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Mood fluctuations, projection bias, and volatility of equity prices

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Abstract

This paper focuses on the potential effects of small fluctuations in investors' subjective preferences (specifically, their discount factors and attitudes towards risk) on the volatility of equity prices. We briefly summarize some of the arguments and evidence regarding the fluctuations in subjective preferences. Our analysis indicates that such fluctuations may have significant implications for understanding the volatility of the prices of financial assets.

We derive a closed-form expression for equilibrium equity prices, and use this expression to map the fluctuations in investors' subjective preferences to the fluctuations in equity prices. Our analysis suggests that small fluctuations in the discount factor have potentially large effects on the latter. For example, if the standard deviation of the fluctuations in the discount factor is of the order of 1/10th of one percent, then this by itself can induce a 3–4% standard deviation in the fluctuations in equity prices. The fluctuations in the attitude towards risk have a smaller, but nevertheless non-negligible effect. We present the intuition underlying our conclusions. © 2002 Published by Elsevier Science B.V.

Keywords: Equity pricing; Volatility; Discount factor; Fluctuation; Projection bias

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1. Introduction

As Hayek presciently noted in his writings, prices are perhaps the most crucial mechanisms of a market economy. In particular, prices of financial assets influence a wide range of individual and organizational decisions, including those entailing the creation of productive assets, and the allocation of resources to alternative assets. It is thus important to try to understand the sources of fluctuations in the prices of financial assets.

Within this larger issue, the present paper examines a relatively narrow question, namely, can small fluctuations in investors' subjective parameters induce large fluctuations in equity prices? We consider two subjective parameters: the discount factor, and the level of risk-aversion. To our knowledge, a formal analysis of these relationships has not been undertaken in the literature.

What we mean by fluctuations in investors' subjective parameters is perhaps better understood from some of the arguments and evidence in this regard that are presented later in this section. Briefly stated, the three central hypotheses on which this paper is based are that: (i) an individual's subjective parameters fluctuate over time, (ii) there is some correlation among these fluctuations across particular groups of individuals, and (iii) when making his current choices, an individual behaves as if his current subjective parameters are likely to persist to a significant degree in the future. This bias of an individual has been referred to in the literature as the 'projection bias'.

Using a variant of the Lucas (1978) model, we derive a closed-form expression for equity prices. We use this expression to map the fluctuations in investors' subjective parameters to the fluctuations in equity prices. Our analysis suggests that fluctuations in the discount factor have a large effect on the fluctuations in equity prices. For example, a standard deviation of 1/10th of one percent in the fluctuations in the discount factor can induce, *by itself*, a standard deviation of 3–4% in the fluctuations in equity prices. 'By itself' means that these fluctuations in equity prices exclude all other sources of fluctuations, including the effects of changes in the economy's fundamentals. Our analysis also indicates that fluctuations in investors' attitudes towards risk (measured by the parameter of relative risk-aversion) have a smaller, but nevertheless non-negligible effect on the fluctuations in equity prices. We analyze both local (small) and non-local (large) fluctuations in investors' subjective parameters.

The present paper focuses on the fluctuations in individuals' subjective parameters. In contrast, as has rightly been done in the literature, a large number of economic questions can be analyzed while ignoring such fluctuations. For example, the incorporation of these fluctuations is unlikely to alter the estimation of the system of household demand equations for the annual consumption of goods and services. However, if the object is to understand the fluctuations in the prices of financial assets, then our analysis suggests that the fluctuations in investors' subjective parameters may play a significant role.

Our analysis is entirely positive. It maps one set of fluctuations (namely, in investors' subjective parameters) to another set of fluctuations, namely, in equity prices. We do not, therefore, address a vast literature, originated by Shiller (see Shiller, 1989), that examines whether the observed volatility of equity prices is or is not excessive, when compared to the predictions generated by particular sets of models and empirical specifications. For an overview of this literature, see Lo (1997).

1.1. Remarks on fluctuations in investors' subjective parameters

We motivate these fluctuations in several different ways: [1]

1. Becker and Mulligan (1997) have developed a framework in which an individual endogenously determines his discount factor, in part through how much effort and resources he chooses to spend on creating his appreciation of the future. They show that an individual's discount factor is affected by a variety of variables, including interest rates and the individual's income. Some of these variables are random by nature, and this randomness is faced by more than one individual at the same time. Hence, this framework supports our maintained hypotheses that there are some fluctuations in an individual's discount factor, and that there is some correlation among these fluctuations for particular groups of individuals.

2. Blanchard (1985) has shown that the fluctuations in the probabilities of death will effectively cause the discount factors to fluctuate. Insofar as there are population-wide processes (such as changing future prospects of epidemics, armed conflicts, and new discoveries in medical treatments) that stochastically affect the probabilities of death, the discount factors of individuals will fluctuate and display some degree of correlation.

