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Junior must pay: pricing the implicit put in privatizing Social Security*

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Summary. Proposals that a portion of the Social Security Trust Fund assets be invested in equities entail the possibility that a severe decline in equity prices will render the Fund's assets insufficient to provide the currently mandated level of benefits. In this event, existing taxpayers may be compelled to act as insurers of last resort. The cost to taxpayers of such an implicit commitment equals the value of a put option with payoff equal to the benefit's shortfall. We calibrate an OLG model that generates realistic equity premia and value the put. With 20 percent of the Fund's assets invested in equities, the highest level currently under serious discussion, we value a put that guarantees the currently mandated level of benefits at one percent of GDP, or a temporary increase in Social Security taxation of, at most, 20 percent. We value a put that guarantees 90 percent of benefits at .03 percent of GDP. In contrast to the earlier literature, our results account for the significant changes in the distribution of security returns resulting from Trust Fund purchases.

Keywords and Phrases: Social Security Trust Fund, Privatized Social Security, Government warranties, Put options, Overlapping generations.

JEL Classification Numbers: D91, E2, E60, G11, G13, H55.

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1 Introduction

This paper is a contribution to the current debate on the privatization of Social Security. The term *privatization* covers a gamut of policy definitions and implications. We use the term to refer to the proposal to invest a portion of Social Security Trust Fund assets in the stock market. Currently, the Social Security Trust Fund (hereafter, SSTF) is restricted to invest only in United States Treasury securities.

Unlike capital market investments, Social Security is a form of social insurance, in that it *implicitly is designed to guarantee a minimum consumption level* for its participants. Suppose that a large fraction of Social Security taxes (Trust Funds) is invested in equities. Given the volatility of the United States stock market over the past 70-odd years, there is a distinct possibility that Social Security funds invested in the stock market may decline in value to a level such that they are inadequate to provide a subsistence level of benefits. In such a situation, the government may be compelled to remedy the shortfall by raising taxes on the younger working generations.

We argue that any *time-consistent* discussion of privatizing Social Security must take into account this *de facto* role of the younger working generations as insurers of last resort. By providing a consumption floor for the retired cohort, current wage earners effectively grant themselves a put option on their risky portfolio, to be honored by succeeding generations. We consider these issues in the context of a dynamic OLG model where cohorts live for three periods. Three related topics guide our discussion:

1. The explicit pricing of the aforementioned put option. We undertake this exercise in order to give planners and policy makers an idea of the order of magnitude of the costs involved. When 20 percent of Trust Fund assets are invested in equities, the highest level currently under serious discussion, we find that a put that guarantees the currently mandated level of benefits is priced at one percent of GDP for reasonable model parameterizations. This corresponds to a temporary increase in Social Security taxation vis-à-vis the current level of, at most, 20 percent[1]. A put that guarantees only 90 percent of the currently mandated benefits is priced at roughly .03 percent of GDP, which is proportionately much less.

2. The replication of the observed pattern of security returns. The impetus for Social Security privatization, as we have defined it, is to allow SSTF beneficiaries to benefit from the historically higher return on the stock market as compared to government debt (the equity premium). A comprehensive discussion of the potential costs and benefits of privatization must therefore occur in the context of a model that replicates this premium[2]. Our paradigm satisfies this requirement, while also reasonably approximating observed return volatilities. Furthermore, all of our results

[1] The time period in the model that we are about to describe corresponds to twenty years of calendar time. This statement thus implies that social security taxes will have to be increased by the indicated amounts for roughly twenty years.

[2] The question as to why the historical equity premium is so high and the real rate of interest is so low was first raised in Mehra and Prescott (1985). For a current survey of the literature on the equity premium see Mehra (2003) and Mehra and Prescott (2003).

fully take into account the non-trivial changes in the distributions of security returns attendant to the initiation of SSTF equity purchases.

3. A narrowing of the old-aged income distribution. Another justification for privatization is the assertion that it will reduce old-aged income inequality by empowering lower income individuals currently constrained from investing in the stock market to have stock market investments undertaken on their behalf[3]. Our model displays this feature and does so for an entirely straightforward reason: when the SSTF undertakes equity investments, it partially crowds out high income middle aged investors who, as a result, necessarily acquire fewer securities and pay more for them. Ceteris paribus, their income and consumption decline in both middle age and as old persons. The opposite is true for the low-income middle aged previously outside the equity markets: the high returns afforded by equity investing increases their expected income and consumption when they are old, without diminishing their middle aged consumption level. Old age income and consumption inequality is thus reduced. The full welfare consequences of SSTF equity investing, per se, are also discussed.

The direct antecedents of our work are the models analyzed in Feldstein et al. [11], Smetters [25], and Pennacchi [24]. These papers evaluate a "benefits guarantee" or put in the context of partial equilibrium models that take as fixed the distribution of equilibrium returns. Our general equilibrium approach allows this distribution to evolve endogenously. In related work, Abel [1,2] and Diamond and Geanakoplos [10] explore the implications of investing SSTF assets in the stock market for aggregate investment and the time path of the capital stock. Our exchange economy shares many features with Abel's [2] production economy; however, he focuses on capital stock dynamics.

More recently, Abel [3] explores the effects of a high birth rate (baby boom) on the dynamics of the price of capital. While the model considered in this paper does not admit population growth, it can be easily adapted to do so.

Our conclusions are also broadly consistent with those of Diamond and Geanakoplos [10]. In particular, we also find that diversification by the SSTF into equities reduces the equity premium and presents the opportunity for welfare enhancements. Diamond and Geanakoplos [10] do not, however, present numerical estimates of the magnitude of the effects they detail. Albeit in a simpler context, this estimation is a primary focus of our work and, in that sense, it complements many of their conclusions. While recognizing the nature of the risks involved, they also do not seek to value the Social Security put. In a series of papers, Bohn [4–6] and Campbell, Cocco, Gomes, and Maenhout [7] study the risk reallocation characteristics of various Social Security financing and payout arrangements, especially defined benefit versus defined contribution systems. We consider similar issues, but in the context of comparing pay-as-you-go versus partial equity SSTF financing schemes. Excellent discussions of many of the practical issues surrounding

[3] Contrary to the predictions of the standard model, up to 60% of the US population does not participate in the equities market. Clearly the model does not capture some of the key features of the economy that inhibits this participation. In this paper we do not endogonize non participation but rather capture the phenomena by imposing a non participation constraint on some agents. See Section 2 below.

