JUNIOR CAN'T BORROW: A NEW PERSPECTIVE ON THE EQUITY PREMIUM PUZZLE*

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Ongoing questions on the historical mean and standard deviation of the return on equities and bonds and on the equilibrium demand for these securities are addressed in the context of a stationary, overlapping-generations economy in which consumers are subject to a borrowing constraint. The key feature captured by the OLG economy is that the bulk of the *future* income of the young consumers is derived from their wages forthcoming in their middle age, while the bulk of the *future* income of the middle-aged consumers is derived from their savings in equity and bonds. The young would like to borrow and invest in equity but the borrowing constraint prevents them from doing so. The middle-aged choose to hold a diversified portfolio that includes positive holdings of bonds, and this explains the demand for bonds. Without the borrowing constraint, the young borrow and invest in equity, thereby decreasing the mean equity premium and increasing the rate of interest.

I. INTRODUCTION

The question as to why the historical equity premium is so high and the real rate of interest is so low was addressed in Mehra and Prescott [1985]. They demonstrated that the equilibrium of a reasonably parameterized, representative-consumer exchange economy is able to furnish a mean annual premium of equity return over the riskless rate of, at most, 0.35 percent, in contrast to its historical level of 6 percent in U. S. data. Furthermore, the equilibrium annual riskless rate of interest is consistently too high, about 4 percent, as opposed to the observed 1 percent in U. S. data.[1] Further, in econometric tests, the condi-

1. This point is emphasized in Weil [1989].

tional Euler equation of per capita consumption is also rejected by Hansen and Singleton [1982], Hansen and Jagannathan [1991], Ferson and Constantinides [1991], and others.

Several generalizations of key features of the Mehra and Prescott [1985] model have been proposed to better reconcile observations with theory. These include alternative assumptions on preferences,[2] modified probability distributions to admit rare but disastrous events,[3] incomplete markets,[4] and market imperfections;[5] none have fully resolved the anomalies. Cochrane and Hansen [1992] and Kocherlakota [1996] provide excellent surveys of this literature.

The novelty of this paper lies in incorporating a life-cycle feature to study asset pricing. The idea is appealingly simple. The attractiveness of equity as an asset depends on the correlation between consumption and equity income. If equity pays off in states of high marginal utility of consumption, it will command a higher price (and consequently a lower rate of return) than if its payoff is in states where marginal utility is low. Since the marginal utility of consumption varies inversely with consumption, equity will command a high rate of return if it pays off in states when consumption is high, and vice versa.[6]

A key insight of our paper is that as the correlation of equity income with consumption *changes* over the life cycle of an individual, so does the attractiveness of equity as an asset. Consump-

2. For example, Abel [1990], Benartzi and Thaler [1995], Boldrin, Christiano, and Fisher [2001], Campbell and Cochrane [1999], Constantinides [1990], Daniel and Marshall [1997], Epstein and Zin [1991], and Ferson and Constantinides [1991].
3. See, Reitz [1988] and Mehra and Prescott [1988].
4. For example, Bewley [1982], Constantinides and Duffie [1996], Detemple and Serrat [1996], Heaton and Lucas [1997, 2000], Krusell and Smith [1998], Lucas [1994], Mankiw [1986], Marcet and Singleton [1999], Mehra and Prescott [1985], Storesletten, Telmer, and Yaron [1999], and Telmer [1993]. Empirical papers that investigate the role of incomplete markets on asset prices include Brav, Constantinides, and Geczy [2002], Cogley [1999], Jacobs [1999], and Vissing-Jorgensen [2002].
5. For example, Aiyagari and Gertler [1991], Alvarez and Jermann [2000], Bansal and Coleman [1996], Basak and Cuoco [1998], Brav and Geczy [1995], Danthine, Donaldson, and Mehra [1992], He and Modest [1995], Heaton and Lucas [1996], Luttmer [1996], McGrattan and Prescott [2000, 2001], and Storesletten, Telmer, and Yaron [1999]. Empirical papers that investigate the role of *limited participation,* as a manifestation of market imperfections, on asset prices include Attanasio, Banks, and Tanner [2002], Brav and Geczy [1995], Brav, Constantinides, and Geczy [2002], Cogley [1999], Jacobs [1999], Mankiw and Zeldes [1991], and Vissing-Jorgensen [2002].
6. This is precisely the reason why high-beta stocks in the simple CAPM framework have a high rate of return. In that model, the return on the market is a proxy for consumption. High-beta stocks pay off when the market return is high, i.e., when marginal utility is low, hence their price is (relatively) low and their rate of return high.

tion can be decomposed into the sum of wages and equity income. A young person looking forward in his life has uncertain future wage *and* equity income; furthermore, the correlation of equity income with consumption will not be particularly high, so long as stock and wage income are not highly correlated. This is empirically the case, as documented by Davis and Willen [2000]. Equity will thus be a hedge against fluctuations in wages and a "desirable" asset to hold as far as the young are concerned.

The same asset (equity) has a very different characteristic for the middle-aged. Their wage uncertainty has largely been resolved. Their future retirement wage income is either zero or deterministic, and the innovations (fluctuations) in their consumption occur from fluctuations in equity income. At this stage of the life cycle, equity income is highly correlated with consumption. Consumption is high when equity income is high, and equity is no longer a hedge against fluctuations in consumption; hence, for this group, it requires a higher rate of return.

The characteristics of equity as an asset therefore change, depending on who the predominant holder of the equity is. Life-cycle considerations thus become crucial for asset pricing. If equity is a "desirable" asset for the marginal investor in the economy, then the observed equity premium will be low, relative to an economy where the marginal investor finds it unattractive to hold equity. The deus ex machina is the *stage* in the life cycle of the marginal investor.

In this paper we argue that the young, who should be holding equity in an economy without frictions and with complete contracting, are effectively shut out of this market because of borrowing constraints. They are characterized by low wages; ideally, they would like to smooth lifetime consumption by borrowing against future wage income (consuming a part of the loan and investing the rest in higher return equity). However, as is well recognized, they are prevented from doing so because human capital alone does not collateralize major loans in modern economies for reasons of moral hazard and adverse selection.

In the presence of borrowing constraints, equity is thus exclusively priced by the middle-aged investors since the young are effectively excluded from the equity markets and we observe a high equity premium. If the borrowing constraint is relaxed, the young will borrow to purchase equity, thereby raising the bond yield. The increase in the bond yield induces the middle-aged to shift their portfolio holdings from equity to bonds. The increase in

the demand for equity by the young and the decrease in the demand for equity by the middle-aged work in opposite directions. On balance, the effect is to increase both the equity and the bond return while simultaneously shrinking the equity premium. Furthermore, the relaxation of the borrowing constraint reduces the net demand for bonds, and the risk-free rate puzzle reemerges.