3. *Projection bias*. Based on many empirical studies (including Loewenstein and Adler, 1995), Loewenstein et al. (2000) analyze and summarize several patterns that are pertinent to the present paper. They show that an individual's preferences change with time, including due to exogenous factors. They demonstrate that typically an individual mispredicts his future sequence of preferences. Further, they show that there is a marked bias in this misprediction. The nature of this bias is that an individual systematically exaggerates the degree to which his future preferences will resemble his present preferences. These empirical studies provide direct evidence for our hypothesis that, while facing an intertemporal choice, a person behaves as if his present tastes

[1] Krusell and Smith (1998) employ random discount factors in a model of which an objective is to generate outcomes that match the key features of the observed distribution of wealth. Their motivation for the preceding randomness seems to be that genes affect the discount factor, and that genes are passed on imperfectly from parents to children in an infinitely lived dynasty.

are likely to persist into the future with a greater intensity, and for a longer period of time, than what the persistence of the present tastes will actually be.

We fully recognize that the foregoing bias may or may not be consistent with one or another conceptual model of an individual's ability to introspect and predict. However, as is the case in Loewenstein et al. (2000), the objective of the present paper is to use simplified models to capture some aspects of how individuals might be behaving rather than how they ought to behave.

4. Saunders' (1993) empirical work demonstrates that the weather on Wall Street has a significant effect on stock prices. His causal link is that weather fluctuations (in particular, bright versus non-bright days) induce fluctuations in human moods, which in turn affect stock prices. [2] This work provides support for each of the three main hypotheses, described earlier, on which the present paper is based: (i) The causal link just stated bolsters the hypothesis that an investor's subjective parameters fluctuate over time. (ii) Saunders' study suggests that mood fluctuations have some correlation among particular groups of individuals; because if this were not the case, the effects of individual moods would diversify away and the aggregate effect on stock prices would likely be insignificant. (iii) It is revealing here that a significant impact on stock prices is produced by a phenomenon as repetitive and transient as the prevailing weather conditions. This finding, once again, supports the perspective that agents project their current mood into what their future sequence of moods will be. Put differently, if the prediction of the future sequence of moods were accurate and complete then the Wall Street weather should have virtually no impact on stock prices. For modeling simplicity, our paper makes the working assumption that an investor behaves as if his present subjective parameters will persist into the future. Based on the arguments presented above, as well as the material from the psychology literature cited later, we believe that this working assumption is more realistic than another simplifying assumption that an investor fully and accurately takes into account his future sequence of subjective parameters when making his current choices.

5. *Observations from the psychology literature*. These observations on human mood fluctuations are based on intertemporal recording and analyses of the moods of the subjects. [3] Among the observations are: (a) An awake adult's mood fluctuates, putting the person in different states of moods, attitudes and perceptions at different times. (b) The etiology of these fluctuations,

[2] Kamstra et al. (2000) show that the reduced length of the day during winters has an impact on stock prices. Kamstra et al. (1999) find that changes in sleep patterns, induced by daylight-savings time changes, influence stock prices. See also the paper by Hirshleifer and Shumway (2001).

[3] The data collection is done over a period of time, while the subjects are in their natural environment involved in their own respective routine activities. The phrase 'experience sampling' or 'time sampling' is generally used to describe this method of data collection. See Csikszentmihalyi et al. (1977) for an exposition and early application of this methodology.

including the role of changes (often random, on an ex ante basis) in an individual's environment, is not fully understood. From an economic viewpoint, at a reduced-form level, it is therefore perhaps appropriate to treat these mood fluctuations as random. (c) Within a range, such fluctuations do not indicate any pathological disequilibrium (Csikszentmihalyi and Larson, 1984, p. 123), and they are an integral part of human nature. (d) With mood shifts, one's 'entire consciousness becomes revised and reconstructed' (Csikszentmihalyi and Larson, 1984, p. 120). An interpretation of observations such as the preceding one, which is specially relevant in the context of the present paper, is as follows. At any given point in time, an individual is unaware, to a significant degree, of the sequence of moods that will unfold in the future, or even of the statistical properties of future fluctuations. (e) While psychologists have examined numerous dimensions of mood fluctuations, these do not in general have a one-to-one correspondence to the subjective parameters that are of immediate interest to economists, such as the discount factor, the attitude towards risk, and the perceptions of pleasure or non-pleasure in work and leisure. However, see Constans and Mathews (1993), Leith and Baumeister (1996), and Brandstatter (1994) for examples of some partial exceptions.

6. *Other observations*. A separate set of psychological arguments and some related evidence suggest that the fluctuations in subjective parameters might be correlated to some degree across individuals in particular groups of people, especially among investors. [4] First, there are extensive ongoing interactions among investors, direct as well as indirect. Second, it appears that human interactions have a general tendency to induce a greater similarity of attitudes and perceptions among the participants than what would have existed without such interactions (see Shiller (1989) and references cited therein). In its extreme form, a part of this phenomenon has been referred to as 'Groupthink' (Janis, 1982).

1.2. Organization of the paper

Section 2 derives a closed-form solution for equity prices. Section 3 examines the effects of local distribution-free fluctuations in the discount factor; Section 4 does the same for non-local fluctuations. Section 5 examines the effects of local distribution-free fluctuations in the risk-aversion parameter; Section 6 does the same for non-local fluctuations. Section 7 concludes the paper.

2. A closed-form solution for equity prices

We base our analysis on what is perhaps the simplest general equilibrium model suitable for the purpose at hand. The simplicity of the model allows

[4] The response of investors to changes in the index of consumer confidence would seem to partially capture this sentiment.

us to: (i) focus on the narrow question stated in the last section; (ii) derive several explicit, though illustrative, conclusions; and (iii) develop intuitions and insights that might be obscured if additional features are incorporated in the model.