Trust Fund stock market investing and Social Security reform in general can be found in the volumes edited by Campbell and Feldstein [8] and Mitchell et al. [22]. Many of the issues underlying the need for Social Security reform of some type are well documented in Geanakoplos et al. [13], Kotlifkoff et al. [14] and Mitchell and Zeldes [23].

The paper is organized as follows: In Section 2 we describe the economy and define equilibrium. In Section 3 we explain how the Social Security put is valued. We calibrate the economy in Section 4 and detail our results in Section 5. Our conclusions are presented in Section 6.

2 The economy

We consider a one-good overlapping-generations, pure exchange economy. It is a version of the economy studied in Constantinides et al. [9], generalized to account for Social Security transfers. Each generation consists of a continuum of consumers who live for three periods as *young*, *middle- aged* and *old*. Three is the minimal number of periods that captures the heterogeneity of consumers across the age groups that we wish to emphasize: the *borrowing-constrained* young, the *wage-earning* middle-aged, and the *dis-saving* old. In the calibration, each period is taken to represent 20 years. By restricting our attention to an exchange setting, we focus on the purely re-distributive effects. That is, we assume that the technological or other mechanisms that give rise to the yearly aggregate income process of the United States economy are invariant to the manner by which Social Security payments are financed. Abel [2] provides a different perspective.

There is one consumption good in each period which perishes at the end of the period. Wages, Social Security taxes, consumption, payments from, and prices of all securities traded in the model are denominated in units of the consumption good. We admit two types of securities in positive net supply, a consol *bond* and *equity*. Consumers may purchase these securities but realistic borrowing and short sale restrictions are in effect. We consider various scenarios under which the SSTF may purchase either security or a combination thereof.

The bond is default-free and pays one unit of consumption in every period in perpetuity. We think of the bond as a proxy for long-term government debt. Its supply is fixed at B units in perpetuity[4]. The *ex coupon* bond price in period t is denoted by q_t^b and represents the price of the claim to the unit coupon paid in perpetuity, beginning with period $t+1$.

Equity in this model is a claim to a residual dividend stream and pays a net dividend d_t^e in period t. We think of equity as the sum total of the claims to firms including net corporate debt payments, net rental payments, etc. The issue and repurchase of equities and bonds is implicitly accounted for by the fact that the equity is defined as the claim to the *net* dividend. The *ex dividend* price of equity in period t is denoted by q_t^e. This equity security represents ownership of the dividend

[4] We focus on consol bonds for two reasons. First, it is specious to introduce a short-term (say, one-year) bond in this economy because the length of one period is assumed to be 20 years. Second, the Trust Fund is invested primarily in coupon bearing Treasury securities rather than discount T-bills, and we would like the model to reflect this fact.

stream in perpetuity, beginning with period $t+1$; and its supply is fixed at one share in perpetuity.

2.1 Wage and income profiles

Consumers born in period *t* receive deterministic wage income $w_0 > 0$, when young. We interpret w_0 to be net of all income (as opposed to social insurance) taxes. Without loss of generality, the Social Security taxes levied on the young are divided into three components, τ_0, τ_0^e, and τ_0^b, according to the manner by which they are invested. The tax τ_0 is that portion paid out immediately to the current old generation under a pay-as-you-go system. This is largely the form of current Social Security finance. The second and third components, τ_0^e and τ_0^b, represent the taxes invested in equity and bonds, respectively. The actual quantities purchased of each security are thus τ_0^e/q_t^e (equity) and τ_0^b/q_t^b (bonds).

We assume that the young do not privately participate in the financial markets[5]. Their budget constraint is

$$c_{t,0} \leq w_0 - \tau_0 - \tau_0^e - \tau_0^b \tag{2.1}$$

where $c_{t,0}$ denotes the consumption of the young born in period *t*.

When entering middle age, a young consumer either becomes a high-wage earner with probability *h* or a low wage earner with probability $1-h$. Consumers become aware of their high or low wage status only upon entering middle age, and it is an event against which it is impossible for them to insure. Since we assume a continuum of consumers, *h* also represents the fraction of the middle-aged cohort who are high-wage earners; accordingly, the fraction of low wage earners is $1-h$. A high-wage middle-aged consumer born in period t receives stochastic real wage income $\tilde{w}_{t,1}^H$, out of which he pays Social Security taxes, makes private investments in equity and bonds and consumes.

We denote by $c_{t,1}^H$ the consumption in period $t+1$ of a high-wage middle-aged consumer born in period *t*. We denote by $z_{t,1}^e$ and $z_{t,1}^b$ this consumer's private purchases of equity and bonds in period $t+1$. The three Social Security tax components, $\tau_{H,1}, \tau_{H,1}^e$ and $\tau_{H,1}^b$ are directly analogous to the corresponding quantities levied on the young: $\tau_{H,1}$ denotes the pay-as-you-go component while $\tau_{H,1}^e$ and $\tau_{H,1}^b$ denote the amounts invested in equity and bonds, respectively. The actual quantities purchased of each security are $\tau_{H,1}^e/q_{t+1}^e$ (stock) and $\tau_{H,1}^b/q_{t+1}^b$ (bonds), and the budget constraint for the high-wage middle-aged consumer is thus

$$c_{t,1}^H + q_{t+1}^e z_{t,1}^e + q_{t+1}^b z_{t,1}^b \leq \tilde{w}_{t,1}^H - \tau_{H,1} - \tau_{H,1}^e - \tau_{H,1}^b. \tag{2.2}$$

[5] This assumption reflects an implicit borrowing constraint in the following sense: under our calibration, consumers experience a very steep consumption profile while passing from young to middle age. The young would not wish to save under these conditions. Consumption smoothing considerations suggest they would rather wish to borrow against their future (higher) middle-aged income. In practice, this is difficult to execute without holding other assets (collateral). As we exclude bequests, there is no provision for an accumulation of collateral by the young. We summarize these constraints by the indicated assumption.