In order to systematically illustrate these ideas, we construct an overlapping-generations (OLG) exchange economy in which consumers live for three periods. In the first period, a period of human capital acquisition, the consumer receives a relatively low endowment income. In the second period, the consumer is employed and receives wage income subject to large uncertainty. In the third period, the consumer retires and consumes the assets accumulated in the second period. We explore the implications of a borrowing constraint by deriving and contrasting the stationary equilibria in two versions of the economy. In the *borrowing-constrained* version, the young are prohibited from borrowing and from selling equity short. The *borrowing-unconstrained* economy differs from the borrowing-constrained one only in that the borrowing constraint and the short-sale constraint are absent.

Our model introduces two forms of market incompleteness. First, consumers of one generation are prohibited from trading claims against their future wage income with consumers of another generation.[7] Second, consumers of one generation are prohibited from trading bonds and equity with consumers of an unborn generation. Our model suppresses a third and potentially important form of market incompleteness that arises from the inability of an age cohort of consumers to insure via pooling the risks of their persistent, heteroskedastic idiosyncratic income shocks.[8] Specifically, we model each generation of consumers with a *representative consumer*. This assumption is justified only if there exists a complete set of claims through which heteroge-

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7. Being homogeneous within their generation, consumers have no incentive to trade claims with consumers of their own generation.
8. This perspective is emphasized in Storesletten, Telmer, and Yaron [1999]. They provide empirical evidence that shocks to the wage income process indeed have these properties and introduce this type of shocks in their model. They find that the interaction of life-cycle effects and the uninsurable wage income shocks play an important role in generating their results. Although they have a borrowing constraint in their model, as we do, it is the uninsurable wage income shocks that drive their results by deterring the young consumers from investing in equity. By contrast, in our model, it is the borrowing constraint exclusively that deters the young consumers from investing in equity.

neous consumers within a generation can pool their idiosyncratic income shocks. Absent a complete set of such claims, consumer heterogeneity in the form of *uninsurable, persistent,* and heteroskedastic idiosyncratic income shocks, with countercyclical conditional variance, has the potential to resolve empirical difficulties encountered by representative-consumer models.[9] Nevertheless, consumer heterogeneity *within* a generation is downplayed in our model in order to isolate and explore the implications of heterogeneity *across* generations in a parsimonious paradigm.

The paper is organized as follows. The economy and equilibrium are defined in Section II. In Section III we discuss the calibration of the economy. In Section IV we present and discuss the equilibrium results in both the borrowing-constrained and the unconstrained economies for a plausible range of parameter values. Extensions are discussed in Section V. Section VI concludes the paper. Technical aspects on the definition of equilibrium, existence of equilibrium, and the numerical calculations are detailed in the appendixes available from the authors.

II. The Economy and Equilibrium

We consider an overlapping-generations, pure exchange economy.[10] Each generation lives for three periods as *young, middle-aged,* and *old.* Three is the minimal number of periods that captures the heterogeneity of consumers across age groups, which we wish to emphasize: the *borrowing-constrained* young, the *saving* middle-aged, and the *dissaving* old. In the calibration, each period is taken to represent twenty years. We model each generation of consumers with a *representative consumer*. As explained in the introduction, consumer heterogeneity *within* a generation is downplayed in our model in order to isolate and explore the implications of heterogeneity *across* generations in a parsimonious paradigm.

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9. See, Mehra and Prescott [1985], Mankiw [1986], and Constantinides and Duffie [1996].

10. There is a long tradition of OLG models in the literature. Auerbach and Kotlikoff [1987] employ a deterministic OLG model in their study of fiscal policy. Rios-Rull [1994] employs a stochastic OLG model in his investigation of the role of market incompleteness on equilibrium allocations. Kurz and Motolese [2000] use the framework to examine rational beliefs. See also Cocco, Gomes, and Maenhout [1998], Huggett [1996], Jagannathan and Kocherlakota [1996], and Storesletten [2000].

There is one consumption good in each period, and it perishes at the end of the period. Wages, consumption, dividends, and coupons, as well as the prices of the bonds and equity are denominated in units of the consumption good.

There are two types of securities in the model, *bonds* and *equity*. Both are infinitely lived. We think of bonds as a proxy for long-term government debt. Each bond pays a fixed coupon of one unit of the consumption good in every period in perpetuity.[11] The supply of bonds is fixed at b units. The aggregate coupon payment is b in every period and represents a portion of the economy's capital income. We denote by q_t^b the ex-coupon bond price in period t.

One perfectly divisible equity share is traded. It is the claim to the net dividend stream $\{d_t\}$, the sum total of all private capital income (stocks, corporate bonds, and real estate). We denote by q_t^e the *ex-dividend* share price in period t. With the supply of equity fixed at one share in perpetuity, the issue and repurchase of equities and bonds is implicitly accounted for by the fact that the equity is defined as the claim to the *net* dividend. We do not model the method by which firms determine and finance the net dividend—firms are exogenous to the exchange economy.

The consumer born in period t receives deterministic wage income $w^0 > 0$ in period t, when young; stochastic wage income $w_{t+1}^1 > 0$ in period $t + 1$, when middle aged; and zero wage income in period $t + 2$, when old. By making the wage income process of the middle-aged consumer exogenous, we abstract from the labor-leisure trade-off. Claims on a consumer's future wage income are not traded.

A consumer born in period t enters life with zero endowment of the equity and bond. The consumer purchases $z_{t,0}^e$ shares of stock and $z_{t,0}^b$ bonds when young. The consumer adjusts these holdings to $z_{t,1}^e$ and $z_{t,1}^b$, respectively, when middle aged. Since we rule out bequests, the consumer liquidates his/her entire portfolio when old.[12] Thus, $z_{t,2}^e = 0$, and $z_{t,2}^b = 0$.

We study and contrast two versions of the economy. In the *unconstrained* economy, consumers are permitted to borrow by

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11. We also report the *shadow price* of a one-period (twenty-year) bond in *zero net supply*. Note that it is infeasible to introduce a one-year bond in this economy because the length of one period is assumed to be twenty years.

12. Ruling out bequests provides a parsimonious way to emphasize the effect of a borrowing constraint on consumers' life-cycle behavior. This admittedly controversial assumption is extensively discussed in Section V.

shorting the bonds. They are also permitted to short the shares of stock. (Negative holdings of either the bonds or equity are interpreted as short positions.) In the *constrained* economy, the consumers are forbidden to borrow by shorting the bonds. It is irrelevant whether or not we allow the consumers to short the equity because a restriction on shorting the equity is nonbinding for the particular range of parameters value with which we calibrate the economies.

We denote by $c_{t,j}$ the consumption in period $t + j(j = 0, 1, 2)$ of a consumer born in period t. The budget constraint of the consumer born in period t is

$$(1) \qquad c_{t,0} + z_{t,0}^b q_t^b + z_{t,0}^e q_t^e \leq w^0$$

when young;

$$(2) \quad c_{t,1} + z_{t,1}^b q_{t+1}^b + z_{t,1}^e q_{t+1}^e \leq w_{t+1}^1 + z_{t,0}^b(q_{t+1}^b + b) + z_{t,0}^e(q_{t+1}^e + d_{t+1})$$

when middle-aged; and

$$(3) \qquad c_{t,2} \leq z_{t,1}^b(q_{t+2}^b + b) + z_{t,1}^e(q_{t+2}^e + d_{t+2})$$

when old.