We view this paper as an initial step towards understanding the role of fluctuations in individuals' subjective parameters in the context of equity prices. Accordingly, we abstract from a number of issues, one of which is as follows. In an economy consisting of a large number of individuals, the fluctuations in a subjective parameter (say, the discount factor) will, through some non-linear relationship, exert an effect on equity prices. As noted earlier, a maintained hypothesis of this paper is that the fluctuations in the discount factors across individuals are correlated to some small degree. Therefore, even after balancing out the effects of the non-correlated parts of the fluctuations in the discount factors of individuals, it is likely that the preceding fluctuations will induce some fluctuations in the equity prices.

We abstract from modeling an economy of the kind just described, consisting of heterogeneous individuals.[5] This is because it would then not be possible to obtain explicit solutions and conduct analysis that sheds some light on the issues on which we focus. Instead, we mimic such an economy by considering a much simpler one in which there is one individual, with fluctuating subjective parameters, who behaves competitively.

The individual's expected utility from any random consumption path is given by

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t)\right\},\quad 0<\beta<1, \tag{1}$$

where c_t is the per capita consumption in period t, $u(\ )$ is the utility function, β is the subjective discount factor, and $\mathrm{E}_0\{\ \}$ is the expectations operator conditional upon the information available at time zero, that is, the present.

There is one productive unit producing a stochastic quantity of a perishable consumption good. There is one equity share of ownership to this productive unit. This share is competitively traded. Since only one productive unit is considered, the return on this share of equity is also the return on the market. The firm's output in period t is the same as the firm's dividend payout in that period, and it is denoted by d_t. The consumption in period t is constrained to be less than or equal to d_t.

We follow the convention of pricing securities ex-dividend at time t, in terms of the time t consumption good. Let u_c denote the derivative of u with respect to its argument. Then, in competitive equilibrium, given the process

[5] For an illustration of how this may be done, see Kraus and Sagi (2000).

$\{d_{t+j}\}$ on dividend payouts, the equity price in period t is

$$p_t = \mathrm{E}_t\left\{\sum_{j=1}^{\infty} \beta^j \frac{u_c(y_{t+j})}{u_c(y_t)} d_{t+j}\,\middle|\, y_t\right\}, \tag{2}$$

where, in equilibrium, $\{y_{t+j}\} = \{d_{t+j}\}$. Expression (2) follows from an extension of Lucas (1978) by Mehra and Prescott (1985). This model is stationary in the growth rates of consumption (thus enabling us to capture the non-stationarity in the consumption series associated with the large increases in per capita consumption over time) rather than being stationary in consumption levels, as in Lucas. In (2), the variable y_t is sufficient relative to the entire history of shocks up to and including time t for predicting the subsequent evolution of the economy.

We work with the iso-elastic utility function:

$$u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \quad 0 < \alpha < \infty, \tag{3}$$

where α represents the parameter of relative risk-aversion. Substitution of $u_c(c) = c^{-\alpha}$ into (2) yields

$$p_t = \mathrm{E}_t\left\{\sum_{j=1}^{\infty} \beta^j y_t^{\alpha} y_{t+j}^{1-\alpha}\,\middle|\, y_t\right\}. \tag{4}$$

Define

$$G_{t+j} \equiv y_{t+j}/y_t.$$

That is, G_{t+j} is the (stochastic) cumulative growth multiple for j periods. Accordingly, (4) can be rewritten as

$$p_t = y_t \sum_{j=1}^{\infty} \beta^j \mathrm{E}_t\{(G_{t+j})^{1-\alpha}\}. \tag{5}$$

Let $x(\tau; \tau \geqslant 0)$ represent the geometric Brownian process

$$\mathrm{d}x = \mu x\, \mathrm{d}t + \sigma x\, \mathrm{d}z, \tag{6}$$

where z is the unit Weiner process, $x(0) = 1$, μ is the drift per unit of time, and σ is the volatility per unit of time. We identify G_{t+j} as the discrete observation of $x(\tau)$ at $\tau = j$. Here j's are integers and $j = 1$ to ∞. The "unit of time", with respect to which μ and σ are defined, is the time interval (a year, in the analysis below) between j and $j+1$.[6]

[6] In general, the unit of time will be the time difference between two successive flows of expected utilities to the individual in expression (1).

Then, the equity price p_t can be expressed, in a closed-form manner, as a function of the parameters:

$$p_t = y_t \frac{\beta e^{(1-\alpha)(\mu-(1/2)\alpha\sigma^2)}}{1-\beta e^{(1-\alpha)(\mu-(1/2)\alpha\sigma^2)}}. \tag{7}$$

Appendix A contains a derivation of (7). For brevity, the time subscript of p in (7) is suppressed in the rest of the paper.

The validity of (7) requires that the summation on the right-hand side of (5) converge. A sufficient condition for this is

$$\frac{1}{2}\alpha\sigma^2 - \mu > \frac{1}{1-\alpha}\ln\beta. \tag{8}$$

Throughout the paper, we use the following parameter values for the US economy: $\mu = 0.018$ and $\sigma = 0.035$.[7] For these parameter values, the restriction (8) will typically be satisfied if the household's risk-aversion is at least logarithmic. The accumulated evidence on the parameter of relative risk-aversion suggests that this parameter is larger than that for the logarithmic risk-aversion (see Mehra and Prescott, 1985; Barsky et al., 1997). In the rest of this paper, we work with the restriction (8).

