ON THE FINANCING AND INVESTMENT DECISIONS OF
MULTINATIONAL FIRMS IN THE PRESENCE OF EXCHANGE RISK

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In recent years, several papers [Mossin [13], Hamada [7], Rubinstein [14]]
have addressed the normative implications of the CAPM (developed by Sharpe [15],
Lintner [9] and Mossin [12]) for the capital budgeting and capital structure
decisions of a value maximizing firm.¹ The model has been extended by Chen and
Boness [3] to analyze the effects of uncertain inflation and by Adler and Dumas
[1] to study optimal international acquisitions.

This paper, with the aid of a two-country model, attempts to investigate
the effects of exchange risk on the investment and financing decisions of
multinational firms. Recent work suggests that international capital markets
are efficient though the evidence is far from being conclusive (see Agmon [2],
Farber [6], Jacquillant and Solnik [8] and Solnik [16]). In view of this, the
paper assumes that the capital markets of the two countries are not segmented.
We assume that the investors in each country can freely invest in the bond and
stock markets of both countries.

The paper consists of five parts I through V. Section I develops an
equilibrium model for pricing securities in the presence of exchange risk. It
is shown that in this case the "beta" consists of two terms; one involving the
covariance of the security with the "world market portfolio" and another term
to account for the exchange risk. An interesting result is that the effect of
the exchange risk depends on the net investment position of a country (i.e.,
whether it is a surplus or a deficit country).

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¹For a discussion of the appropriateness of the value-maximizing principle,
see Fama and Miller [5]. For a further discussion in the context of incomplete
markets, see papers by Leland and by Ekern and Wilson in the Symposium on
Capital Markets [18].
Section II establishes a relationship between the interest rates in the two countries and the exchange rate. It is shown that the forward rate is a biased estimator of the future expected spot rate. Sections III and IV deal with investment and financing decisions. Criteria are developed for evaluating both domestic and foreign investments. An implication of the model is that failure to account for exchange risk can lead to a systematic bias in the investments accepted. MM propositions 1 and 2 [10], [11] are shown to hold even in the presence of exchange risk. Finally Section V discusses international mergers. In the case where investors in each country can freely invest in both countries, the irrelevance proposition is known to hold (Rubinstein [14]).

I. Asset Pricing under Uncertain Exchange Rates

We make the following assumptions:
1. Perfect markets [see Fama [4]];
2. Homogeneous expectations;
3. Investors rational in the von Neumann-Morgenstern sense;
4. Investors with quadratic utility of terminal wealth;\(^2\)
5. Riskless debt in both countries;
6. No stochastic inflation in both countries.\(^3\)

Notation: There are two countries A and B. A is the domestic country and B is the foreign country. The notation for country A is presented below the notation for country B is symmetric with A replaced by B. The domestic currency is \(\mathcal{F}\) and the foreign currency is \(\mathcal{B}\).

\[ S^A: \text{the vector of market values of the stock of country A firms.} \]
\[ S^A = \{S^A_j\}, \quad j = 1, 2, \ldots, N^A. \]

\(^2\)The use of quadratic utility is not particularly restrictive since any result derived under the assumption of quadratic utility can also be derived in a continuous time framework. It is well known that a quadratic utility function may result in negative marginal utility of wealth. In the analysis to follow the domain of the utility function is limited such that \(EU' > 0\), to avoid the objectional property listed above.

\(^3\)In this case as Adler and Dumas [1] point out "exchange rate can remain stochastic only if we assume stochastic tariffs or transportation costs . . . or if we postulate some short-term lack of adjustment of international prices to parity. That latter circumstance, which is admittedly a disequilibrium one, is very realistic since we can expect the flows of goods to react to exchange rates and prices less rapidly than flows of capital."
$Y^A$: the matrix of fractional holdings of the stock of country A firms by country A investors.

$$Y^A = [y^A_{ij}] \quad i = 1, 2, \ldots, N^A$$
$$j = 1, 2, \ldots, M^*.$$

$z^A$: the matrix of fractional holdings of the stock of country B firms by country A investors.

$$z^A = [z^A_{ij}] \quad i = 1, 2, \ldots, N^A$$
$$j = 1, 2, \ldots, M^B.$$

$B^A$: the vector of market value of the bonds of country A firms.

$$B^A = \{B^A_j\} \quad j = 1, 2, \ldots, M^A.$$

$P^A$: the matrix of fractional holdings of the bonds of country A firms by country A investors.