We also impose the constraints,

$$(4) \qquad c_{t,0} \geq 0, \quad c_{t,1} \geq 0, \quad \text{and} \quad c_{t,2} \geq 0,$$

that rule out negative consumption and personal bankruptcy. They are sometimes referred to as *positive-net-worth* constraints.

Underlying the economy is an increasing sequence $\{\mathfrak{I}_t: t = 0, 1, \dots\}$ of information sets available to consumers in period t. The information set $\mathfrak{I}_t$ contains the wage income and dividend histories up to and including period t. It also contains the consumption, bond investment, and stock investment histories of all consumers up to and including period $t - 1$. Most of this information turns out to be redundant in the particular stationary equilibria explored in Section III.

Consumption and investment policies are such that decisions made in period t depend only on information available in period t. Formally, a consumption and investment policy of the consumer born in period t is defined as the collection of the $\mathfrak{I}_t$-measurable $(c_{t,0}, z_{t,0}^b, z_{t,0}^e)$, the $\mathfrak{I}_{t+1}$-measurable $(c_{t,1}, z_{t,1}^b, z_{t,1}^e)$, and the $\mathfrak{I}_{t+2}$-measurable $c_{t,2}$.

The consumer born in period t has expected utility,

$$(5) \qquad E(\sum_{i=0}^{2} \beta^i u(c_{t,i})|\Im_t),$$

where β is the constant subjective discount factor.

In period t, the old consumers sell their holdings in equity and bonds and consume the proceeds. By market clearing, the demand for equity and bonds by the young and middle-aged consumers must equal the fixed supply of equity and bonds:

$$(6) \qquad z_{t,0}^e + z_{t-1,1}^e = 1$$

and

$$(7) \qquad z_{t,0}^b + z_{t-1,1}^b = b.$$

We conclude the description of the economy by specifying the joint stochastic process of the wage income and dividend. As noted earlier, the wage income of the young is a constant w^0, and the wage income of the old is equal to zero. Instead of specifying the joint process of the wage income of the middle-aged consumer and the dividend, (w_t^1, d_t), we choose to specify the joint process of the aggregate income and the wages of the middle-aged, (y_t, w_t^1), where the aggregate income y_t is defined as

$$(8) \qquad y_t = w^0 + w_t^1 + b + d_t.$$

Our definition of aggregate income includes the (constant) coupon payment on government debt.[13] For simplicity, we model the joint process of the (detrended) aggregate income and the wage income of the middle-aged as a time-stationary Markov chain with a nondegenerate, unique, stationary probability distribution.[14] In

13. This definition appears to differ from the standard definition of the GDP, which does not include the coupon payment on government debt. We justify our definition of the GDP as follows. In a more realistic model that takes into account the taxation of wages and dividend by the government to service its debt, $w^0 + w_t^1 + d_t$ stands for the sum of the *after-tax* wages and dividend. The sum of the *before-tax* wages and dividend is obtained by adding b to the after-tax wages and dividend, as in equation (8). In any case, the interest on government debt in the United States is about 3 percent of the GDP, and the calibration remains essentially unchanged whether the definition of the GDP includes the term b or not.

14. In the spirit of Lucas [1978], the model abstracts from growth, and considers an economy that is stationary in levels. The average growth in total output is thus zero. Mehra and Prescott [1985], however, study an economy that is stationary in growth rates and has a unit root in levels. In their model, the effect of the latter generalization is to increase the mean return on all financial assets relative to what would prevail, ceteris paribus, in a stationary-in-levels economy, but the return differentials across different securities are not much affected. The intuition is as follows: with growth creating preordained increases in future

the calibration, y_t and w_t^1 assume two values each. The four possible realizations of the pair (y_t, w_t^1) are represented by the state variable $s_t = j, j = 1, \ldots, 4$. We denote the corresponding 4×4 transition probability matrix as Π.

We consider stationary rational expectations equilibria as in Lucas [1978]. Equilibrium is defined as the set of consumption and investment policies of the consumers born in each period and the $\mathfrak{I}_t$-measurable bond and stock prices q_t^b and q_t^e in all periods such that (a) each consumer's consumption and investment policy maximizes the consumer's expected utility from the set of admissible policies while taking the price processes as given; and (b) bond and equity markets clear in all periods.[15]

It is beyond the scope of this paper to characterize the full set of such equilibria. It turns out, however, that in the borrowing-unconstrained economy there exists a stationary rational expectations equilibrium in which decisions made in period t and prices in period t are measurable with respect to the current state $s_t = j, j = 1, \ldots, 4$, and the one-period lagged state. These additional state variables are present because in every period, a middle-aged consumer participates in the securities market and his/her actions are influenced by the securities acquired when young.

In the borrowing-constrained economy and for the particular range of parameters that we calibrate the economy, there exists a stationary rational expectations equilibrium in which the young consumers do not participate in the equity and bond markets. Decisions made in period t and prices in period t are measurable with respect to the current state $s_t = j, j = 1, \ldots, 4$, alone.

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consumption relative to the present, investors require greater mean returns from all securities across the board in order to be induced to postpone consumption. The point of all this is that our life-cycle considerations can be examined in either context; we choose the stationary-in-levels because it is marginally simpler computationally, and better matches observed mean return data. It is also consistent with zero population growth, another feature of our model. We have constructed an analogous model where both output and population grow (output stochastically). The general results of this paper are duplicated in that setting as well. Generalizations to incorporate production could be done along the lines in Donaldson and Mehra [1984] and Prescott and Mehra [1980].

15. The characterization of the equilibrium and the proof of existence of a stationary equilibrium are in Appendix A of the unabridged version of this paper, available from the authors. The numerical routine for calculating the equilibrium in both the borrowing-constrained and the unconstrained economies is outlined in Appendix C of the same paper.

Lagged state variables are absent because middle-aged consumers do not participate in the securities market when young.[16]

Since our main results depend crucially on the assumption that borrowing is ruled out, this assumption merits careful examination. The borrowing constraint may be challenged because in reality consumers have the opportunity to purchase equities on margin and purchase index futures with small initial and maintenance margins. They may also borrow indirectly by purchasing the equity of highly levered firms and by purchasing index options. We investigate these possibilities in the context of the equilibrium of borrowing-constrained economies. In Section V we report that a very small margin suffices to deter a borrowing-unconstrained young consumer from purchasing equity on margin, index futures, and highly levered forms of equity. Essentially, *a young consumer is unwilling to sacrifice even a small amount of immediate consumption to put up as margin for the purchase of equity.*

For both versions of the model, the common stylized assumptions made on the income processes enable us to capture three key aspects of reality in a parsimonious way.[17] First, the wage income received by the young and the old is small compared with the income received by the middle-aged. Therefore, the young would like to *borrow* against future income, and the middle-aged would like to *save.* However, the young cannot borrow because of the borrowing constraint. Second, the major future income uncertainty is faced by the young. It turns out that, in the equilibrium of most of our borrowing-constrained economies, the equity premium has low correlation with the wage income that the young expect to receive in their middle age. The young would like to borrow and invest in equity but the borrowing constraint prevents them from doing so. Third, the saving middle-aged face no wage uncertainty.[18] Therefore, they save by investing in a port-

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16. See Appendix B of the unabridged version of this paper for the proof of existence of an equilibrium and discussion.