Now consider (7), assuming for a moment that the subjective parameters (β,α) are non-random. Then, it is apparent from (7) that the equity price in any given period is non-random, once the dividend payout for that period has been revealed. The equity prices will evolve over time as a function of the evolution of the external economic fundamentals (captured here through the stochastic nature of the dividend payout process). In particular, the relationship (7) will translate the latter evolution into the volatility of equity prices.

Further note that if (β,α) are non-random, then the price-earnings ratio (where the earning in a period is identified with the dividend payout in that period) is a constant through time. This ratio is easily calculated from (7). For example, for $\beta = 0.98$, and $\alpha = 2$, the price-earnings ratio is 26. Note however that this ratio is markedly sensitive to the values of the parameters β and α. This ratio can be in the range of 15–30 for reasonable values of these parameters.

Next, suppose that the discount parameter is random, and that it is drawn in each period from an intertemporally i.i.d. distribution. Then, after the dividend payout for a particular period has been revealed, the equity price in that period will still be random, and the price-earnings ratio will also be random. The observed price in that period will be determined by (7) after the value of β for that period has been revealed. The observed intertemporal variability of equity prices will thus be jointly determined by the intra-period randomness

[7] We thank Robert Shiller for providing the data on per capita real consumption of non-durables and services.

of prices just noted, and the intertemporal evolution of the external economic fundamentals.

As a source of the volatility of equity prices, the present paper abstracts from the intertemporal evolution of the external economic fundamentals, while fully recognizing that this source of volatility is an inescapable part of the economy. Instead, we use the relationship (7) to examine the fluctuations in equity prices that arise solely due to the fluctuations in the investors' subjective parameters (β, α). We turn to this task in the next section.

Finally, note in expression (1) and the subsequent analysis that we have used the discrete subjective discount factor, β, to discount the individual's flow of future expected utilities. This approach is used widely in general equilibrium modeling in finance and macroeconomics. There are other ways to achieve the same discounting, for example, by using a discrete or continuous subjective discount rate, or the equilibrium interest rate. However, in the present context of representing the individual's preferences, β has a clear economic meaning; it is the marginal rate of substitution between two successive flows of expected utilities to the individual. Accordingly, by using β, we are focusing on some of the effects of the fluctuations in this marginal rate of substitution. In comparison, the subjective discount rates do not have analogously direct economic meanings in the present context. Further, a difficulty with the interest rate is that it is not known a priori, and is determined in equilibrium.

3. Local distribution-free fluctuations in the discount factor

In this section, we trace the effects of local fluctuations in the discount factor on the fluctuations in equity prices. The fluctuations in the discount factor are distribution-free, in that no assumption is made concerning the statistical distribution underlying these fluctuations.

We first present an expression for the elasticity of the price of equity with respect to the discount factor. It is shown that this elasticity, denoted by $\mathscr{E}_\beta$, is a simple but powerful summary representation of the effect of fluctuations in the discount factor on the fluctuations in equity prices. Specifically, if the fractional fluctuations in the discount factor have a one percent standard deviation, then the standard deviation of the induced fractional fluctuation in equity prices will be $|\mathscr{E}_\beta|$ percent. We then present some illustrative values of $|\mathscr{E}_\beta|$.

As shown in the appendix, using the definition $\mathscr{E}_\beta \equiv \partial \ln p / \partial \ln \beta$, expression (7) yields

$$\mathscr{E}_\beta = \frac{1}{1 - \beta e^{(1-\alpha)(\mu-(1/2)\alpha\sigma^2)}} > 0. \tag{9}$$

Since $\mathscr{E}_\beta$ is positive, we often do not distinguish it from $|\mathscr{E}_\beta|$.

3.1. Economic meaning of $\mathscr{E}_\beta$

Let $p = f(\beta)$ summarize the relationship (7) between β and p. Let f_β denote the derivative of f with respect to its argument. Consider a small interval around β, within which $f(\ )$ can be treated as a linear function. Let $\tilde{\beta}$ denote a value of the discount factor inside this interval, and let $\tilde{p}$ denote the corresponding equity price. Then $\tilde{p} - p = (\tilde{\beta} - \beta) f_\beta$, where f_β is evaluated at β. This yields $(\tilde{p} - p)/p = [(\tilde{\beta} - \beta)/\beta]\mathscr{E}_\beta$. Next, treat $\tilde{\beta}$ as one of the possible random values of the distribution of the discount factor, with mean β, such that the entire distribution falls within the interval noted above. Let $\varsigma\{\ \}$ denote the operator for the standard deviation. Then, the last equality yields

$$\varsigma\{(\tilde{p} - p)/p\} = |\mathscr{E}_\beta| \varsigma\{(\tilde{\beta} - \beta)/\beta\}. \tag{10}$$

That is, the standard deviation of the fractional fluctuations induced in equity prices by fluctuations in the discount factor is $|\mathscr{E}_\beta|$ times the standard deviation of the fractional fluctuations in the discount factor. The standard deviations on both sides of (10) have the desirable property that they are unit-free. Thus, $|\mathscr{E}_\beta|$ can be viewed as a convenient metric that captures β-induced fluctuations in equity prices. An interpretation analogous to that of (10) holds for the elasticity of equity price with respect to the risk-aversion parameter, to be derived and examined later in the paper.