$$P^A = [p^A_{ij}] \quad i = 1, 2, \ldots, N^A$$
$$j = 1, 2, \ldots, M^A.$$

$Q^A$: the matrix of fractional holdings of the bonds of country B firms by country A investors.

$$Q^A = [q^A_{ij}] \quad i = 1, 2, \ldots, N^A$$
$$j = 1, 2, \ldots, M^B.$$

$\tilde{R}^A$: the vector of the return on the stocks of country A firms.

$$\tilde{R}^A = \{\tilde{R}^A_j\} \quad j = 1, 2, \ldots, M^A.$$

$\tilde{R}_e$: the return on a position in foreign exchange (for an investor in country A).

$$\tilde{R}_e = \frac{\tilde{s} - s^O}{s^O} \quad s^O: \text{is the current exchange rate } \tilde{E}/\tilde{E}^*$$
$$\tilde{s}_1: \text{is the exchange rate at the end of the period } (\tilde{E}/\tilde{E}^*).$$

229
r^A_A: risk-free rate in country A.

\( \mathbf{w}^A \): the vector of initial wealth of investors in country A.

\[
\mathbf{w}^A_i = \{w^A_i\}, \quad i = 1, 2, \ldots, N^A.
\]

The budget constraint for investor \( i \) of country A is

(1)

\[
\frac{\mathbf{w}^A_i}{s^A_o} = \sum_j y_{ij}^A s_{ij}^A + \sum_j z_{ij}^A s_{ij}^B + \sum_j p_{ij}^A b_{ij}^A + \sum_j q_{ij}^A b_{ij}^B.
\]

The first and third terms represent the investor's holding of stocks and bonds in country A while the second and fourth terms represent his investment in the foreign country. [Note: We assume that the current exchange rate is 1.]

The investor's end-of-period wealth\(^4\) is

(2)

\[
\frac{z^A_i}{s^A_o} = \sum_j y_{ij}^A s_{ij}^A (1 + \gamma^A_j) + \sum_j z_{ij}^A s_{ij}^B (1 + \gamma^B_j + \gamma^e) + \sum_j p_{ij}^A b_{ij}^A (1 + r_A^B) + \sum_j q_{ij}^A b_{ij}^B (1 + r_B^A + \gamma^e).
\]

The budget constraint for an investor in country B is

(3)

\[
\frac{w^B_i}{s^B_o} = \sum_j y_{ij}^B s_{ij}^B + \sum_j z_{ij}^B s_{ij}^A + \sum_j p_{ij}^B b_{ij}^B + \sum_j q_{ij}^B b_{ij}^A
\]

\(^4\)If \( \$1 \) is invested in security \( j \) of country B, the return is

\[
\frac{1}{s^B_o} (1 + \gamma^B_j) s_{ij}^B = (1 + \gamma^B_j) (1 + \gamma^e) \approx 1 + \gamma^B_j + \gamma^e
\]

Note that \( \gamma^e > 0 \) implies the currency of Country A (\( \$ \)) has depreciated.

Although we do not have an explicit forward market, a forward transaction is equivalent to borrowing in one country and lending in the second. The model allows for this type of transaction.
and his end-of-period wealth is

\[
\tilde{Z}_i = \sum_j y_{ij} S_j^A (1 + R_j^B) + \sum_j z_{ij} S_j^A (1 + R_j^A - \tilde{r}_e)
+ \sum_j p_{ij} S_j^A (1 + r_B) + \sum_j q_{ij} S_j^A (1 + r_A - \tilde{r}_e).
\]

(4)

Since the individuals are assumed to obey the von Neumann-Morgenstern axioms, the problem facing a typical country A investor is

\[
\max_{y_{ij}, z_{ij}, p_{ij}, q_{ij}} \{ EU_i(\tilde{Z}_i) \}
\]

(5)

subject to equation (1).

The first-order conditions are

\[
E[U_i'(\tilde{Z}_i)](1 + R_j^A)S_j^A = -\lambda S_j^A
\]

(6)

\[
E[U_i'(\tilde{Z}_i)](1 + R_j^A + \tilde{r}_e)S_j^B = -\lambda S_j^B
\]

(7)

\[
E[U_i'(\tilde{Z}_i)](1 + r_A)B_j^A = -\lambda B_j^A
\]

(8)

\[
E[U_i'(\tilde{Z}_i)](1 + r_B + \tilde{r}_e)B_j^B = -\lambda B_j^B
\]

(9)

(\(\lambda\) is a Lagrange multiplier.)