Circumstances are similar for the low-income middle-aged consumers, except that we assume that they do not privately participate in the financial markets. The notation is analogous to that detailed above, except for the superscript/subscript L which identifies this group of consumers. We assume that the wage income of this group, $w^L_{t,1}$, is deterministic. The possibilities that $\tau^e_{H,1} \neq \tau^e_{L,1}$ and $\tau^b_{H,1} \neq \tau^b_{L,1}$ and that consumers of different income levels pay different aggregate Social Security taxes and receive different levels of benefits (which is indeed the case in the United States) are also allowed. The budget constraint for the low-wage middle-aged consumer is

$$c^L_{t,1} \leq w^L_{t,1} - \tau_{L,1} - \tau^e_{L,1} - \tau^b_{L,1}.^{6} \tag{2.3}$$

For simplicity, we rule out bequests. For a period- t-born consumer of low middle-aged income status, the period $t+2$ retirement consumption, $c^L_{t,2}$, is limited to the wage in retirement, w_2, plus Social Security benefits

$$c^L_{t,2} \leq w_2 + \tau_{L,1} + \tau_0 + \left(q^e_{t+2} + d^e_{t+2}\right)\left(\frac{\tau^e_{L,1} + \tau^e_0}{q^e_{t+1}}\right) + \left(q^b_{t+2} + 1\right)\left(\frac{\tau^b_{L,1} + \tau^b_0}{q^b_{t+1}}\right). \tag{2.4}$$

The latter quantity includes not only the elements arising from the pay-as-you-go aspects of the system $(\tau_{L,1} + \tau_0)$ but also the proceeds of the sales of securities held on the consumer's behalf under the proposed financing alternatives.

In the case of a previously high-income middle-aged consumer, consumption in old age, $c^H_{t,2}$, is further augmented by the proceeds of the sale of the consumer's privately held securities:

$$\begin{aligned} c^H_{t,2} \leq\ & w_2 + \tau_0 + \tau_{H,1} + \left(q^e_{t+2} + d^e_{t+2}\right)\left(z^e_{t,1} + \frac{\tau^e_{H,1} + \tau^e_0}{q^e_{t+1}}\right) \\ & + \left(q^b_{t+2} + 1\right)\left(z^b_{t,1} + \frac{\tau^b_{H,1} + \tau^b_0}{q^b_{t+1}}\right).^{7} \end{aligned} \tag{2.5}$$

Equations (2.4) and (2.5) imply that the taxes on the young which are invested in equity and bonds benefit the generation that is old in the next period rather than the young themselves, when they become old in two periods. Thus, we do not have the feature of generation-specific accounts that are held and augmented for multiple periods, whereby a consumer's benefits are determined by his contributions alone. Rather, our model is one of aggregate equity investment by the Trust Fund on behalf of all beneficiaries collectively. Since the level of public debt outstanding is $3.5 trillion, but the assets of the fund are expected to peak at $7.5 trillion, it is clear

[6] Our assumption of non-market participation for this class of consumers is fully exogenous, as their income profile looking forward to old age is rapidly diminishing under our calibration. This is also an empirical fact for a large subset of the United States population, yet they do not save. We offer no explanation for this phenomenon but model it as an exogenous fact in the manner indicated.

[7] Equations (2.4) and (2.5) suggest that taxes paid by the low-income middle-aged exclusively fund the retirement of the low-income old, and analogously for the high-income middle-aged, as though the low and high-income "accounts" were segregated from one another. This is not the case in reality; the current representation rather is intended to capture the fact that under the present Social Security system, the level of benefits is, at least in part, determined by the level of prior contributions.

that other assets besides bonds must be purchased by the fund at large and equities are a natural first choice[8]. We thus focus on modeling this aggregate phenomenon and retain the current formulation.

Negative consumption and personal bankruptcy are ruled out by imposing the following constraints which are easily satisfied under our parameterization:

$$z^e_{t,1} \geq 0, \quad z^b_{t,1} \geq 0, \quad c^H_{t,1} \geq 0, \quad c^L_{t,1} \geq 0, \quad c^H_{t,2} \geq 0, \text{ and } c^L_{t,2} \geq 0. \tag{2.6}$$

We study the equilibrium security prices and the value of the implied Social Security put as we alter the relative mix of $\left\{\tau^e_0, \tau^b_0, \tau_{H,1}, \tau^e_{H,1}, \tau^b_{H,1}\right\}$ and $\left\{\tau^e_0, \tau^b_0, \tau_{L,1}, \tau^e_{L,1}, \tau^b_{L,1}\right\}$, maintaining constant their respective total values.

A consumer born in period *t* maximizes expected utility

$$\max E\left[\sum_{i=0}^{2} \beta^i u(c_{t,i}) | \Im_t\right] \tag{2.7}$$

subject to conditions (2.1), (2.3), and (2.4), if he is of low-income in middle age; or subject to conditions (2.1), (2.2), and (2.5), if he is of high-income in middle age. Under formulation (2.7) $\Im_t$ denotes the information available to the consumer at date *t*, and β is the subjective discount factor. The period utility function is of the standard form $u(c) = (1-\gamma)^{-1} c^{1-\gamma}$, $\gamma > 0$. It is clear that there are no decisions to be made in the first and final periods of a consumer's life: only the high-income consumers have any decision to make, and then only when middle-aged. Problem (2.7) effectively reduces to a one-period problem[9].

Aggregate income is given by

$$\tilde{y}_t = w_0 + h\tilde{w}^H_{t-1,1} + (1-h)\, w^L_{t-1,1} + w_2 + B + d^e_t. \tag{2.8}$$

Note that *B* captures the aggregate coupon on government debt.

In our numerical work to follow, we assume that there are $j = 1, 2, \ldots, J$, states of nature. From our prior discussion we note that there are two fundamental sources of uncertainty in this economy as captured by the stochastic processes on aggregate income, $\tilde{y}_t \equiv y(j)$, and the high income middle-aged wage, $\tilde{w}^H_{t,1} \equiv w_{H,1}(j)$. We model these as a joint time-stationary Markov chain with transition matrix $\Pi = \{\pi_{ij}\}$ and a unique stationary probability distribution.[10] Since the process is

[8] An alternative to equity investing is simply for the SSTF to maintain "journal entries" indicating the extent of the present value of future tax liabilities earmarked for it. These entries would equal the magnitude of tax revenues not immediately paid to beneficiaries (and thus presumably transferred to the general fund). Such a practice would allow the continued trading of government debt to facilitate the conduct of monetary policy.

[9] In particular, the consumer is unable while young to insure against the event that he will receive a low-income realization when in middle age. The likelihood of this event is thus not present in (2.7), and equilibrium is invariant to whether or not the consumer knows the future income state.

[10] In the spirit of Lucas, the model abstracts from growth, and considers an economy that is stationary in levels. The average growth in total output is thus zero. Mehra [16] and Mehra and Prescott [19], however, study an economy that is stationary in growth rates and has a unit root in levels. In their model, the effect of the latter generalization is to increase the mean return on all financial assets relative to

time stationary, there exists an equilibrium with time stationary pricing and security demand functions.