3.2. Some numerical values of $\mathscr{E}_\beta$

Table 1 presents some values of $\mathscr{E}_\beta$ for plausible values of β and α.

Note that the elasticity is large in all of the cases considered in the above table. The elasticity is larger if β (which is the mean value of the discount factor) is larger. This relationship holds regardless of the values of the parameters. Further, for the range of parameters considered in Table 1, the elasticity is larger if the risk-aversion parameter is smaller.

Table 1
Values of $\mathscr{E}_\beta$ for some values of β and α

Risk-aversion parameter, α	Discount factor, β		
	0.99	0.98	0.96
1.1	85.2	46.0	24.0
2.0	37.2	27.2	17.7
3.0	23.5	19.1	14.0

4. Non-local fluctuations in the discount factor

The discount factor, which is now random, is denoted as $\tilde{\beta}$, with the range

$$0 < \tilde{\beta} < 1. \tag{11}$$

The randomness of $\tilde{\beta}$ will in general induce a randomness in the equity price. Accordingly, the latter is now denoted as $\tilde{p}$. Define $\bar{\beta} \equiv \mathrm{E}\{\tilde{\beta}\}$, and $\bar{p} \equiv \mathrm{E}\{\tilde{p}\}$. For later use, define Var{ } as the variance operator, and recall that $\varsigma\{\ \} \equiv [\mathrm{Var}\{\ \}]^{1/2}$ is the operator for the standard deviation.

Define

$$\tilde{\beta} \equiv \bar{\beta}(1+\tilde{b}) \quad \text{or equivalently, } \tilde{b} = (\tilde{\beta}-\bar{\beta})/\bar{\beta}. \tag{12}$$

That is, $\tilde{b}$ is the fractional fluctuation of $\tilde{\beta}$ around its mean. Define s_β as the standard deviation of the fractional fluctuations of $\tilde{\beta}$ around its mean. Then, from the second part of (12),

$$s_\beta \equiv \varsigma\{\tilde{b}\}. \tag{13}$$

Analogously, define

$$S_\beta \equiv \varsigma\{(\tilde{p}-\bar{p})/\bar{p}\} \tag{14}$$

as the standard deviation of the fractional fluctuations in equity prices.

To construct a mapping from s_β to S_β, we posit that $\tilde{b}$ is a continuous uniform variate with parameters $(-\eta, \eta)$, where $\eta > 0$. By definition,

$$-\eta \leqslant \tilde{b} \leqslant \eta. \tag{15}$$

Then, (13) yields

$$s_\beta = \eta/\sqrt{3}; \tag{16}$$

see Johnson et al. (1995, Vol. 2, p. 277). A closed-form solution for S_β is derived in Appendix A. This yields

$$S_\beta = \left\{ \left[\frac{\ell_1 - \ell_2}{(-1+\ell_2/2)^2} \right] - 1 \right\}^{1/2}, \tag{17}$$

where

$$\ell_1 \equiv \frac{2+\varphi^2[1-\eta^2]-2\varphi}{1+\varphi^2[1-\eta^2]-2\varphi},$$

$$\ell_2 \equiv \frac{1}{\eta\varphi} \ln \left[\frac{1-\varphi+\varphi\eta}{1-\varphi-\varphi\eta} \right] \quad \text{and } \varphi \equiv \bar{\beta} \mathrm{e}^{(1-\alpha)(\mu-(1/2)\alpha\sigma^2)}. \tag{18}$$

Further, given (15), it is shown in Appendix A that a sufficient condition for expression (11) to be satisfied is:

$$\eta < -1 + 1/\bar{\beta} \quad \text{and} \quad \bar{\beta} > 0.5. \tag{19}$$

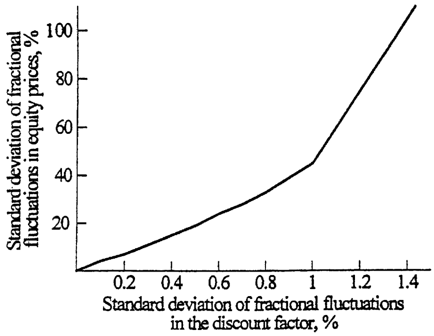

Fig. 1.

Now consider expressions (16) and (17) for s_β and S_β, respectively. For any given values of the parameters $(\bar{\beta}, \alpha, \mu, \sigma)$, both of these expressions are functions of η. Hence, by varying the values of η, it is possible to obtain a mapping from s_β to S_β. In doing so, it is obviously essential that the values of η and the parameters $(\bar{\beta}, \alpha, \mu, \sigma)$ be consistent with expressions (8) and (19).

Using the approach just described, we construct mappings from s_β to S_β for various plausible values of $\bar{\beta}$ and α. A typical graph, in which $\bar{\beta} = 0.99$, and $\alpha = 2$, is presented in Fig. 1. It is readily seen that small fluctuations in the discount factor induce large fluctuations in equity prices. This confirms and complements the local results presented in the last section.

5. Local distribution-free fluctuations in the risk-aversion parameter

Analogous to what was done in Section 3, we now examine the effects of local distribution-free fluctuations in α on the fluctuations in equity prices.