From (6) and (8) on simplification we get

\[
E[U_i'(\tilde{Z}_i)](R_j^A - r_A) = 0
\]

(10)

and from (7) and (9) we get

\[
E[U_i'(\tilde{Z}_i)](R_j^B - r_B) = 0.
\]

(11)

\(\text{If} x_i \text{ is invested in security } j \text{ of country } A, \text{ the return is}
\]

\[
\frac{S_0}{S_1} \left(1 + \tilde{r}_j^A \right) = \frac{1 + \tilde{r}_j^A}{1 + \tilde{r}_e} = 1 + \tilde{r}_j^A - \tilde{r}_e.
\]
(10) implies

\[ (12) \quad E(U_i'(\tilde{Z}_i^A)) \cdot (\tilde{R}_j^A - r_A) + \text{Cov}(U_i', \tilde{R}_j^A) = 0 \]

where \( \tilde{R}_j^A = E(\tilde{R}_j^A) \).

Since we have assumed quadratic utility of the form

\[ U(Z) = \tilde{Z} - c\tilde{Z}^2 \]

we have

\[ (13) \quad U_i'(\tilde{Z}_i^A) = 1 - 2c_i\tilde{Z}_i^A. \]

Therefore

\[ (14) \quad E(U_i'(\tilde{Z}_i^A)) \cdot (\tilde{R}_j^A - r_A) - 2c_i\tilde{Z}_i^A \text{Cov}(\tilde{Z}_i^A, \tilde{R}_j^A) = 0 \]

or

\[ (15) \quad (\tilde{R}_j^A - r_A) \left[ \frac{E(U_i')}{{\tilde{Z}_i^A}} \right] = \text{Cov}(\tilde{Z}_i^A, \tilde{R}_j^A) \quad i = 1, 2, \ldots, N^A \]

\[ j = 1, 2, \ldots, M^A. \]

Similarly from (11) one can get

\[ (16) \quad (\tilde{R}_j^B - r_B) \left[ \frac{E(U_i')}{{\tilde{Z}_i^B}} \right] = \text{Cov}(\tilde{Z}_i^A, \tilde{R}_j^B) \quad i = 1, 2, \ldots, N^A \]

\[ j = 1, 2, \ldots, M^B. \]

Solving the optimization problem for investor \( i \) of country \( B \), we can get conditions similar to (15) and (16). These are

\[ (17) \quad (\tilde{R}_j^A - r_A) \left[ \frac{E(U_i')}{{\tilde{Z}_i^A}} \right] = \text{Cov}(\tilde{Z}_i^B, \tilde{R}_j^A) \quad i = 1, 2, \ldots, N^B \]

\[ j = 1, 2, \ldots, M^A. \]

\[ (18) \quad (\tilde{R}_j^B - r_B) \left[ \frac{E(U_i')}{{\tilde{Z}_i^B}} \right] = \text{Cov}(\tilde{Z}_i^B, \tilde{R}_j^B) \quad i = 1, 2, \ldots, N^B \]

\[ j = 1, 2, \ldots, M^B. \]

We can sum equations (15)-(18) over all individuals to get
(19) \( (\bar{r}_j^A - r_A) = \lambda^A \text{Cov}(\bar{z}_j^A, \bar{r}_j^A) \)

(20) \( (\bar{r}_j^B - r_B) = \lambda^B \text{Cov}(\bar{z}_j^A, \bar{r}_j^B) \)

(21) \( (\bar{r}_j^A - r_A) = \lambda^B \text{Cov}(\bar{z}_j^B, \bar{r}_j^A) \)

(22) \( (\bar{r}_j^B - r_B) = \lambda^B \text{Cov}(\bar{z}_j^B, \bar{r}_j^B) \)

(23) \( \frac{1}{\lambda^A} = \sum_{i=1}^{N^A} \frac{1}{2c_i} \quad \frac{1}{\lambda^B} = \sum_{i=1}^{N^B} \frac{1}{2c_i} \)

where \( \bar{z}_i^A = \sum \bar{z}_i^A \), etc.