The aggregate dividend is the residual

$$d_t^e = \tilde{y}_t - w_0 - h\tilde{w}_{t-1,1}^H - (1-h)w_{t-1,1}^L - B - w_2. \tag{2.9}$$

As a function of the state *j*, the aggregate dividend is denoted as $d_t^e = d(j)$.[11]

Two additional comments are in order. First, our model identifies the middle-aged consumers with a high wage income as the only cohort that invests in equity and bonds. We acknowledge that private ownership of a large amount of securities need not necessarily be coincident with high wage income but in a world without bequests this is a reasonable assumption. Second, the assumption of a non-stochastic wage income for the non-stockholding class allows the model to reflect the empirical observation that the consumption of stockholders is more variable than that of non-stockholders; see Mankiw and Zeldes [15].

2.2 Equilibrium

We consider the set of stationary rational expectations equilibria for which the consumption and investment policies of the consumers born in each period and the bond and equity prices $q^b(j)$ and $q^e(j)$ are measurable with respect to the current state (j) and are such that: (a) each consumer's consumption and investment policy maximizes the consumer's expected utility from the set of admissible policies while taking the price processes as given; and (b) bond and equity markets clear in all periods.

Specifically, let $z_1^e(j)$ and $z_1^b(j)$ denote the holdings of equity and bonds, respectively, by a high-income middle-aged consumer in state *j*, and let $w_{H,1}(j)$ denote his wage income. The first-order necessary conditions, which are also sufficient, are stated below:

$$\begin{aligned}
&u_1(w_{H,1}(j) - z_1^e(j)q^e(j) - z_1^b(j)q^b(j) - \tau_{H,1}^e - \tau_{H,1}^b - \tau_{H,1})\, q^b(j) \\
&\quad = \beta \sum_{k=1}^{J} u_1((q^e(k) + d^e(k))[z_1^e(j) + (\tau_{H,1}^e + \tau_0^e)/q^e(j)] \\
&\qquad + (q^b(k) + 1)[z_1^b(j) + (\tau_{H,1}^b + \tau_0^b)/q^b(j)] + w_2 + \tau_0 + \tau_{H,1})[q^b(k) + 1]\pi_{jk}
\end{aligned} \tag{2.10}$$

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what would prevail, ceteris paribus, in a stationary-in-levels economy, but the return differentials across different securities are not much affected. The intuition is as follows: with growth creating preordained increases in future consumption relative to the present, investors require greater mean returns from all securities across the board in order to be induced to postpone consumption. The point of all this is that our life cycle considerations can be examined in either context; we choose the stationary-in-levels one because it is marginally simpler computationally, and better matches observed mean return data. It is also consistent with zero population growth, another feature of our model.

[11] We implicitly interpret d_t^e as the after tax income to corporate capital. McGrattan and Prescott (2000) provide data that suggest that over 90 percent of corporate capital is equity capital; we use the indicated approximation (100 percent of corporate capital is equity). We also assume that income taxes exactly cover government expenditures (a balanced budget) so that neither appears in the economy-wide budget constraint.

and

$$
\begin{aligned}
& u_1(w_{H,1}(j) - z_1^e(j)\, q^e(j) - z_1^b(j)\, q^b(j) - \tau_{H,1}^e - \tau_{H,1}^b - \tau_{H,1})\, q^e(j) \\
& \quad = \beta \sum_{k=1}^{J} u_1((q^e(k) + d^e(k))[z_1^e(j) + (\tau_{H,1}^e + \tau_0^e)/q^e(j)] \\
& \qquad + (q^b(k) + 1)[z_1^b(j) + (\tau_{H,1}^b + \tau_0^b)/q^b(j)] + w_2 + \tau_0 + \tau_{H,1}) \\
& \qquad \times [q^e(k) + d^e(k)]\pi_{jk}.
\end{aligned} \tag{2.11}
$$

The market clearing conditions are

$$B = h\, z_1^b(j) + h(\tau_{H,1}^b + \tau_0^b)/q^b(j) + (1-h)\,(\tau_{L,1}^b + \tau_0^b)/q^b(j). \tag{2.12}$$

and

$$1 = h\, z_1^e(j) + h(\tau_{H,1}^e + \tau_0^e)/q^e(j) + (1-h)\,(\tau_{L,1}^e + \tau_0^e)/q^e(j). \tag{2.13}$$

Definition: *An equilibrium for the economy described by equations (2.1) – (2.8) is a set of security demand and price functions* $\left\{ z_1^e(j), z_1^b(j), q^e(j), q^b(j) \right\}$, $j = 1, 2, \ldots, J$ *which satisfy equations (2.10)–(2.13).*

Under standard conditions, equilibrium exists and is easily computable.[12]

3 Valuing the social security put option

We consider a scenario in which high-income middle-aged retirees are guaranteed a level of benefits M_H and low-income middle-aged retirees are guaranteed a level of benefits M_L, where $M_L \le M_H$. We argue that the current system is effectively pay-as-you-go because the assets of the Social Security Trust Fund were \$798 billion on December 31, 2000, while the present value of all liabilities is estimated to be in excess of \$9 trillion. The current system is thus reasonably captured in our scenario by requiring that

$$\tau_0 + \tau_{H,1} = M_H \quad (\tau_{H,1}^e = \tau_{H,1}^b = 0)$$

and

$$\tau_0 + \tau_{L.1} = M_L \quad (\tau_{L,1}^e = \tau_{L,1}^b = 0). \tag{3.1}$$

Using the specifications in (3.1) as the benchmark for comparison, the current proposals to privatize Social Security can be broadly classified under two operational definitions of the term “privatize”.

[12] Note that we define (and subsequently compute) steady state equilibria. A proof of existence is available from the authors. We do not consider equilibrium transition paths between steady states. Feldstein et al. (2001) and Smetters (2001) explore this issue in detail.

Plan 1. Leave the current level of Social Security taxation unchanged while diverting some fraction of the tax revenues to equities and/or long-term bonds. This amounts to requiring

$$\tau_0 + \tau_0^e + \tau_0^b + \tau_{H,1} + \tau_{H,1}^e + \tau_{H,1}^b = M_H$$

and

$$\tau_0 + \tau_0^e + \tau_0^b + \tau_{L,1} + \tau_{L,2}^e + \tau_{L,2}^b = M_L, \tag{3.2}$$

with at least one of $\{\tau_H^e, \tau_H^b\}$ and at least one of $\{\tau_L^e, \tau_L^b\}$ being strictly positive. Under Plan 1, the resources available for benefits disbursement vary across states. In particular, if the current state is j and the state next period is k, then next period the payments to the formerly high- and formerly low-income retirees are, respectively,

$$\tau_0 + \tau_{H,1} + \frac{(\tau_o^e + \tau_{H,1}^e)}{q^e(j)}\left[q^e(k) + d^e(k)\right] + \frac{(\tau_0^b + \tau_{H,1}^b)}{q^b(j)}\left[q^b(k) + 1\right]$$

and

$$\tau_0 + \tau_{L,1} + \frac{(\tau_o^e + \tau_{L,1}^e)}{q^e(j)}\left[q^e(k) + d^e(k)\right] + \frac{(\tau_0^b + \tau_{L,1}^b)}{q^b(j)}\left[q^b(k) + 1\right]. \tag{3.3}$$

If the model is calibrated to reflect the high mean equity returns observed in the United States over the past 70 years, in most states the available funds substantially exceed the mandated benefit levels. In the event of a severe downturn in the securities markets, however, the assets of the Trust Fund may fall short of the level necessary to fund promised benefits. This discrepancy would need to be offset with additional taxes. Opponents to privatization emphasize this latter possibility.