17. The simplifying assumption that the wage income of the young is deterministic and common across the young of the same generation may be relaxed to allow this income to be stochastic and different across the young of the same generation. Whereas this generalization would certainly increase the realism (and complexity) of the model, it would not change the basic message of our paper, as long as a sufficiently large fraction of the young were to remain borrowing-constrained.

18. The simplifying assumption that the wage income of the old is zero may be relaxed to allow for pension income and social security benefits. This income and benefits are deterministic from the perspective of the middle-aged consumers;

folio of equities and bonds, driven primarily by the motive of diversification of risk.

III. CALIBRATION

Period utility is assumed to be of the form,

$$(9) \qquad u(c) = (1 - \alpha)^{-1}(c^{1-\alpha} - 1),$$

where $\alpha > 0$ is the (constant) relative risk aversion coefficient. We adopt a conventional specification of preferences in order to focus attention on a different issue—the role of the borrowing constraint in the context of an overlapping-generations economy—as well as to make our results directly comparable to the prior literature.

We present results for values of $\alpha = 4$, and 6. We set $\beta = 0.44$ for a period of length 20 years. This corresponds to an *annual* subjective discount factor of 0.96, which is standard in the macroeconomic literature.[19]

The calibration of the joint Markov process on the wage income of the middle-aged consumers, w^1, and the aggregate income y is simplified considerably by the observation that the equilibrium security prices in the borrowing-constrained economy are linear scale multiples of the wage and income variables. This follows from the homogeneity introduced by the constant-RRA preferences.[20]

This property of equilibrium security prices implies that the equilibrium joint probability distribution of the bond and equity *returns* is invariant to the *level* of the exogenous macroeconomic variables *for a fixed y, w^1 correlation structure*. Rather, the distribution depends on a set of fundamental ratios and correlations: (i) the average share of income going to labor, $E[w^1 + w^0]/E[y]$; (ii) the average share of income going to the labor of the

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when incorporated into our analysis, they increase the demand for equity by the middle-aged and reduce the mean equity premium. Specifically, the mean equity premium decreases approximately by the factor $1 - x$, where x is the fraction of consumption of the old consumers that is derived from these benefits.

19. In the OLG literature there has been a trend toward calibrating the models with the subjective discount factor β greater than one. Unlike in an infinite horizon setting in an OLG framework, $\beta < 1$ is not necessary for the existence of equilibrium. Hence, we also investigate the equilibrium in economies with annual subjective discount factor equal to 1.04. The results are insensitive to the value of the subjective discount factor.

20. For the unconstrained economy, this statement is proved in Lemma A.1, Appendix A of the unabridged version of this paper, available from the authors.

young, $w^0/E[y]$; (iii) the average share of income going to interest on government debt, $b/E[y]$; (iv) the coefficient of variation of the twenty-year wage income of the middle aged, $\sigma(w^1)/E(w^1)$; (v) the coefficient of variation of the twenty-year aggregate income, $\sigma(y)/E(y)$; (vi) the twenty-year autocorrelation of the labor income, $\text{corr}(w^1_t, w^1_{t-1})$; (vii) the twenty-year autocorrelation of the aggregate income, $\text{corr}(y_t, y_{t-1})$; and (viii) the twenty-year cross-correlation, $\text{corr}(y_t, w^1_t)$.

Accordingly, we calibrate the model on ranges of the above moments (i)–(viii). There are enough degrees of freedom to permit the construction of a 4×4 transition matrix that exhibits a particular type of symmetry. Specifically, the joint process on income (y) and wage of the middle-aged (w^1) is modeled as a simple Markov chain with transition matrix:

$$
(10) \quad
\begin{array}{c}
 \\
(Y_1, w^1_1) \\
(Y_1, w^1_2) \\
(Y_2, w^1_1) \\
(Y_2, w^1_2)
\end{array}
\begin{array}{cccc}
(Y_1, w^1_1) & (Y_1, w^1_2) & (Y_2, w^1_1) & (Y_2, w^1_2) \\
\left[\begin{array}{cccc}
\phi & \pi & \sigma & H \\
\pi + \Delta & \phi - \Delta & H & \sigma \\
\sigma & H & \phi - \Delta & \pi + \Delta \\
H & \sigma & \pi & \phi
\end{array}\right],
\end{array}
$$

where the condition,

$$
(11) \qquad \phi + \pi + \sigma + H = 1,
$$

ensures that the row sums of the elements of the transition matrix are one. There are *nine* parameters to be determined: $Y_1/E[y]$, $Y_2/E[y]$, $w^1_1/E[y]$, $w^1_2/E[y]$, ϕ, π, σ, Δ, and H.[21] These parameters are chosen to satisfy the eight target moments and the condition (11). As it turns out, these parameters are such that all the elements of the transition matrix are positive.

The single most serious challenge to the calibration is the estimation of the above unconditional moments. Recall that the wage income of the middle-aged and the aggregate income are twenty-year aggregates. Thus, even a century-long time series provides only five nonoverlapping observations, resulting in large standard errors of the point estimates. Standard econometric

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21. In Tables II, III, and VI, the matrix parameters corresponding to the indicated panels are as follows: $\phi = 0.5298$, $\pi = 0.0202$, $\sigma = 0.0247$, $H = 0.4253$, and $\Delta = 0.01$ (top left). $\phi = 0.8393$, $\pi = 0.0607$, $\sigma = 0.0742$, $H = 0.0258$, and $\Delta = 0.03$ (top right). $\phi = 0.5496$, $\pi = 0.0004$, $\sigma = 0.0034$, $H = 0.4466$, and $\Delta = 0.03$ (bottom left). $\phi = 0.8996$, $\pi = 0.0004$, $\sigma = 0.0034$, $H = 0.0966$, and $\Delta = 0.03$ (bottom right). We do not report our choice of Y, W etc. because the returns in this economy are scale invariant and thus these values are not uniquely determined.

methods designed to extract more information from the time series, such as the utilization of overlapping observations or the fitting of high-frequency, high-order, time-series models, only marginally increase the effective number of nonoverlapping observations and leave the standard errors large.

We thus rely in large measure on an extensive sensitivity analysis, with the range of values considered as follows.

i. The average share of income going to labor, $E[w^1 + w^0]/E[y]$. In the U. S. economy this ratio is about .66 to .75, depending on the historical period and the manner of adjusting capital income.