Equation (19) can be rewritten as

\[
(\bar{r}_j^A - r_A)/\lambda^A = \text{Cov}(\sum_{k} y_{ik}^A \bar{s}_j^A, \bar{r}_j^A) + \text{Cov}(\sum_{k} z_{ik}^A \bar{s}_j^A, \bar{r}_j^A) \\
+ \text{Cov}(\sum_{k} z_{ik}^B \bar{s}_j^B, \bar{r}_j^A) + \text{Cov}(\sum_{k} q_{ik}^B \bar{s}_j^B, \bar{r}_j^A)
\]

and (21) as

\[
(\bar{r}_j^A - r_A)/\lambda^B = \text{Cov}(\sum_{k} y_{ik}^B \bar{s}_j^B, \bar{r}_j^A) + \text{Cov}(\sum_{k} z_{ik}^B \bar{s}_j^B, \bar{r}_j^A) \\
- \text{Cov}(\sum_{k} z_{ik}^A \bar{s}_j^A, \bar{r}_j^B) - \text{Cov}(\sum_{k} q_{ik}^A \bar{s}_j^A, \bar{r}_j^B)
\]

where

\( y_{ik}^A = \sum_{i} y_{ik}^A, \quad q_{ik}^A = \sum_{i} q_{ik}^A, \quad z_{ik}^A = \sum_{i} z_{ik}^A \), etc.

Since all securities must be held by investors of countries A and B, we have the following closure conditions.

\( y_{ik}^A + z_{ik}^B = 1 \quad k = 1, 2, \ldots, M^A \)

\( y_{ik}^B + z_{ik}^A = 1 \quad k = 1, 2, \ldots, M^B \).

---

\( \frac{1}{\lambda^A} \) can be interpreted as \( \frac{1}{\lambda} \) \( E(\frac{1}{R_i^A}) \) where \( R_i^A = \frac{U''}{U'} \) is the Arrow-Pratt coefficient of absolute risk aversion for individual i.

233
Adding (24) and (26) and making use of the closure conditions (27) and (28), we get

$$(29) \quad (R_j^A - r_A^j)(1/\lambda^A + 1/\lambda^B) = \text{Cov}(\sum_k S_k^A, R_j^A) + \text{Cov}(\sum_k S_k^B, R_j^A) + (\sum_k S_k^A - \sum_k S_k^B)\text{Cov}(e_j^A, R_j^A) + (\sum_k q_k^B - \sum_k q_k^A)\text{Cov}(e_j^B, R_j^A).$$

Define

$$R_m^A = \sum_k S_k^A \quad \text{and} \quad R_m^B = \sum_k S_k^B$$

as the return on the market in country A.

Note that $\sum_k S_k^A$ is the total stock of country B held by investors of country A.

Therefore

$$\sum_k S_k^A - \sum_k S_k^B$$

is the net investment by country A in country B's stocks.

Let

$$\Delta S^A = \sum_k S_k^A - \sum_k S_k^B \quad (= -\Delta S^B).$$

Similarly let

$$\Delta B^A = \sum_k q_k^B - \sum_k q_k^A \quad (= -\Delta B^B)$$

be the net investment by country A in country B's bonds. Therefore we get

$$(30) \quad (R_j^A - r_A^j)(1/\lambda^A + 1/\lambda^B) = S_m^A \text{Cov}(R_j^A, R_m^A) + S_m^B \text{Cov}(R_j^A, R_m^B) + (\Delta S^A + \Delta B^A)\text{Cov}(e_j^A, R_j^A).$$

Define

$$\tilde{R}_{AB}^m = \frac{S_m^A + S_m^B}{S_m^A + S_m^B}$$

as the return on the market in the currency of country A.
as the return on the "world market"

\[ \frac{1}{\lambda^A} = \frac{1}{\lambda^A} + \frac{1}{\lambda^B} \quad S_{AB} = S^A + S^B \]

\[ \Delta S^A = \Delta S^A + \Delta B^A = -\Delta S^B \]

as the net investment by country A in country B (in both stocks and bonds). Therefore

\[ (31)^7 \quad \tilde{R}_j^A = r_A + \lambda^{AB} [S^{AB} \text{Cov}(\tilde{R}_j^A, \tilde{R}_m^B) + \Delta S^A \text{Cov}(\tilde{R}_j^A, \tilde{R}_e^B)] \]

The risk-return relationship is

\[ (32) \quad \tilde{R}_j^A = r_A + \lambda^{AB} \beta_j^A \]

where

\[ (33) \quad \beta_j^A = S^{AB} \text{Cov}(\tilde{R}_j^A, \tilde{R}_m^B) + \Delta S^A \text{Cov}(\tilde{R}_j^A, \tilde{R}_e^B) \]

Equation (33) shows that the risk of security j can be decomposed into two parts: a) the covariance of the security with the "world market portfolio," and b) the covariance of the security with the return on a position in foreign exchange. It is interesting to note that, if the net foreign investment is zero (i.e., \( \Delta S^A = 0 \)), then the exchange risk vanishes.