For simplicity of presentation, let us temporarily focus on the formerly high-wage old consumers, and let k^* denote such a disaster state, where the preceding state is j. The shortfall in state $k^*$is

$$\tau_0 + \tau_{H,1} + \frac{(\tau_{H,1}^e + \tau_0^e)}{q^e(j)}\left[q^e(k^*) + d^e(k^*)\right] + \frac{(\tau_{H,1}^b + \tau_0^b)}{q^b(j)}\left[q^b(k^*) + 1\right] - M_H < 0\,. \tag{3.4}$$

A tax surcharge to cover the shortfall is essentially the payoff to a put option written by the young and middle-aged and given to the old, with exercise price $M_H - \tau_0 - \tau_{H,1}$, when the underlying portfolio is $\left\{\tau_{H,1}^e + \tau_0^e)/q^e(j),\, (\tau_{H,1}^b + \tau_0^b)/q^b(j)\right\}$. From the perspective of the high-income middle-aged, (the consumers most likely to have to cover the shortfall), this future guarantee is an asset and its value is the value of the implied Social Security put.[13] This value differs across states because

[13] We argue that this burden would fall on the high-income consumers because it would most likely be financed by dramatically increasing the range of income subject to Social Security taxes. For low-income consumers, their entire wage income is already subject to the tax.

different current states give different conditional expectations on income and security price levels next period. The put issued to the prior period's middle aged is, of course, a liability of the current middle aged because they must fund it.

For the high-wage middle-aged consumers, the value in state j of this implied benefit, $V^H_{SSP}(j)$, is given by[14]

$$V^H_{SSP}(j) = \beta \sum_{k=1}^{J} \pi_{jk} \frac{u_1(c_2^H(k))}{u_1(c_1^H(j))} \max\{0, M_H - \tau_0 - \tau_{H,1} - \frac{(\tau_0^e + \tau_{H,1}^e)}{q^e(j)} [q^e(k) + d^e(k)] - \frac{(\tau_0^b + \tau_{H,1}^b)}{q^b(j)} [q^b(k) + 1]\}, \quad (3.5)$$

where

$$c_2^H(k) = [q^e(k) + d^e(k)]\{z_1^e(j) + (\tau_0^e + \tau_{H,1}^e)/q^e(j)\} + [q^b(k) + 1]\{z_1^b(j) + (\tau_0^b + \tau_{H,1}^b)/q^b(j)\} + w_2 + \tau_0 + \tau_{H,1} \quad (3.5a)$$

and

$$c_1^H(j) = w_{H,1}(j) - \tau_{H,1} - \tau_{H,1}^e - \tau_{H,1}^b - z_1^e(j)\, q^e(j) - z_1^b(j)\, q^b(j).^{15} \quad (3.5b)$$

Note that equation (3.5) implies that we undertake a marginal analysis from the perspective of a high income middle-aged consumer when the put is not yet in place (i.e., his consumption as middle aged and old, respectively, $c_1^H(j)$ and $c_2^H(k)$, do not reflect put payments). Since the put increases consumption when the consumer is old and reduces it when the consumer is middle-aged, the put reduces the marginal rate of substitution in the relevant states. In this regard, our computations overstate the true value of the put.

By way of contrast, and to get a better understanding of the range of valuations across the entire population, we also value the Social Security put extended to the formerly low-wage old by the currently low-wage middle aged.

If the current state is j, this quantity, $V^L_{SSP}(j)$, is given by

[14] This expression assumes that the rich insure their fellow rich and no one else. It is difficult to use marginal analysis if the rich insure everyone, as the benefits paid to the low income old do not enhance the utility of the elderly rich under our (standard) model scheme. If we view the payments to the elderly low income agents by the high income middle aged as a separate security priced as though it provided those very same benefits to the high income old, then the value of the two securities together would be (taking account of relative population differences) $(1+\frac{1-h}{h})V^H_{SSP}(j)$. In our calibration h=.45, so the factor is 2.22. Under this interpretation, all valuation associated with the high income middle aged must be increased by this factor.

[15] Note that (3.5a) implies that if SSTF assets should exceed the mandated benefits, the incremental difference is paid to the old. To assume otherwise would be to add a 'pseudo-bequest', something we have ruled out previously.

$$V_{SSP}^L(j) = \beta \sum_{k=1}^{J} \pi_{jk} \frac{u_1(c_2^L(k))}{u_1(c_1^L(j))} \max\{0, M_L - \tau_0 - \tau_{L,1} - \frac{(\tau_{L,1}^e + \tau_0^e)}{q^e(j)} (q^e(k) + d^e(k)) - \frac{(\tau_{L,1}^b + \tau_0^b)}{q^b(j)} (q^b(k) + 1)\} \quad (3.6)$$

with $c_2^L(k)$ and $c_1^L(j)$ defined in equations (2.3) and (2.4).[16,17]

Our formulation allows us to value the put for all possible Social Security financing schemes; that is, for all possible combinations of $\{\tau_0, \tau_0^e, \tau_0^b, \tau_{H,1}, \tau_{H,1}^e, \tau_{H,1}^b\}$ and $\{\tau_0, \tau_0^e, \tau_0^b, \tau_{L,1}, \tau_{L,1}^e, \tau_{L,1}^b\}$. It also allows us to see how these alternatives influence equilibrium returns. However, we require that $M_H - \tau_0 - \tau_{H,1} = M_L - \tau_0 - \tau_{L,1}$, $\tau_{L,1}^e = \tau_{H,1}^e$ and $\tau_{L,1}^b = \tau_{H,1}^b$. This standardization has two implications. First, the put extended to the low-income old is valued by the high-income middle-aged identically as their own put (identical strike price and underlying portfolio). The same is true from the perspective of the low-income middle-aged consumer. Together these identifications allow us to report only two put values for each set of parameters rather than four. The second implication is that the low-income consumers have a higher proportion of their Social Security Trust Fund assets invested in securities. This further reduces the inequality of income in old age.[18] We maintain these conventions throughout the paper.