The model considered in this paper, however, is implicitly concerned only with the fraction of the population that owns financial assets, at least at some stage of their life cycle, and it is the labor income share of that group that should matter. For the time period for which the equity premium puzzle was originally stated, about 25 percent of the population held financial assets [Mankiw and Zeldes 1991; Blume and Zeldes 1993]; that fraction has risen to its current level of about 40 percent. In our borrowing-constrained economy, the fraction of the population owning financial assets is .33, midway between the aforementioned estimates. We acknowledge, however, that age is not the sole determinant of ownership of financial assets. Nevertheless, it is likely that the share of income to labor is probably lower for the security-owning class than the population at large. In light of these comments, we set the ratio $E[w^1 + w^0]/E[y]$ in the lower half of the documented range (.66, .70).

ii. The average share of income going to the labor of the young, $w^0/E[y]$. This share is set in the range (.16, .20), sufficiently small to guarantee that the young have the propensity to borrow and render the borrowing constraint binding in the borrowing-constrained economy.

Our model presumes a high ratio of expected middle-aged income to the income of the young, one that implies a 4.5 percent per year annual real wage growth (twenty-year time period). Campbell et al. [2001] report that the age profile of labor income is much less upwardly sloped for less well-educated groups (see their Figure 1). We would argue that this group is less likely to own stocks or long-term government bonds, so that we are in effect

modeling the age profiles of labor income only of the well-educated stockholding class. Assuming no trend in factor shares, overall labor income will grow at the same rate as national income, which is about 3 percent. Assuming that the 50 percent of the population who do not own stock experience only a 1.5 percent per year (as suggested by the Campbell et al. figure) average increase in wage income, the wage growth of the more highly educated stockholding class must then be in the neighborhood of 4.5 percent per year which is what we assume.

We have constructed a model in which there are stockholders and nonstockholders with the latter experiencing slower labor income growth. The general results of the present paper are unaffected by this generalization.

iii. The average share of income going to interest on government debt, $b/E[y]$. This is set at .03, consistent with the U. S. historical experience.

iv. The coefficient of variation of the twenty-year wage income of the middle-aged, $\sigma(w^1)/E(w^1)$. The comparative return distributions generated by the constrained and the unconstrained versions of the model depend crucially on this coefficient. Ideally, we would like the calibration to reflect the fact that the young face large idiosyncratic uncertainty in their future labor income, generated by uncertainty in the choice of career and on their relative success in their chosen career. Nevertheless, consumer heterogeneity *within* a generation is disallowed in our formal model in order to isolate and explore the implications of heterogeneity *across* generations in a parsimonious way.

We are unaware of any study that estimates the coefficient of variation of the twenty-year (or, annual) wage income of the middle aged, $\sigma(w^1)/E(w^1)$. Creedy [1985], in a study of select "white-collar" professions in the United Kingdom, estimates that the annual coefficient $\sigma(w)/E(w)$ is in the range 0.31–0.57; in a study of women, Cox [1984] estimates the coefficient to be about 0.25. Gourinchas and Parker [forthcoming] estimate the annual *cross-sectional* coefficient of variation to be about 0.5. Considering the above estimates, we calibrate the coefficient of variation to be 0.25.

v. The coefficient of variation of the twenty-year aggregate income, $\sigma(y)/E(y)$. This coefficient captures the variation in *detrended,* twenty-year aggregate income. In the U. S. economy the log of the detrended (Hodrick-Prescott filtered) *quarterly* aggregate income is highly autocorrelated and has standard deviation of about 1.8 percent. This information provides little guidance in choosing the coefficient of variation of the twenty-year aggregate income. We consider the values 0.20 and 0.25.

vi. The 20-year autocorrelations and cross-correlation of the labor income of the middle-aged and the aggregate income, $\mathrm{corr}(y_t, w_t)$, $\mathrm{corr}(y_t, y_{t-1})$, and $\mathrm{corr}(w_t^1, w_{t-1}^1)$. Lacking sufficient time-series data to estimate the twenty-year autocorrelations and cross correlation, we present results for a variety of autocorrelation and cross-correlation structures.

In Table I we report empirical estimates of the mean and standard deviation of the annualized, twenty-year holding-period return on the S&P 500 total return series; and on the Ibbotson U. S. Government Treasury Long-Term bond yield. For years prior to 1926, the series was augmented using Shiller's S&P 500 series and the twenty-year geometric mean of the one-year bond returns. Real returns are CPI adjusted. The annualized mean (on equity or the bond) return is defined as the sample mean of [log{20-year holding period return}]/20. The annualized standard deviation of the (equity or bond) return is defined as the sample standard deviation of [log{20-year holding period return}]/$\sqrt{20}$. The annualized mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of [log{20-year nominal equity return} − log{the 20-year nominal bond return}]/$\sqrt{20}$. Estimates on returns cover the sample period 1/1889–12/1999, with 92 overlapping observations and the sample period 1/1926–12/1999 with 55 overlapping observations. We do not report standard errors, as these are large: on nominal returns, we have only four; and on real returns, we have only two *nonoverlapping* observations.

In Table I the real mean equity return is 6–7 percent with a standard deviation of 13–15 percent; the mean bond return is about 1 percent; and the mean equity premium is 5–6 percent. Since the equity in our model is the claim not *just* to corporate

TABLE I

	Real returns (in percent)					
	1/1889–12/1999			1/1926–12/1999		
	Equity	Bond	Premium	Equity	Bond	Premium
MEAN	6.15	0.82	5.34	6.71	0.14	6.58
STD	13.95	7.40	14.32	15.79	7.25	15.21

	Nominal returns (in percent)					
	1/1889–12/1999			1/1926–12/1999		
	Equity	Bond	Premium	Equity	Bond	Premium
MEAN	9.20	3.86	5.34	10.55	3.97	6.58
STD	13.88	7.27	14.32	14.47	8.49	15.21

We report empirical estimates of the mean and standard deviation of the annualized, twenty-year holding-period-return on the S&P 500 total return series; and on the Ibbotson U. S. Government Treasury Long-Term Bond yield. For years prior to 1926 the series was augmented using Shiller's S&P 500 series and the twenty-year geometric mean of the one-year bond returns. Real returns are CPI adjusted. The annualized mean (on equity or the bond) return is defined as the sample mean of the [log{20-year holding period return}]/20. The annualized standard deviation of the (equity or bond) return is defined as the sample standard deviation of the [log{20-year holding period return}]/$\sqrt{}$ 20. The mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of the [log{20-year nominal return on equity} − log{the 20-year nominal return on the bond}]/$\sqrt{}$ 20. Estimates on returns cover the sample period 1/1889–12/1999, with 92 overlapping observations and the sample period 1/1926–12/1999 with 55 overlapping observations.

dividends but also to *all* risky capital in the economy, the mean equity premium that we aim to match is about 3 percent.

IV. RESULTS AND DISCUSSION

The properties of the stationary equilibria of the calibrated economies are reported in Tables II and III. In Table II we set $RRA = 6$, $\sigma(y)/E[y] = 0.20$, and $\sigma(w^1)/E[w^1] = 0.25$; and in Table III we set $RRA = 4$, $\sigma(y)/E[y] = 0.25$, and $\sigma(w^1)/E[w^1] = 0.25$.

Our terminology is the same for both the constrained and the unconstrained economies. The one-period (twenty-year) bond is referred to as the bond. The bond is in zero net supply and its price is defined as the private valuation of the bond by the middle-aged consumer.[22] The consol bond, which is in positive net supply, is referred to as the consol.