From equation (30) it is clear that if the value of one particular stock market is much larger than the other (e.g., \( S^A >> S^B \)), then the security will be priced on the larger market. So if the value of the NYSE >> than that of the other exchanges, it is reasonable to assume that the securities are priced on the NYSE. (This lends some support to the studies testing the CAPM on the NYSE.)

The contribution of the exchange risk depends on the net investment position of the country and the covariance of the firm's return with \( \tilde{R}_e^B \). If for country A, \( \Delta S^A \) is positive, the Sharpe-Lintner-Mossin (SLM) CAPM will overstate (understate) the risk of a firm if its return is negatively (positively) correlated with \( \tilde{R}_e^B \). Since a negative correlation corresponds to the case where a

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\(^7\text{For a firm in Country B we have}\)

\[ \tilde{R}_j^B = r_B + \lambda^{AB} [S^{AB} \text{Cov}(\tilde{R}_j^B, \tilde{R}_m^B) + \Delta S^B \text{Cov}(\tilde{R}_j^B, \tilde{R}_e^B)] \]
firms' return is expected to increase due to a revaluation by Country A (devaluation by Country B), the SLM-CAPM, for instance overstates the risk of a firm if its return is expected to increase due to a revaluation by Country A.  

The tables below show the cases in which the SLM-CAPM understates [-] or overstates [+] the risk of Country A and B firms.

<table>
<thead>
<tr>
<th>COUNTRY A FIRMS</th>
<th>Firms whose returns are likely to increase (decrease) due to a devaluation (revaluation) by country A, i.e., COV(\tilde{R}_j, \tilde{R}_e) &gt; 0</th>
<th>Firms whose returns are likely to decrease (increase) due to a devaluation (revaluation) by country A, i.e., COV(\tilde{R}_j, \tilde{R}_e) &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country A surplus, i.e., \Delta SB^A &gt; 0</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Country A deficit, i.e., \Delta SB^A &lt; 0</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

It should be emphasized that in carrying out the comparison the same "market" is used in both models. The SLM-CAPM in this case would be

\[
\tilde{R}_j^A = r_A + \lambda^A [s^A AB \text{ Cov}(\tilde{R}_j^A, \tilde{R}_m^A)]
\]

and not the one in the single country case, i.e.,

\[
\tilde{R}_j^A = r_A + \lambda^A [s^A \text{ Cov}(\tilde{R}_j^A, \tilde{R}_m^A)].
\]
COUNTRY B FIRMS

<table>
<thead>
<tr>
<th>Firms whose returns are likely to decrease (increase) due to a devaluation (revaluation) by country B, i.e.,</th>
<th>Firms whose returns are likely to increase (decrease) due to a devaluation (revaluation) by country B, i.e.,</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{COV}(\tilde{R}_j^B, \tilde{R}_e) &gt; 0$</td>
<td>$\text{COV}(\tilde{R}_j^B, \tilde{R}_e) &lt; 0$</td>
</tr>
</tbody>
</table>

Country B surplus
i.e.,
$\Delta SB^B > 0$

Country B deficit
i.e.,
$\Delta SB^B < 0$

II. Equilibrium in the Bond Market

From equations (8) and (9) we get

$$E[U'(Z^A)](r_A - r_B - \tilde{R}_e) = 0.$$  

(34)

After some straightforward simplification and summing over all investors in country A, we get

$$(-r_A + r_B + \tilde{R}_e)/\lambda^A = \text{Cov}(Z^A, \tilde{R}_e).$$  

(35)

Similarly one can get an equation for investors in country B,

$$(r_B - r_A + \tilde{R}_e)/\lambda^B = \text{Cov}(Z^B, \tilde{R}_e).$$  

(36)