If either of the puts is to have value, there must exist a depression state that the economy can enter with positive probability. This follows from the fact that the value of taxes invested in equity alone will increase, on average, by nearly a factor of 4.5 over 20 years (the length of our period), if the average annual real return on equity is eight percent, which is typical of our calibration. Under standard normality assumptions on returns, the probability of a twenty-year negative return realization is exceedingly small. We avoid this unrealistically optimistic scenario by introducing the low-probability depression state. There is, however, another perspective on the put which eliminates the need for introducing a depression state, and one that is perhaps more typical of the current privatization debate.

Plan 2. Rather than distribute a surplus, reduce the Social Security taxes. Investing Social Security tax revenues in positive return investments would require, on

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[16] Expressions (3.5) and (3.6) require that the high-income middle-aged insure only the high-income old and similarly for the low-income groups. Since the put payoffs are identical in structure, however, if we set $M_L = M_H$, $\tau_{H,1}^e = \tau_{L,1}^e$ and $\tau_{H,1}^b = \tau_{L,1}^b$, expression (3.5) can be used to value a put extended by the high-income middle-aged to the low-income old (regarding the put by extended to the old as an asset the high-income middle-aged might themselves acquire). In a like fashion we could use (3.6), properly parameterized, to compute the value of a put extended by the low-income middle-aged to the high-income old. If either group insures the whole population the payoff must take into account the measure of the two groups. In particular the valuation formulae should be scaled by a factor $1 + \frac{(1-h)}{h}/h$ as discussed above in footnote 14.

[17] Note that in a complete market $V_{SSP}^L(j) = V_{SSP}^H(j)$ if $M_L - \tau_0 - \tau_{L,1} = M_H - \tau_0 - \tau_{H,1}$, something we require in all our simulations. Our markets are clearly incomplete because of the "limited participation" phenomena, and the excess of states over securities.

[18] But it is not necessary for reduced income inequality. If both income groups have the same proportionate Social Security equity investments, but different absolute levels (the low income middle aged having less) old aged income inequality is still reduced for our calibrations.

average, less to be taxed today vis-à-vis the full pay-as-you-go system where the return is implicitly zero.[19] The crucial issue here is what mean return we may assume. Given the practical difficulties in forecasting conditional returns, it is reasonable to assume that the government would take the mean unconditional returns under the full pay-as-you-go system as the benchmark. After all, the pay-as-you-go system is the only one for which unconditional historical return data is available on which to base an estimate. Under this plan, the quantities $\tau_0^e, \tau_0^b, \tau_{H,1}^e$, and $\tau_{H,1}^b$ are chosen to satisfy

$$E\left[\tau_0 + \tau_{H,1} + (\tau_{H,1}^e + \tau_0^e)(1 + r^e(j,k)) + (\tau_{H,1}^b + \tau_0^b)(1 + r^b(j,k))\right] = M_H, \tag{3.7}$$

and the quantities $\tau_0^e, \tau_0^b, \tau_{L,1}^e$ and $\tau_{L,1}^b$ are chosen to satisfy

$$E\left[\tau_0 + \tau_{L,1} + (\tau_{L,1}^e + \tau_0^e)(1 + r^e(j,k)) + (\tau_{L,1}^b + \tau_0^b)(1 + r^b(j,k))\right] = M_L. \tag{3.8}$$

The latter expectation is the unconditional one under the pure pay-as-you-go system. Consistent with this convention, $r^e(j,k)$ and $r^b(j,k)$ are, respectively, the equilibrium equity and bond returns when the pay-as-you-go economy passes from state *j* to state *k*. The two Social Security put values are then computed exactly as in equations (3.5) and (3.6), using equilibrium return data under the actual level of Trust Fund taxation and securities investment. Under Plan 2, it is unnecessary to introduce a disaster state. In order to estimate a robust upper bound for the put's value, in our numerical work we consider Plan 2 cases with and without a disaster state.

4 Calibration

We present results for values of the risk aversion coefficient $\gamma = 4$ and 6. The (twenty-year) subjective discount factor is fixed at $\beta = .44$, which corresponds to an annualized value of .96, as typically assumed in business cycle studies. We set the fraction of the population owning stock at $h = .45$, which implies that 30 percent of the population have non-trivial investments in the stock market, the high-income middle-aged and the high-income old.

As noted earlier, the economy-wide state variables are the level of output, $\tilde{y}$, and the wage of the high-income middle-aged, $\tilde{w}_{H,1}$. Calibration is considerably simplified by the observation that equilibrium security prices are linear scale multiples

[19] This presumes that the new steady state has been achieved. During the transition from the full pay-as-you-go system to the invest-something-in securities regime, however, taxes will have to be increased as the working consumers will have to provide for the then-current old as well as financing security purchases for themselves when they are old in the subsequent period. In our calibration (to follow), we limit ourselves to the case in which a maximum 20 percent of benefits is financed by Trust Fund security purchases. For this scenario, taxes imposed on the middle-aged would have to rise by a maximum of 1 percent of their income during the transition year, assuming that year did not correspond to a disaster state. In the following year, taxes imposed on the middle-aged would decline and remain permanently below their pre reform level. See Feldstein et al. (2001) for a careful analysis of these issues.

of these wage and income variables and economy wide parameters. This follows from the homogeneity introduced by the constant-relative-risk-aversion preferences and implies that the equilibrium joint probability distribution of bond and equity *returns* is invariant to the *level* of the exogenous macro-economic variables *for a fixed* $\tilde{w}_{H,1}, \tilde{y}$ *joint probability structure*. The scale of the economy is thus irrelevant. Measured as a fraction of the expected output, the value of the Social Security put is also scale invariant. Accordingly, we parameterize the model around the following fundamental ratios:

(i) The average share of income to labor,

$$\frac{E\left[w_0 + h\,\tilde{w}_{H,1} + (1-h)\,w_{L,1} + w_2\right]}{E\left[\tilde{y}\right]} \tag{4.1}$$

For the United States economy, this ratio lies in the range [.66, .75], depending on the historical period and the manner of adjusting capital income. For most cases, we choose a value of .7, but also undertake a sensitivity analysis.