22. Specifically, it is the shadow price of the bond determined by the marginal rate of substitution of the middle-aged consumer. It would be meaningless to report

TABLE II

	Correlation $(y,w^1) = 0.1$			
	Low serial autocorr. of y and of w^1 (0.1)		High serial autocorr. of y and of w^1 (0.8)	
	Borrowing constrained	Borrowing unconstrained	Borrowing constrained	Borrowing unconstrained
MEAN EQUITY RET.	8.4	10.2	9.4	12.2
STD OF EQUITY RET.	23.0	42.0	26.5	26.5
MEAN BOND RET.	5.1	9.0	6.7	11.1
STD OF BOND RET.	15.4	27.6	20.8	11.9
MEAN PRM/BOND	3.4	1.1	2.6	1.0
STD PRM/BOND	18.4	31.6	12.8	2.5
MEAN CONSOL RET.	3.7	9.9	4.5	11.1
STD OF CONSOL RET	19.1	27.6	19.0	11.9
MEAN PRM/CONSOL	4.7	0.3	4.9	1.0
STD PRM/CONSOL	10.5	5.2	10.1	9.2
MARGIN^{-1}, M	530	N.A.	170	N.A.
CORR(w^1,d)	−0.43	−0.43	−0.42	−0.42
CORR(w^1, PRM/BOND)	−0.02	0.00	0.13	0.58
	Correlation $(y,w^1) = 0.8$			
MEAN EQUITY RET.	7.9	12.7	8.3	13.9
STD OF EQUITY RET.	18.6	29.8	14.9	16.2
MEAN BOND RET.	5.8	11.3	6.9	12.9
STD OF BOND RET.	15.2	25.8	10.6	12.6
MEAN PRM/BOND	2.1	1.4	1.4	0.9
STD PRM/BOND	12.6	13.4	9.8	10.4
MEAN CONSOL RET.	6.1	11.8	6.7	12.9
STD OF CONSOL RET	16.7	26.6	10.3	12.8
MEAN PRM/CONSOL	1.8	0.56	1.6	1.0
STD PRM/CONSOL	7.2	3.8	7.4	1.2
MARGIN^{-1}, M	827	N.A.	156	N.A.
CORR(w^1,d)	0.35	0.55	0.36	0.91
CORR(w^1, PRM/BOND)	0.05	−0.04	0.19	0.13

We set $RRA = 6$, $\sigma(y)/E[y] = 0.20$, and $\sigma(w^1)/E[w^1] = 0.25$. The variables are defined in the main text of the paper. The consol bond is in positive net supply, and the one-period (twenty-year) bond is in zero net supply.

For all securities the annualized mean return is defined as mean of log{20-year holding period return}/20. The annualized standard deviation of the (equity, bond, or consol) return is de-

the private valuation of the bond by the young consumer because the young consumer would like to sell the bond short (borrow) but the borrowing constraint is binding. The private valuation of the bond by the young consumer is also well defined. We have calculated both private valuations of the bond, and they agree to the second decimal point. Essentially the two traded securities, the equity and the consol, come close to *completing the market* and the private valuation of the (one-period) bond by the young and the middle-aged practically coincide, *even though the bond is not traded in the equilibrium.*

TABLE III

| | Correlation $(y, w^1) = 0.1$ | | | |
| | Low serial autocorr. of y and of w^1 (0.1) | | High serial autocorr. of y and of w^1 (0.8) | |
	Borrowing unconstrained	Borrowing unconstrained	Borrowing constrained	Borrowing unconstrained
MEAN EQUITY RET.	8.9	8.4	9.6	10.1
STD OF EQUITY RET.	25.3	29.3	27.5	29.7
MEAN BOND RET.	5.0	7.4	6.9	9.4
STD OF BOND RET.	13.6	21.1	19.7	12.9
MEAN PRM/BOND	3.9	1.0	2.7	0.7
STD PRM/BOND	22.9	33.2	15.0	24.4
MEAN CONSOL RET.	3.7	8.1	4.5	9.3
STD OF CONSOL RET	17.4	21.7	18.2	12.8
MEAN PRM/CONSOL	5.2	0.3	5.1	0.8
STD PRM/CONSOL	9.5	4.7	10.0	12.8
MARGIN^{-1}, M	188	NA	390	NA
CORR(w^1,d)	−0.30	−0.30	−0.31	−0.31
CORR(w^1, PRM/BOND)	−0.02	0.30	0.11	0.41
	Correlation $(y, w^1) = 0.8$			
MEAN EQUITY RET.	8.4	10.4	8.8	11.8
STD OF EQUITY RET.	18.9	26.7	15.8	18.3
MEAN BOND RET.	5.8	8.8	7.4	10.9
STD OF BOND RET.	12.9	19.3	8.4	11.1
MEAN PRM/BOND	2.6	1.6	1.4	0.9
STD PRM/BOND	15.8	18.4	12.1	13.1
MEAN CONSOL RET.	6.1	9.3	6.9	10.8
STD OF CONSOL RET	14.5	20.0	8.9	11.1
MEAN PRM/CONSOL	2.3	1.1	1.9	1.0
STD PRM/CONSOL	6.3	3.6	7.2	13.1
MARGIN^{-1}, M	262	NA	330	NA
CORR(w^1,d)	0.51	0.51	0.53	0.53
CORR(w^1, PRM/BOND)	0.04	0.74	0.16	−0.47

We set $RRA = 4$, $\sigma(y)/E[y] = 0.25$, and $\sigma(w^1)/E[w^1] = 0.25$. The variables are defined in the main text of the paper. The consol bond is in positive net supply, and the one-period (twenty-year) bond is in zero net supply.

fined as the standard deviation of [log{20-year holding period return}]/$\sqrt{20}$. The mean annualized equity premium return over the bond return, "MEAN PRM/BOND," is defined as the difference between the mean return on equity and the mean return on the bond. The standard deviation of the premium of equity return over the bond return, "STD PRM/BOND," is defined as the standard deviation of [log{20-year nominal equity return} − log{the 20-year nominal bond return}]/$\sqrt{20}$. The mean premium of equity return over the consol return, "MEAN PRM/CONSOL," and the

TABLE IV

	State 1	State 2	State 3	State 4	Unconditional
PROBABILITY	0.275	0.225	0.225	0.275	1
CONSUMPTION OF THE YOUNG	19,000	19,000	19,000	19,000	19,000
CONSUMPTION OF THE MIDDLE-AGED	36,967	33,003	27,335	28,539	31,591
CONSUMPTION OF THE OLD	62,232	26,594	71,864	31,058	47,834
MEAN EQUITY RETURN	4.7	5.4	12.9	11.0	8.4
MEAN BOND RETURN	2.5	0.8	7.4	9.2	5.1
MEAN PRM/BOND	2.2	4.6	5.5	2.1	3.4
MEAN CONSOL RETURN	2.3	−1.4	4.7	8.8	3.7
MEAN PRM/CONSOL	2.4	6.9	8.2	2.5	4.7
MARGIN^{-1}, M	1212	386	178	373	530

We set $RRA = 6$, $\sigma(y)/E[y] = 0.20$, $\sigma(w^1)/E[w^1] = 0.25$, $\text{corr}(y_t,y_{t-1}) = \text{corr}(w_t^1,w_{t-1}^1) = 0.1$, and $\text{corr}(y_t,w_t) = 0.1$. The variables are defined in the main text of the paper. The consol bond is in positive net supply, and the one-period (twenty-year) bond is in zero net supply.

standard deviation of the premium of equity return over the consol return, "STD PRM/CONSOL," are defined in a similar manner.