Therefore

$$r_B - r_A + \tilde{R}_e (1/\lambda^A + 1/\lambda^B) = \text{Cov}(\tilde{Z}^A + \tilde{Z}^B, \tilde{R}_e)$$  

(37)

which on simplification gives

$$\tilde{R}_e = r_A - r_B + \lambda^{AB} [S^{AB} \text{Cov}(\tilde{R}_e, \tilde{R}_m^{AB}) + \Delta SB^A \sigma^2 (\tilde{R}_e)].$$  

(38)

237
From the interest rate parity theorem, we know that

\[
\frac{F - s_o}{s_o} = r_A - r_B
\]

where \(F\) is the forward rate at the beginning of the period (\(S/F\)).

Since

\[
\bar{r}_e = \frac{s_1 - s_o}{s_o},
\]

we get

\[
\frac{s_1}{s_o} = F + s_o \lambda^{AB} [s^{AB} \text{Cov}(\bar{R}^e, \bar{R}^m) + \Delta S^A \sigma^2(\bar{R}^e)]
\]

which shows that the forward rate is a biased estimator of the expected future spot rate, a fact observed by Solnik [16]. Equation (38) gives an equilibrium relationship between \(\bar{r}_e\), \(r_A\), and \(r_B\). It should, however, be clear that though intuitively appealing \(\bar{r}_e\) cannot be replaced by \(r_A - r_B\).

III. Asset Expansion

Rubinstein [14] has shown that a firm should accept a project if

\[
E(R^0) > r + \lambda \text{Cov}(R^0, R_m)
\]

where \(R^0\) is the rate of return of the project. In an international setting this is easily translated to the condition that a project in country A should be accepted if and only if

\[
E(\bar{R}^A) > r_A + \lambda^{AB} [s^{AB} \text{Cov}(\bar{R}^A, \bar{R}^m) + \Delta S^A \text{Cov}(\bar{R}^A, \bar{R}^e)]
\]

or

\[
\frac{E(\bar{R}^A) - r_A}{s^{AB} \text{Cov}(\bar{R}^A, \bar{R}^m) + \Delta S^A \text{Cov}(\bar{R}^A, \bar{R}^e)} > \lambda^{AB}.
\]

So in accepting a project, the relevant factors are its covariance with the world market portfolio and its covariance with the exchange rate. The effect of the latter depends on the net investment position of the country (i.e., \(\Delta S^A > 0\) or \(< 0\)). If \(\Delta S^A\) is > 0, a project whose returns decrease due to a devaluation (by A) would be "preferred" in the sense that it would have a lower

\[9\text{Assuming the denominator is positive.}\]
cost of capital. For a country like Canada which has $\Delta SB^B < 0$, a "preferred" project would be one whose returns increase due to a domestic devaluation. So in a deficit country, even if two projects have the same covariance with the market, the one that is favored by a devaluation will have a lower cost of capital.

The above analysis suggests that, if the effects of exchange risk are not considered explicitly in capital budgeting decisions, a systematic bias will develop. In the case of a deficit country the project acceptance criteria would be biased against devaluation favored projects, resulting in an economy-wide under investment in projects aided by a devaluation.

**Foreign Investments**

Consider the case where a firm in country A is evaluating an investment in country B whose rate of return in country B is $\tilde{R}^B$. However, viewed from country A, the return on this investment is $\tilde{R}^B + R_e$. The project should, therefore, be accepted if

$$\tilde{R}^B + R_e > r_A + \lambda^{AB} [S^{AB} \text{Cov}(R^B, R_m) + \Delta SB^A \text{Cov}(\tilde{R}^B, R_e)]$$

$$+ \lambda^{AB} [S^{AB} \text{Cov}(R_e, R_m) + \Delta SB^A \sigma(\tilde{R}_e)].$$

This can be simplified using equation (38) to give

$$\tilde{R}^B > r_B + \lambda^{AB} [S^{AB} \text{Cov}(\tilde{R}^B, R_m) + \Delta SB^A \text{Cov}(\tilde{R}^B, R_e)]$$

which is the acceptance criteria a country B firm would have used. In other words, a country A firm evaluating a project in country B should use the same acceptance criteria as a country B firm. (See also Section V.)