(ii) The coefficient of variation of twenty-year aggregate income, $\sigma(\tilde{y})/E(\tilde{y})$**.**

The first major challenge to our calibration exercise is the estimation of this unconditional moment. Unfortunately, a century-long time series provides only five non-overlapping observations, resulting in large standard errors of the point estimates. Standard econometric methods designed to extract more information from the time series, such as the utilization of overlapping observations only marginally increase the effective precision and still leave large standard errors. We thus consider a wide range of potential values in the range $[.10, .30]$.

(iii) The coefficient of variation of twenty-year wage income for the middle-aged,

$$\frac{\sigma(h\tilde{w}_{H,1} + (1-h)\,w_{L,1})}{E(h\tilde{w}_{H,1} + (1-h)\,w_{L,1})}. \tag{4.2}$$

This vital statistic represents another calibration challenge, for the same reasons mentioned above. Ideally, we would like our calibration to reflect the fact that the young experience large idiosyncratic uncertainty in their future labor income, and this is captured in the choice of $\tilde{w}_{H,1}$ versus $w_{L,1}$. Accordingly, we invoke consumer heterogeneity as the justification for also being liberal in estimating these moments. In particular, we assume that the coefficient of variation lies in the range $[.10, .25]$ and again conduct a sensitivity analysis.

(iv) The average share of income going to interest on government debt, $B/E(\tilde{y})$.

The United States government interest expense in 1999 was $230 billion, which corresponds to 2.5 percent of GDP ($9254 billion). Since our calibration is normalized at $E(\tilde{y}) = 122,000$ in the absence of a disaster state, we choose $B = 3,000$, which matches this statistic almost exactly, for current levels of government debt. Recall that our consol bonds are a proxy for long term debt, and that each bond pays one unit of consumption every period; hence aggregate interest payments are also 3000 and 3000/122,000 $\approx$ .0247.

(v) The mean level of social insurance benefits economy-wide.

In 1999, Federal Social Security, Medicare, and other income security payments totaled \$818 billion, which represented 8.8 percent of GDP. We match this figure by setting the respective exercise prices (promised benefit levels) at M_H = 12,000 and M_L = 10,000. The level of benefits expressed as a fraction of national income then works out to be $\{(12,000)h + (10,000)(1-h)\}/122,000$ = .089 or 8.9 percent, with $h = 0.45$.

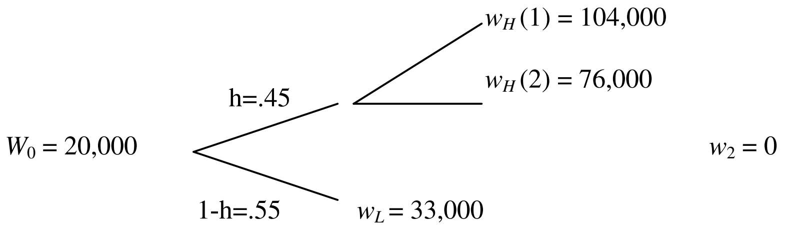
(vi) The average share of income going to the labor of the young, $\frac{E(w_0)}{E(\tilde{y})}$.

Somewhat arbitrarily, we fix the income of the young as $w_0 = 20,000$, which corresponds to the ratio $E(w_0)/E(\tilde{y}) \approx .16$.

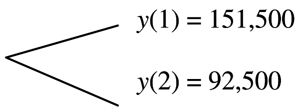
(vii) The ratio of the average income of the high-income middle-aged to the low-income middle-aged, $E\left[\tilde{w}_{H,1}\right]/w_{L,1}$.

This ratio is fixed at 2.25. In order to compute upper bounds for the put values we assume that the old receive no wage income, $w_2 = 0$. Their entire income comes either as Social Security payments, (for those who were low income as middle-aged), or as Social Security payments plus private security holdings (in the case of the high-income middle-aged).

Our base-case calibration is summarized as follows:



(4.3)



For these values, $E\left[w_0 + h\tilde{w}_{H,1} + (1-h)w_{L,1} + w_2\right]/E\left[\tilde{y}\right] = .695 \approx .7, \sigma(\tilde{y})/E(\tilde{y})$ = .30, and $\sigma(h\tilde{w}_{H,1} + (1-h)w_{L,1})/E\left[h\tilde{w}_{H,1} + (1-h)\,w_{L,1}\right]$ = .18.

It remains to consider the probability structure. Of special relevance to security pricing are **(vii) the auto-correlation** $corr\left(\tilde{y}_t, \tilde{y}_{t-1}\right)$, **(viii) the auto-correlation** $corr\left(\tilde{w}^H_{t,1}, \tilde{w}^H_{t-1,1}\right)$, and **(ix) the cross-correlation** $corr\left(\tilde{y}_t, \tilde{w}^H_{t-1,1}\right)$. Lacking sufficient time-series data to estimate these statistics, we present results for a variety of correlation structures. In particular, we consider four possible structures: $corr\left(\tilde{y}_t, \tilde{y}_{t-1}\right) = corr\left(\tilde{w}^H_{t,1}, \tilde{w}^H_{t-1,1}\right)$ = .1 or .8 in conjunction with $corr\left(\tilde{y}_t, \tilde{w}^H_{t-1,1}\right)$ = .1 or .8.

There are enough degrees of freedom for the above possibilities to be captured by the following 4×4 transition matrix Π :

$$
\begin{array}{ll}
 & \begin{array}{cccc} (y(1), w_{H,1}(1)) & (y(1), w_{H,1}(2)) & (y(2), w_{H,1}(1)) & (y(2), w_{H,1}(2)) \end{array} \\
\Pi: \begin{array}{c} (y(1), w_{H,1}(1)) \\ (y(1), w_{H,1}(2)) \\ (y(2), w_{H,1}(1)) \\ (y(2), w_{H,1}(2)) \end{array} & \begin{bmatrix} \phi & \pi & \sigma & H \\ \pi + \Delta & \phi - \Delta & H & \sigma \\ \sigma & H & \phi - \Delta & \pi + \Delta \\ H & \sigma & \pi & \phi \end{bmatrix}
\end{array}
\tag{4.4}
$$

Given the assumed symmetry of the transition matrix, there are only five parameters to be determined, $\phi, \pi, \sigma, \Delta$, and H, subject to the condition that the row sums equal one, $\phi + \pi + \sigma + \mathrm{H} = 1$.

In total, there are *twelve* parameters to be determined: the five matrix parameters $\phi, \pi, \sigma, \Delta$, and H plus the seven parameters $w_0, w_L, w_{H,1}(1), w_{H,1}(2), y(1), y(2)$, and B. (Recall that the income of the old is set equal to zero, $w_2 = 0$.) The parameters are chosen to satisfy the following *eleven* conditions: the seven target moments, (i)–(vii), the three conditions, and the normalization $E[y]$= 122,000. That leaves one extra degree of freedom that is chosen to ensure that all the elements of the transition matrix are positive. The precise values of $\phi, \pi, \sigma, \Delta$, and H are given below. In the discussion and tables, we uniquely identify each transition structure with the corresponding value of the parameter ϕ.