The single most important observation across all the cases reported in Tables II–IV is that *the mean (20-year or consol) bond return roughly doubles when the borrowing constraint is relaxed.* This observation is robust to the calibration of the correlation and autocorrelation of the labor income of the middle-aged with the aggregate income. In these examples, the borrowing constraint goes a long way, albeit not *all* the way, toward resolving the *risk-free rate puzzle.* This, of course, is the first part of the thesis of our paper: if the young are able to borrow, they do so and purchase equity; the borrowing activity of the young raises the bond return, thereby exacerbating the risk-free rate puzzle.

The second observation across all the borrowing-constrained cases reported in Tables II and III is that the *minimum* mean equity premium over the twenty-year bond is about half the target of 3 percent. Furthermore, the premium *decreases* when the borrowing constraint is relaxed, in some cases quite substantially. This is the second part of the thesis of our paper: if the young are able to borrow, the increase in the bond return induces the middle-aged to shift their portfolio holdings in favor of the

bond; the increase in the demand for equity by the young and the decrease in the demand for equity by the middle-aged work in opposite directions; on balance, the effect is to increase the return on both equity and the bond while simultaneously shrinking the equity premium. Although the mean equity premium decreases in all the cases when the borrowing constraint is relaxed, the *amount* by which the premium decreases is the largest in the top panels of Tables II and III in which the labor income of the middle-aged and aggregate income are *negatively* correlated.

The third observation across all the cases reported in Tables II–III is that the correlation of the labor income of the middle-aged and the equity premium over the twenty-year bond, $\text{corr}(w^1, PRM/BOND)$, is much smaller in absolute value[23] than the exogenously imposed correlation of the labor income of the middle-aged and the dividend, $\text{corr}(w^1, d)$. Thus, equity is attractive to the young because of the large mean equity premium *and* the low correlation of the premium with the wage income of the middle-aged, thereby corroborating another important dimension of our model. In equilibrium, it turns out that the correlation of the wage income of the middle-aged and the equity return is low.[24] The young consumers would like to invest in equity because equity return has low correlation with their future consumption, if their future consumption is derived from their future wage income. However, the borrowing constraint prevents them from purchasing equity on margin. Furthermore, since the young consumers are relatively poor and have an incentive to smooth their intertemporal consumption, they are unwilling to decrease their current consumption in order to save by investing in equity. Therefore, the young choose not participate in the equity market.

The fourth observation is that the borrowing constraint results in standard deviations of the annualized, twenty-year equity and bond returns which are *lower* than in the unconstrained case and which are *comparable* to the target values in Table I.

In Table IV we present the consumption of the young, middle-aged, and old and the *conditional* first moments of the returns at the four states of the borrowing-constrained economy. The economy is calibrated as in the first two columns of the top left

23. This is consistent with the low correlation between the return on equity and wages reported by Davis and Willen [2000].
24. The low correlation of the wage income of the middle-aged and the equity return is a property of the equilibrium and obtains for a wide range of values of the assumed correlation of the wage income of the middle-aged and the dividend.

panel of Table II and corresponds to the case where $RRA = 6$, $\sigma(y)/E[y] = 0.20$, $\sigma(w^1)/E[w^1] = 0.25$, $\text{corr}(y_t, y_{t-1}) = \text{corr}(w_t^1, w_{t-1}^1) = 0.1$ and $\text{corr}(y_t, w_t) = 0.1$. This is our base case. As expected, the young simply consume their endowment, which in our model is constant across states. The consumption of the middle-aged is also smooth. The consumption of the old is surprisingly variable; it is this variability that induces the middle-aged to invest partly in bonds, despite the high mean premium of equity over bonds. The conditional first moments of the returns are substantially different across the states.

V. EXTENSIONS

V.1. Limited Consumer Participation in the Capital Markets

Our life-cycle economy induces a type of limited participation, that of young consumers in the stock market and that of old consumers in the labor market. However, *all consumers* participate in the capital markets in two out of the three phases of the life cycle: as savers in their middle age, and as dissavers in their old age.[25] In this section we introduce a second type of consumers, the *passive* consumers, who *never* participate in the capital markets. The passive consumers are introduced in order to accommodate, albeit in an ad hoc fashion, a different type of limited participation of consumers in the capital markets, that addressed in Mankiw and Zeldes [1991], Blume and Zeldes [1993], and Haliassos and Bertaut [1995]. We refer to the consumers who participate in capital markets in two out of the three phases of the life cycle as active consumers.

In calibrating this alternative economy, we assume that 60 percent of the consumers are passive and 40 percent are active. Since only two-thirds of the active consumers participate in the capital markets in any period, the percentage of the population (of active *and* passive consumers) that participate in the capital markets in any period is 26 percent.

We assume that both passive and active consumers receive wage income $19,000 when young, and $0 when old. The passive consumers receive income $33,000 when middle-aged. The active consumers receive income either $90,125 or $34,125 when middle-aged. The results are presented in Table V and are contrasted

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25. For the unconstrained version, all ages participate.

TABLE V

MEAN EQUITY RETURN	17.4
STD OF EQUITY RETURN	47.6
MEAN BOND RETURN	12.6
STD OF BOND RETURN	43.5
MEAN PRM/BOND	4.8
STD PRM/BOND	23.5
MEAN CONSOL RETURN	9.7
STD OF CONSOL RETURN	45.2
MEAN PRM/CONSOL	7.7
STD PRM/CONSOL	12.2
MARGIN^{-1}, M	927
CORR(w^1, d)	−0.42
CORR(w^1, PRM/BOND)	−0.03

The serial autocorrelation of y and of w^1 is 0.1. The table presents the borrowing-constrained case. We set $RRA = 6$, $\sigma(y)/E[y] = 0.20$, $\sigma(w^1)/E[w^1] = 0.25$, $W(0)/E[y] = 0.19$, $w_{\text{passive}}(1)/E[y] = 0.20$, $E[w_{\text{active}}(1)]/E[y] = 0.25$, $W(2)/E[y] = 0$, and the proportion of active consumers 40 percent. The variables are defined in the main text of the paper. The consol bond is in positive net supply, and the one-period (twenty-year) bond is in zero net supply.

to our "prime" case in Table II, the upper left panel. The results are essentially unchanged—the premium is somewhat higher—attesting to the robustness of the model along this dimension of limited participation.