**IV. Financing Decision**

In this section we examine the effect of exchange rate fluctuation on the financing decision of firms. [It should be clear that except for the exchange rate risk the form of the International CAPM (ICAPM) we have derived is similar to the SLM-CAPM. Indeed if we ignore the term containing $R_e$, we get

$$E(R_j^A) = r_A + \lambda^{AB} S^{AB} \text{Cov}(R_j^A, R_m^A)$$

which is similar to the asset pricing model derived by Solnik [16] in continuous time and to the SLM-CAPM.]
Let $\bar{X}_j^A$ be the value of the net operating income of the $j^{th}$ firm in country A.

$$\bar{R}_j^A = \frac{X_j^A - r_{A,j}^A}{S_j^A}.$$ 

Let $u$ stand for an unlevered firm, i.e.,

$$\bar{R}_j^A = \frac{X_j^{Au}}{S_j^{Au}}, \quad r_j^{Au} = s_j^{Au}.$$ 

Substituting for $\bar{R}_j^A$ in equation (31) we get

$$E(X_j^A) = r_j^A v_j^A + \lambda^{AB} [s_j^{AB} \text{Cov}(\bar{X}_j^A, \bar{R}_m^A) + \Delta S_B A \text{Cov}(\bar{X}_j^A, \bar{R}_e)].$$

For the pure equity firm we get

$$E(X_j^{Au}) = r_j^{Au} v_j^{Au} + \lambda^{AB} [s_j^{AB} \text{Cov}(\bar{X}_j^{Au}, \bar{R}_m^{Au}) + \Delta S_B A \text{Cov}(\bar{X}_j^{Au}, \bar{R}_e)].$$

Therefore if $X_j^A = X_j^{Au}$ it follows that

$$V_j^A = V_j^{Au}.$$ 

As a firm alters its capital structure even in the presence of exchange risk, its value does not change. So the MM [10] theorem on capital structure holds even in the presence of exchange risk.

Following Chen and Boness [3] we can derive MM's second theorem using the ICAPM.

By definition

$$\bar{R}_j^A = \bar{R}_j^{Au} (1 + k_j^A) - r_A k_j^A$$

where $k_j^A = p_j^A / s_j^A$ is the debt equity ratio or

$$\bar{R}_j^A = \bar{R}_j^{Au} (1 + k_j^A) - r_A k_j^A.$$ 

Therefore

$$\bar{R}_j^A = r_A + (1 + k_j^A) \lambda^{AB} [s_j^{AB} \text{Cov}(\bar{R}_j^A, \bar{R}_m^A) + \Delta S_B A \text{Cov}(\bar{R}_j^A, \bar{R}_e)].$$

or

240
\( \tilde{R}_j^A = r_A + \theta + \phi \)

\[
\begin{align*}
\theta &= \lambda \sigma(R_j^\mu) \sigma(R_m^\mu) + \Delta SB^A \sigma(R_e^\mu)
\intertext{and}
\phi &= \lambda \sigma(R_j^\mu) \sigma(R_m^\mu) + \Delta SB^A \sigma(R_e^\mu) \kappa_j^A.
\end{align*}
\]

\( \rho_{jm} \) is the correlation coefficient between \( R_j^\mu \) and \( R_m^\mu \). \( \rho_{je} \) is similarly defined. \( \theta \) is the premium for the business risk of the firm and \( \phi \) is the premium for the financial risk (including exchange risk) of the firm.

**Financing Decision with Corporate Taxes**

The after-tax rate of return on the equity of an unlevered firm is

\[
\tilde{u}_{jA}^\mu = \frac{X_j^A (1-\tau)}{V_j^A}
\]

and on the levered firm it is

\[
\tilde{r}_{jA}^\mu = \frac{(X_j^A - r_{B_j}^A)(1-\tau)}{S_j^A}.
\]

For a pure equity firm we have

\[
E(\tilde{x}_{jA}^\mu) (1-\tau) = r_A [V_j^A] + \lambda \sigma(R_j^\mu) \sigma(R_m^\mu) + \Delta SB^A \sigma(R_e^\mu)
\]

and for the levered firm

\[
E(\tilde{x}_{jA}^\mu) (1-\tau) = r_A [S_j^A + B_j (1-\tau)] + \lambda \sigma(R_j^\mu) \sigma(R_m^\mu)
\]

\[
+ \Delta SB^A \sigma(R_e^\mu).
\]

If \( X_j^A = \tilde{x}_{jA}^\mu \), then (48) and (49) imply that

\[
V_j^A = V_j^A + \tau B_j^A
\]

which is precisely MM's [11] result with taxes.
V. International Mergers

In this section we examine the effect on the market value of a country A firm when it takes over a country B firm.