As is derived in Appendix A, a compact expression for the elasticity $\mathscr{E}_\alpha \equiv \partial \ln p / \partial \ln \alpha$ is

$$\mathscr{E}_\alpha = -\alpha(\mu + \tfrac{1}{2}\sigma^2 - \alpha\sigma^2)\mathscr{E}_\beta, \tag{20}$$

where $\mathscr{E}_\beta$ is given by (9). The interpretation of $\mathscr{E}_\alpha$ is analogous to that of (10). That is, the standard deviation of the fractional fluctuations induced in equity prices by fluctuations in the risk-aversion parameter is $|\mathscr{E}_\alpha|$ times the standard deviation of the fractional fluctuations in the risk-aversion parameter. Note that, unlike $\mathscr{E}_\beta$, the sign of $\mathscr{E}_\alpha$ depends in general on the values of the parameters. The values of $|\mathscr{E}_\alpha|$ for some plausible values of β and α are presented in Table 2.

Table 2
Values of $|\mathscr{E}_\alpha|$ for some values of β and α

Risk-aversion parameter, α	Discount factor, β		
	0.99	0.98	0.96
1.1	1.65	0.89	0.46
2.0	1.25	0.91	0.59
3.0	1.11	0.91	0.66

5.1. *Relative values of $\mathscr{E}_\alpha$ and $\mathscr{E}_\beta$*

It is apparent from Tables 1 and 2 that the values of $\mathscr{E}_\alpha$ are significantly smaller than those of $\mathscr{E}_\beta$. To develop some intuition in this regard, it is useful to consider the polar case of certainty; that is, when $\sigma = 0$. In this case, (7) yields $|\partial p/\partial\alpha|/|\partial p/\partial\beta| = \mu/\beta$. Since $\beta \approx 1$ and $\mu \approx 0.02$, the sensitivity of the equity price to α is about 2% of that to β. Further, since $|\mathscr{E}_\alpha|/\mathscr{E}_\beta = \mu\alpha$, the values of $|\mathscr{E}_\alpha|$ will be much smaller than those of $\mathscr{E}_\beta$, for commonly accepted values of α.

6. Non-local fluctuations in the risk-aversion parameter

The parameter of relative risk-aversion is now random, and is denoted by $\tilde{\alpha}$. This will likely induce fluctuations in equity prices, denoted as $\tilde{p}$. Note that, for notational simplicity, we use the same symbol $\tilde{p}$ to denote the random equity prices as we did in Section 6. However, it is important to observe that the source of randomness in $\tilde{p}$ in the present context is the randomness in $\tilde{\alpha}$; whereas, in Section 4, it was the randomness in the discount factor. In the present analysis, the discount factor is non-random. Define $\bar{\alpha} \equiv \mathrm{E}\{\tilde{\alpha}\}$, and $\bar{p} \equiv \mathrm{E}\{\tilde{p}\}$.

The range of $\tilde{\alpha}$ is

$$0 < \tilde{\alpha} < \infty. \tag{21}$$

Define

$$\tilde{\alpha} \equiv \bar{\alpha}(1 + \tilde{a}) \quad \text{or equivalently,} \;\; \tilde{a} = (\tilde{\alpha} - \bar{\alpha})/\bar{\alpha}. \tag{22}$$

That is, $\tilde{a}$ represents the fractional fluctuation of $\tilde{\alpha}$ around its mean. Further, analogous to what was done earlier, define

$$s_\alpha \equiv \varsigma\{\tilde{a}\} \quad \text{and} \quad S_\alpha \equiv \varsigma\{(\tilde{p} - \bar{p})/\bar{p}\}. \tag{23}$$

The first expression in (23) represents the standard deviation of the fractional fluctuation of $\tilde{\alpha}$. The second expression represents the standard deviation of the induced fractional fluctuations in equity prices.

We assume that $\tilde{a}$ is a continuous uniform variate with parameters $[-\lambda, \lambda]$, where $\lambda > 0$. By definition,

$$-\lambda \leqslant \tilde{a} \leqslant \lambda. \tag{24}$$

It follows that

$$s_\alpha = \lambda/\sqrt{3}. \tag{25}$$

The expression for S_α is derived in Appendix A, and it is

$$S_\alpha = [\mathrm{E}\{(\tilde{p})^2\}/(\bar{p})^2 - 1]^{1/2}, \tag{26}$$

where

$$\bar{p} = y_t \frac{m_1}{2\lambda} \int_{-\lambda}^{\lambda} \frac{\mathrm{e}^{m_2\tilde{a}+m_3(\tilde{a})^2}}{1 - m_1 \mathrm{e}^{m_2\tilde{a}+m_3(\tilde{a})^2}}\, \mathrm{d}\tilde{a}, \tag{27}$$

$$\mathrm{E}\{(\tilde{p})^2\} = y_t^2 \frac{m_1^2}{2\lambda} \int_{-\lambda}^{\lambda} \frac{\mathrm{e}^{2[m_2\tilde{a}+m_3(\tilde{a})^2]}}{[1 - m_1 \mathrm{e}^{m_2\tilde{a}+m_3(\tilde{a})^2}]^2}\, \mathrm{d}\tilde{a}, \tag{28}$$

$$m_1 \equiv \beta \mathrm{e}^{(1-\bar{\alpha})(\mu-(1/2)\bar{\alpha}\sigma^2)}, \tag{29}$$

$$m_2 \equiv -\mu\bar{\alpha} + \bar{\alpha}\sigma^2(\bar{\alpha} - \tfrac{1}{2}) \quad \text{and} \quad m_3 \equiv \tfrac{1}{2}(\bar{\alpha})^2\sigma^2. \tag{30}$$