V.2. Bequests

A simple way to relax the no-bequest assumption is to interpret the "consumption" of the old as the *sum* of the old consumers' consumption, and their bequests. As long as bequests skip a generation and are received by the borrowing-unconstrained middle-aged, as is often the case, the young remain borrowing-constrained, and our results remain intact.

More generally, we distinguish between the old consumers' *actual* consumption and their bequests—the joy of giving. We introduce a utility function for the old consumers that is *separable* over actual consumption and bequests. Furthermore, we specify that the old consumers are satiated at a low level of actual consumption. Such a model would imply that the middle-aged consumers would save primarily to bequeath wealth rather than to consume in their old age. This interpretation is interesting in its own right and makes the OLG model consistent with the empirical observation that the correlation between the (actual) consumption of the old and the stock market return is low.

V.3. Margin Requirements

A novel feature of our paper is that the limited stock market participation by the young consumers arises *endogenously* as the result of an assumed borrowing constraint. The young because of their steep earnings and consumption profile would not choose voluntarily to reduce their period 0 consumption in order to save in the form of equity. They would, however, be willing to borrow against their future labor income to buy equity and increase their period 0 consumption, but this is precluded by the borrowing constraint. The restriction on borrowing against future labor income is realistic. We have motivated it by recognizing that human capital alone does not collateralize major loans in modern economies for reasons of moral hazard and adverse selection. However, the restriction on borrowing to invest in equity may be challenged on the grounds that in reality consumers have the opportunity to purchase equity and stock index futures on margin and purchase a home with a 15 percent down payment. We investigate these possibilities in the context of the equilibrium of the borrowing-constrained economies.

We define M to be the dollar amount that a consumer can borrow for one (twenty-year) period with one dollar down payment and invest $M + 1$ dollars in equity on margin. That is, the margin requirement is $1/(M + 1)$, which is approximately equal to M^{-1} for large M. We report the maximum value of M that still deters young investors from purchasing equity on margin. Tables II–IV display the value of M in the equilibrium of all the borrowing-constrained economies. In all cases, M exceeds the value of 55: a young consumer is unwilling to sacrifice even one dollar of immediate consumption to put up as margin for the purchase of equity worth \$56. This demonstrates that our results remain unchanged, if the borrowing constraint to purchase equity is replaced by even a small margin requirement of 2 percent.

V.4. Firm Leverage

We also investigate the possibility that investors evade the margin requirement by purchasing the equity of a *levered* firm, where the "firm" is the claim to the dividend process. A simple variation of the above calculations shows that a margin requirement of 4 percent suffices to deter the borrowing-constrained young from purchasing the levered equity even if the debt-to-

TABLE VI

| | Correlation $(y,w^1) = 0.1$ | | | |
| | Low serial autocorr. of y and of w^1 (0.1) | | High serial autocorr. of y and of w^1 (0.8) | |
	Borrowing constrained	Borrowing unconstrained	Borrowing constrained	Borrowing unconstrained
MEAN EQUITY RET.	6.9	9.2	8.1	10.7
STD OF EQUITY RET.	18.1	42.9	24.9	26.7
MEAN BOND RET.	4.6	8.2	6.2	9.0
STD OF BOND RET.	15.3	29.3	20.8	19.0
MEAN PRM/BOND	2.3	1.1	1.9	1.7
STD PRM/BOND	10.7	31.3	8.6	17.8
MARGIN^{-1}, M	178	N.A.	170	N.A.
CORR(w^1,d)	-0.43	-0.43	-0.43	-0.43
CORR(w^1, PRM/BOND)	-0.004	-0.004	0.03	0.67
	Correlation $(y,w^1) = 0.8$			
MEAN EQUITY RET.	7.4	11.8	7.9	10.2
STD OF EQUITY RET.	16.7	31.4	11.6	15.8
MEAN BOND RET.	5.5	10.4	6.7	9.2
STD OF BOND RET.	15.2	26.2	10.4	14.7
MEAN PREM/BOND	1.9	1.4	1.2	0.9
STD PRM/BOND	8.9	16.7	6.4	15.3
MARGIN^{-1}, M	827	N.A.	156	N.A.
CORR(w^1,d)	0.35	0.35	0.36	0.36
CORR(w^1, PRM/BOND)	0.07	0.75	0.37	0.05

We set $RRA = 6$, $\sigma(y)/E[y] = 0.20$, and $\sigma(w^1)/E[w^1] = 0.25$. The variables are defined in the main text of the paper. The consol bond is in positive net supply, and the one-period (twenty-year) bond is in zero net supply.

equity ratio is 1:1. We conclude that our results remain effectively unchanged even if we recognize the ability of firms to borrow.

V.5. Other Market Configurations

So far, we have assumed that the equity and the consol bond are in positive net supply, while the one-period (twenty-year) bond is in zero net supply. Here we consider a variation of the economy in which the equity and the one-period (twenty-year) bond are in positive net supply, while the consol bond is in zero net supply. We calibrate the economy using the same parameters as those used in Table II. The properties of the equilibrium are presented in Table VI and are contrasted with the properties of the equilibrium in Table II. It is clear that the major conclusion of the paper remains robust to this variation of the economy: the

borrowing constraint increases the equity premium. Furthermore, security returns in the constrained economy remain uniformly below their unconstrained counterparts.

VI. Concluding Remarks

We have addressed ongoing questions on the historical mean and standard deviation of the returns on equities and bonds and on the equilibrium demand for these securities in the context of a stationary, overlapping-generations economy in which consumers are subject to a borrowing constraint. The particular *combination* of these elements captures the effect of the borrowing constraint on the investors' saving and dissaving behavior over their life cycle. We find in all cases that the *imposition of the borrowing constraint reduces the risk-free rate and increases the risk premium, in some cases quite significantly*. However, the standard deviation of the security returns remains too low relative to the data. On a qualitative basis, our results mirror effects in the larger society: the decline in the premium documented in Blanchard [1992] has been contemporaneous with a substantial increase in individual indebtedness.

The model is intentionally sparse in its assumptions in order to convey the basic message in the simplest possible way. It can be enriched in various ways that enhance its realism. For example, we may increase the number of generations from three to sixty, representing consumers of ages twenty to eighty in annual increments. In such a model, we expect that the youngest consumers are borrowing-constrained for a number of years and invest neither in equity nor in bonds; thereafter they invest in a portfolio of equity and bonds, with the proportion of equity in their portfolio decreasing, as they grow older and the attractiveness of equity diminishes. It is possible to increase the endowment of young consumers to reflect intergenerational transfers, and make the endowment of the young random and different across consumers. These changes will have pricing implications to the extent that the young investors who are currently inframarginal in the equity and bond markets become marginal. We can model the pension income and social security benefits of the old consumers. It is possible to introduce heterogeneity of consumers within a generation. We can model GDP *growth* as a stationary process as in Mehra [1988] rather than modeling (detrended) GDP *level* as a stationary process. We can specify dis-

tinct production sectors, endogenize production, endogenize the labor-leisure trade-off, and model the government sector in a more realistic manner than we have done in the paper. We suspect that in all these cases the essential message of our paper will survive: *the borrowing constraint has the effect of lowering the interest rate and raising the equity premium.*

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