b}, \tau)$, one equation is redundant, and the solution is straightforward. We chose to use the first two equilibrium conditions and then we check that the third one is satisfied as well.

References

Abel, A. B. (1986), "Capital accumulation and uncertain lifetimes with adverse selection." *Econometrica*, 50 (5), 1079–1097. [3]

Abel, A. B. and M. Warshawsky (1988), "Specification of the joy of giving: Insights from altruism." *Review of Economics and Statistics*, 70 (1), 145–149. [6]

Altonji, J. G., F. Hayashi, and L. J. Kotlikoff (1997), "Parental altruism and inter vivos transfers: Theory and evidence." *Journal of Political Economy*, 105 (6), 1121–1166. [5]

Attanasio, O., J. Banks, C. Meghir, and G. Weber (1999), "Humps and bumps in lifetime consumption." *Journal of Business & Economic Statistics*, 17 (1), 22–35. [23]

Barro, R. J. (1974), "Are government bonds net wealth?" *Journal of Political Economy*, 82 (6), 1095–1117. [6]

Becker, G. S. and R. J. Barro (1988), "A reformulation of the economic theory of fertility." *Quarterly Journal of Economics*, 103 (1), 1–25. [6]

Blanchard, O. J. (1985): "Debt, deficits, and finite horizons." *Journal of Political Economy*, 93 (2), 223–247. [6]

Board of Governors of the Federal Reserve System (2008), *Flow of Funds Accounts of the United States*. December Z.1 release. [18]

Braun, G. A., D. Ikeda, and D. H. Joines (2007), "The saving rate in Japan: Why it has fallen and why it will remain low." Working Paper F-535, CIRIE-F. [27]

Cagetti, M. and M. De Nardi (2006): "Entrepreneurship, frictions, and wealth." *Journal of Political Economy*, 114 (5), 835–870. [19, 25]

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003), "Accounting for U.S. earnings and wealth inequality." *Journal of Political Economy*, 111 (4), 818–857. [25]

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007), "A quantitative theory of unsecured consumer credit with risk of default." *Econometrica*, 75 (6), 1525–1589. [25]

Constantinides, G. M., J. B. Donaldson, and R. Mehra (2007): "Junior is rich: Bequests as consumption." *Economic Theory*, 32 (1), 125–155. [6]

De Nardi, M. (2004), "Wealth inequality and intergenerational links." *Review of Economic Studies*, 71 (7), 743–768. [6]

De Nardi, M., S. Imrohoroglu, and T. J. Sargent (1999), "Projected U.S. demographics and social security." *Review of Economic Dynamics*, 2 (3), 575–615. [6]

Díaz-Giménez, J., E. C. Prescott, T. J. Fitzgerald, and F. Alvarez (1992), "Banking in computable general equilibrium economies." *Journal of Economic Dynamics and Control*, 16 (3–4), 533–559. [6]

Fernández-Villaverde, J. and D. Krueger (2002), "Consumption over the life cycle: Some facts from consumer expenditure survey data." Working Paper 9382, NBER. [6]

Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2008), "Altruism, incomplete markets, and tax reform." *Journal of Monetary Economics*, 55 (1), 65–90. [6]

Geanakoplos, J., M. Magill, and M. Quinzii (2004), "Demography and the long-run predictability of the stock market." Discussion Paper 1380R, Cowles Foundation. [27]

Gokhale, J. and K. Smetters (2003), *Fiscal and Generational Imbalances: New Budget Measures for New Budget Priorities*. American Enterprise Institute Press, Washington, District of Columbia. [21]

Gokhale, J. and K. Smetters (2006), "Fiscal and generational imbalances: An update." In *Tax Policy and the Economy*, Volume 20 (J. M. Poterba, ed.), MIT Press, Cambridge, Massachusetts. [21]

Hansen, G. D. and S. Imrohoroglu (2008), "Consumption over the life cycle: The role of annuities." *Review of Economic Dynamics*, 11 (3), 566–583. [6, 23]

Hurd, M. D. (1989), "Mortality risk and bequests." *Econometrica*, 57 (4), 779–813. [5]

Hurd, M. D. and B. G. Mundaca (1989), "The importance of gifts and inheritances among the affluent." In *The Measurement of Saving, Investment and Wealth* (R. E. Lipsey and H. S. Tice, eds.), University of Chicago Press, Chicago. [6]

Kaldor, N. (1961), "Capital accumulation and economic growth." In *The Theory of Capital* (F. A. Lutz and D. C. Hague, eds.), 177–222, St. Martin's Press, New York. [2]

Kopczuk, W. and J. Lupton (2007), "To leave or not to leave: The distribution of bequest motives." *Review of Economic Studies*, 74 (1), 207–235. [6]

Kotlikoff, L. J. and L. H. Summers (1981), "The role of intergenerational transfers in aggregate capital accumulation." *Journal of Political Economy*, 89 (4), 706–732. [22]

Laitner, J. and F. T. Juster (1996), "New evidence on altruism: A study of TIAA-CREF retirees." *American Economic Review*, 86 (4), 893–908. [5]

Laitner, J. and H. Ohlsson (2001), "Bequest motives: A comparison of Sweden and the United States." *Journal of Public Economics*, 79 (1), 205–236. [5, 6]

Lettau, M. and H. Uhlig (2000), "Can habit formation be reconciled with business cycle facts?" *Review of Economic Dynamics*, 3 (1), 79–99. [4]

McGrattan, E. R. and E. C. Prescott (2000), "Is the stock market overvalued?" *Federal Reserve Bank of Minneapolis Quarterly Review*, 24 (4), 20–40. [4]

McGrattan, E. R. and E. C. Prescott (2003), "Average debt and equity returns: Puzzling?" *American Economic Review*, 93 (2), 392–397. [18]

McGrattan, E. R. and E. C. Prescott (2005), "Taxes, regulations, and the value of U.S. and U.K. corporations." *Review of Economic Studies*, 72 (3), 767–796. [18]

Mehra, R. and E. C. Prescott (1985), "The equity premium: A puzzle." *Journal of Monetary Economics*, 15 (2), 145–161. [4, 27]

Mehra, R. and E. C. Prescott (2008), "Non-risk-based explanations of the equity premium." In *Handbook of Investments: The Handbook of the Equity Risk Premium* (R. Mehra, ed.), 101–215, Elsevier, Amsterdam. [27]

Menchik, P. L. and M. David (1983), "Income distribution, lifetime savings, and bequests." *American Economic Review*, 73 (4), 672–690. [5, 21]

Miller, M. H. and C. W. Upton (1974), *Macroeconomics: A Neoclassical Introduction*. Richard D. Irwin, Homewood, Illinois. [8]

Modigliani, F. (1988), "The role of intergenerational transfers and life cycle saving in the accumulation of wealth." *Journal of Economic Perspectives*, 2 (2), 15–40. [21, 22]

Solow, R. M. (2000), *Growth Theory: An Exposition*, 2nd ed. Oxford University Press, New York. [2]

U.S. Department of Commerce, Bureau of Economic Analysis (2008), *National Income and Product Accounts*. August 2008 revision. [18]

U.S. Department of the Treasury, Internal Revenue Service (2007), *Statistics of Income Bulletin* 21 (1). [21]

Wilhelm, M. O. (1996), "Bequest behavior and the effect of heirs' earnings: Testing the altruistic model of bequests." *American Economic Review*, 86 (4), 874–892. [5]

Yaari, M. E. (1965), "Uncertain lifetime, life insurance, and the theory of the consumer." *Review of Economic Studies*, 32 (2), 137–150. [13]

Submitted November, 2009. Final version accepted January, 2011.