Consider two firms, one in country A (firm a) and the other in country B (firm b). The values of the two firms (in country A) can be determined as follows.

For firm a

\[
1 + \frac{\bar{r}_A}{r_A} = \frac{\bar{X}_a + S_a - r_A B_a}{s_a}. \tag{50}
\]

Therefore, using equation (31)

\[
\nu_a = \frac{\bar{X}_a - \lambda \frac{S_a}{r_A} \text{Cov}(\bar{X}_a, \bar{R}_m) + \Delta S_A \text{Cov}(\bar{X}_a, \bar{R}_e)}{r_A}. \tag{51}
\]

The return from firm b as viewed in country A is

\[
1 + \frac{\bar{r}_B}{r_B} = \frac{s_B (\bar{X}_b + \bar{X}_b - r_B B_b)}{S_b}. \tag{52}
\]

This can be written as (see footnote 4)

\[
1 + \frac{\bar{r}_B}{r_B} = \frac{S_b + \bar{X}_b - r_B B_b + r S_b}{S_b}. \tag{53}
\]

Using equations (31) and (38) we get

\[
\nu_b = \frac{\bar{X}_b - \lambda \frac{S_a}{r_A} \text{Cov}(\bar{X}_b, \bar{R}_m) + \Delta S_B \text{Cov}(\bar{X}_b, \bar{R}_e)}{r_B}. \tag{54}
\]

Therefore

\[
\nu_a + \nu_b = \frac{\bar{X}_a + \bar{X}_b - \lambda \frac{S_a}{r_A} \text{Cov}(\bar{X}_a, \bar{R}_m) + \Delta S_B \text{Cov}(\bar{X}_b, \bar{R}_e)}{r_A + \frac{\bar{X}_b - r_B B_b}{r_B}}. \tag{55}
\]

Now consider a situation where firm a takes over firm b, i.e., B becomes a foreign subsidiary of firm A. Let c denote firm a after the takeover. Therefore we have

\[
1 + \frac{\bar{r}_c}{r_c} = \frac{\bar{X}_a + \bar{X}_b - r_A a_A + s_B \left( S_B + \bar{X}_b - r_B B_b \right)}{S_a + S_B}. \tag{56}
\]

\[
\bar{r}_c = \frac{\bar{X}_a - r_A a_A + \bar{X}_b + r_B B_b + s_B}{S_a + S_B}. \tag{57}
\]
Once again using equations (31) and (38) one can get

\[(58) \quad V^A_c + V^B_b(r_B - r_A) = \bar{X}^A_c - \lambda^{AB} [S^A_{AB} \text{Cov}(\bar{X}^A, R^B_m) + \Delta S^B \text{Cov}(\bar{X}^A_c, r_e)]\]

where

\[\begin{align*}
\bar{X}^A_a + \bar{X}^B_b &= \bar{X}^A_c \\
S^A_a + S^B_b &= S^A_c \\
B^A_a + B^B_b &= B^A_c \\
S^A_c + B^A &= V^A_c.
\end{align*}\]

Substituting for \(V^B_b\) from equation (54) in equation (58) we get

\[(59) \quad V^A_c = \frac{\bar{X}^A}{r_A} + \frac{\bar{X}^B}{r_B} - \lambda^{AB} [S^A_{AB} \text{Cov}(\frac{\bar{X}^A}{r_A}, \frac{\bar{X}^B}{r_B}, r^B_m) + \Delta S^B \text{Cov}(\frac{\bar{X}^A}{r_A}, \frac{\bar{X}^B}{r_B}, r_e)].\]

Comparing (59) and (55) we see that

\[V^A_c = V^A_a + V^B_b.\]

The result illustrates that, even in the presence of exchange risk, there is no incentive for firms to merge if individuals can freely invest in both capital markets. This is in contrast to the results of Adler and Dumas [1] where they show that for segmented markets the irrelevance proposition does not hold and the firm in general does have an optimal acquisition decision.

VI. Conclusions

This paper, with the aid of a two-country model, focuses on the effects of exchange risk on the various decisions facing a firm. It is an attempt at synthesizing corporate financial theory in an international setting and derives several normative implications of the model developed in Section I for a value-maximizing firm.
REFERENCES