Further, given (24), it is shown in Appendix A that a sufficient condition for (21) to be satisfied is

$$\lambda < 1. \tag{31}$$

Using (25) and (26), we map the values of s_α into the values of S_α, for a range of values of β and $\bar{\alpha}$, such that the parameter values satisfy (8) and (31). For these mappings, we numerically evaluate the integrals on the right-hand sides of (27) and (28). A typical mapping, in which $\beta = 0.99$, and $\bar{\alpha} = 2$, is presented in Fig. 2. This mapping is nearly linear, which is also

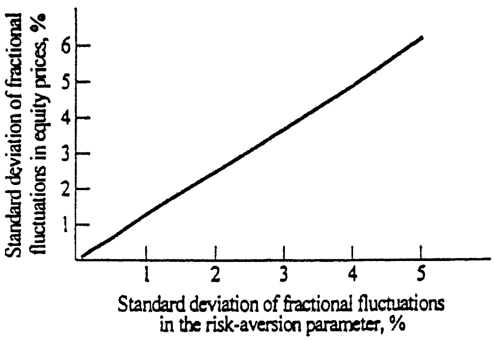

Fig. 2.

the case for other mappings, with several different values of β and $\bar{\alpha}$, that we have examined.

7. Concluding remarks

Over the past two decades, a central concern of the literature on financial economics has been to better understand the volatility of equity prices. This concern is pertinent, given the crucial informational and allocational role that the prices of financial assets play in a modern economy. As some researchers (for example, Black, 1986) have informally suggested, there are perhaps a large number of causes that interactively impact on the volatility of equity prices. Our objective in this paper has been to highlight and assess the potential role of two previously unexplored sources of volatility, namely, the fluctuations in investors' discount factors and in their levels of risk-aversion. We find that, distinct from the role of the evolution of external economic fundamentals, both of these sources induce volatility in equity prices. In addition, we find that small fluctuations in investors' discount factors induce large fluctuations in equity prices.

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Appendix A.

A.1. Derivation of expression (7)

If Z is a lognormal variate with parameters (ξ, h), then, for compactness, we state it as: $Z \sim L(\xi, h)$. Here, $\xi = \mathrm{E}\{\ln Z\}$, and h^2 is the variance of $\ln Z$. Then, for a fixed value of parameter q, we have (see Johnson et al. (1995, Vol. 1, p. 212)):

$$\mathrm{E}\{Z^q\} = \mathrm{e}^{q\xi+(1/2)q^2h^2}. \tag{A.1}$$

Recalling (6), $x(\tau) \sim L((\mu - (1/2)\sigma^2)\tau, \sigma\sqrt{\tau})$. Since G_{t+j} is an observation of $y(\tau)$ at $\tau = j$, it follows that $G_{t+j} \sim L((\mu - (1/2)\sigma^2)j,\ \sigma\sqrt{j})$. Using (A.1), $\mathrm{E}\{G_{t+j}^{1-\alpha}\} = \mathrm{e}^{kj}$, where

$$k \equiv (1-\alpha)(\mu - \tfrac{1}{2}\alpha\sigma^2). \tag{A.2}$$

The preceding definition and (5) yield

$$p_t = y_t \sum_{j=1}^{\infty} (\beta \mathrm{e}^k)^j. \tag{A.3}$$

Define

$$\phi \equiv \beta \mathrm{e}^k. \tag{A.4}$$

The series on the right-hand side of (A.3) converges if

$$\phi < 1, \tag{A.5}$$

and we obtain

$$p = y_t \frac{\phi}{(1-\phi)}. \tag{A.6}$$

Substitution of the definition (A.2) and (A.4) into (A.6) yields the desired expression (7).

A.2. Derivation of expression (9)

From (A.4) and (A.6), it follows that $\partial p/\partial\beta = p/\beta(1-\phi)$. Hence,

$$\mathscr{E}_\beta \equiv (\beta/p)\partial p/\partial\beta = 1/(1-\phi). \tag{A.7}$$

This, using definitions (A.2) and (A.4), yields the middle part of (9). The sign of $\mathscr{E}_\beta$ follows from (A.5) and (A.7).

A.3. Derivation of expression (17)

From (12), (A.4), and (A.6), we have

$$\tilde{p} = y_t \frac{\tilde{\beta}\mathrm{e}^k}{1-\tilde{\beta}\mathrm{e}^k} = y_t \bar{\beta}\mathrm{e}^k \left[\frac{(1+\tilde{b})}{1-\bar{\beta}\mathrm{e}^k(1+\tilde{b})} \right]. \tag{A.8}$$

Define

$$\varphi \equiv \bar{\beta}\mathrm{e}^k \quad \text{and} \quad \rho \equiv (1+\tilde{b}). \tag{A.9}$$

Then, (A.8) becomes

$$\tilde{p} = y_t \varphi \frac{\rho}{1-\varphi\rho}. \tag{A.10}$$

A change in the variable of integration, from $\tilde{b}$ to ρ, yields

$$\begin{aligned} \bar{p} \equiv \mathrm{E}(\tilde{p}) &= y_t \varphi \int_{1-\eta}^{1+\eta} \frac{\rho}{1-\varphi\rho} \frac{1}{2\eta} \, \mathrm{d}\rho \\ &= \frac{y_t \varphi}{2\eta} \int_{1-\eta}^{1+\eta} \frac{\rho}{1-\varphi\rho} \, \mathrm{d}\rho \;\; y_t \left[\frac{\ell_2}{2} - 1 \right], \end{aligned} \tag{A.11}$$

where ℓ_2 is defined in (18).

From (A.10), $(\tilde{p})^2 = (y_t\varphi)^2\rho^2/(1-\varphi\rho)^2$. Hence,

$$\mathrm{E}\{(\tilde{p})^2\} = \frac{(y_t\varphi)^2}{2\eta}\int_{1-\eta}^{1+\eta}\frac{\rho^2}{(1-\varphi\rho)^2}\,\mathrm{d}\rho.$$

This yields

$$\mathrm{E}\{(\tilde{p})^2\} = y_t^2(\ell_1 - \ell_2), \tag{A.12}$$

where ℓ_1 is defined in (18).

Next, the definition of variance implies that $\mathrm{Var}\{(\tilde{p}-\bar{p})/\bar{p}\} = \mathrm{E}\{(\tilde{p})^2\}/(\bar{p})^2 - 1$. Substitution of (A.11) and (A.12) into the preceding expression yields (17).

A.4. Derivation of expression (19)

Consider the first part of (19) and combine it with the second inequality in (15). This yields $1/\bar{\beta} - 1 > \tilde{b}$. In turn, using the definition of $\tilde{\beta}$ in (12), it follows that $1 > \tilde{\beta}$. This satisfies the second inequality in (11).

Next, consider the second part of (19). This yields $1 > 1/\bar{\beta} - 1$. This and the first part of (19) yield $1 > \eta$, or $-\eta > -1$. Thus, from the first part of (15), we obtain $\tilde{b} > -1$, or $1 + \tilde{b} > 0$, or $\bar{\beta}(1+\tilde{b}) > 0$. Thus, recalling the definition of $\tilde{\beta}$ in (12), it follows that the first inequality in (11) is satisfied.

A.5. Derivation of expression (20)

From (A.2) and (A.4), $\partial\phi/\partial\alpha = -\phi(\mu + 1/2\sigma^2 - \alpha\sigma^2)$. This and (A.6) yield $\partial p/\partial\alpha = -y_t\phi(1-\phi)^{-2}(\mu + (1/2)\sigma^2 - \alpha\sigma^2)$. In turn, using (A.6) and (A.7), $\partial p/\partial\alpha = -p(\mu + (1/2)\sigma^2 - \alpha\sigma^2)\mathscr{E}_\beta$. Hence, $\mathscr{E}_\alpha \equiv (\alpha/p)\partial p/\partial\alpha = -\alpha(\mu + (1/2)\sigma^2 - \alpha\sigma^2)\mathscr{E}_\beta$.

A.6. Derivation of expressions (26)–(30)

Recalling (A.2), k is now random, and is denoted as $\tilde{k} = (1-\tilde{\alpha})(\mu - (1/2)\tilde{\alpha}\sigma^2)$. Substitution of the first part of (22) into the preceding expression yields

$$\tilde{k} = (1-\bar{\alpha})(\mu - \tfrac{1}{2}\bar{\alpha}\sigma^2) + [-\mu\bar{\alpha} + \bar{\alpha}\sigma^2(\bar{\alpha} - \tfrac{1}{2})]\tilde{a} + [\tfrac{1}{2}(\bar{\alpha})^2\sigma^2](\tilde{a})^2. \tag{A.13}$$

Thus, using the definitions (29) and (30), the expression (A.4) becomes $\phi = m_1 \mathrm{e}^{m_2\tilde{a} + m_3(\tilde{a})^2}$. Hence, (A.6) yields

$$\tilde{p} = y_t\frac{m_1\mathrm{e}^{m_2\tilde{a} + m_3(\tilde{a})^2}}{1 - m_1\mathrm{e}^{m_2\tilde{a} + m_3(\tilde{a})^2}}. \tag{A.14}$$

Expressions (27) and (28) follow from (A.14). Expression (26) follows directly from the definition of S_α in (23).

A.7. Derivation of expression (31)

Expression (31) implies that $\lambda < \infty$. This and the second inequality in (24) yield $\tilde{a} < \infty$. In turn, $\bar{\alpha}(1 + \tilde{a}) < \infty$, since $\bar{\alpha} > 0$. Thus, using the definition of $\tilde{\alpha}$ in (22), it follows that the second inequality in (21) is satisfied.

Next, (31) implies that $-\lambda > -1$. Thus, from the first inequality in (24), $\tilde{a} > -1$. In turn, $\bar{\alpha}(1 + \tilde{a}) > 0$. Thus, from the definition of $\tilde{\alpha}$ in (22), it follows that the first inequality in (21) is satisfied.

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