Parameter values corresponding to various correlation structures

corr $(\tilde{y}_t, \tilde{y}_{t-1})$ and corr$(\tilde{w}^H_{t,1}, \tilde{w}^H_{t-1,1})$	corr $(\tilde{y}_t, \tilde{w}^H_{t-1,1})$	ϕ	π	σ	η	Δ
.1	.1	.5298	.0202	.0247	.4253	.01
.1	.8	.8393	.0607	.0742	.0258	.03
.8	.1	.5496	.0004	.0034	.4466	.03
.8	.8	.8996	.0004	.0034	.0966	.03

The preceding probability structure lacks a disaster state. As we have noted under Plan 1, the Social Security put is likely to come into force most significantly in a disaster state comparable to the Great Depression of the 1930s, where output fell to 2/3 of its 1929 level. During such economic states, security prices are likely to be low and to persist in that state. We accommodate the potential for this sort of event by modifying our stochastic process on the output and high-income wage to admit the disaster state, $\left(y_3, w_{H,3}\right)$, where $y_3 = .60\, y_2$ and $w_{H,3} = .60 w_{H,2}$. Accordingly, the probability transition matrix is modified as

$$
\begin{array}{ll}
 & \begin{array}{ccccc} (y(1), w_{H,1}(1)) & (y(1), w_{H,1}(2)) & (y(2), w_{H,1}(1)) & (y(2), w_{H,1}(2)) & (y(3), w_{H,1}(3)) \end{array} \\
\begin{array}{c} \\ (y(1), w_{H,1}(1)) \\ (y(1), w_{H,1}(2)) \\ (y(2), w_{H,1}(1)) \\ (y(2), w_{H,1}(2)) \\ (y(3), w_{H,1}(3)) \end{array} & \begin{bmatrix} \phi - \eta_1 & & & & \eta_1 \\ \pi + \Delta & \pi & \sigma & H & \eta_2 \\ \sigma & \phi - \Delta - \eta_2 & H & \sigma & \eta_3 \\ H & H & \phi - \Delta - \eta_3 & \pi + \Delta & \eta_4 \\ & \sigma & \pi & \phi - \eta_4 & \\ A_1 & A_2 & A_3 & A_4 & 1 - \sum_{j=1}^{4} A_j \end{bmatrix}
\end{array}
\tag{4.5}
$$

The η_i parameters govern the likelihood of entering the disaster state while the A_i parameters describe the likelihood of exiting from it. Provided they result in the same stationary probability of disaster, the values of the puts are relatively insensitive to the specific patterns of A_i and η_i. This is not surprising in light of the fact that (European-style) put options depend only on the distribution of the states at expiration, rather than the paths to these states. In all our cases, we choose $\{A_i\}$ and $\{\eta_i\}$ such that the stationary probability of the disaster state is approximately ten percent. In what follows the full stationary distributions are detailed using the identification of (y(1), $\mathrm{w_{H,1}}(1)$) with "state 1", (y(1), $\mathrm{w_{H,1}}(2)$) with "state 2", (y(2), $\mathrm{w_{H,1}}(1)$) with "state 3" and so forth.

5 Results

First, we consider the case $\tau_0^e = \tau_0^b = 0$: the benefits received by the old from security purchases, whether privately or publicly, are exclusively determined by their own contributions as middle-aged consumers. In a representative cohort model, this is as close as we can come to individual specific accounts. Each of the proposed plans is considered. The analysis focuses not only on the value of the put, but also on the consequences for equilibrium security returns, and the distribution of old aged income.

5.1 Social Security Plan 1

We present results for Plan 1 when the SSTF revenues are partially invested in either equity or bonds.[20] In all cases, the amount invested is 2,000 ($\tau_{H,1}^e = \tau_{L,1}^e$ = 2,000, or $\tau_{H,1}^b = \tau_{L,1}^b$ = 2,000), which is approximately 18 percent of SSTF revenues. The latter figure is an upper bound of any proposal that is likely to be accepted in the immediate future. Note that 2000 also represents the maximum possible put payoff at expiration; this is 1.6 percent of the average output in normal times, $E[y]$ = 122,000. In this scenario, the source of value for the put is the presence of the disaster state. For all the cases reported, the stationary probability of a disaster state is about ten percent and represents a 45 percent drop in output relative to the mean level.[21]

5.1.1 Put valuation

When the SSTF revenues are invested partially in stock alone (Table 1, where $\tau_L^e = \tau_H^e = 2000$, $\tau_H^b = \tau_L^b = 0$), the average value of the high-income put, EV_{SSP}^H, is slightly more than one percent of the expected national income, with a standard deviation of 0.75 percent across all the cases when the risk aversion coefficient is 4. For the low-income consumers, the corresponding values are as high as 2.2 percent of the expected national income, with a standard deviation of 1.3 percent. The results are relatively uniform across all the probability structures because the

[20] Combinations of stock and long-term bond financing lead to put values intermediate between the entries reported here.
[21] We say 'approximately' because the different matrix structures give stationary probability distributions which differ slightly from one another.

Table 1. Social security put valuation: Plan 1[(i)] (all valuations computed as a percentage of average national income)

Panel A				
Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1				
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=2000$	$\tau^e_{H,1}=\tau^e_{L,1}=0$ $\tau^b_{H,1}=\tau^b_{L,1}=2000$	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=2000$	$\tau^e_{H,1}=\tau^e_{L,1}=0$ $\tau^b_{H,1}=\tau^b_{L,1}=2000$
Mean V^H_{SSP}	1.0	0.9	1.0	0.9
Std of V^H_{SSP}	0.75	0.8	0.8	0.8
Mean V^L_{SSP}	2.2	2.4	2.2	2.4
Std of V^L_{SSP}	1.3	2.3	1.3	2.2
	Stationary prob. distribution: State 1: .21635, State 2: .20007 State 3: .23103. State 4: .24926 State 5: .10329		Stationary prob. distribution: State 1: .25592, State 2: .16006 State 3: .19111, State 4: .28915 State 5: .10377	
Panel B				
Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.8				
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=2000$	$\tau^e_{H,1}=\tau^e_{L,1}=0$ $\tau^b_{H,1}=\tau^b_{L,1}=2000$	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=2000$	$\tau^e_{H,1}=\tau^e_{L,1}=0$ $\tau^b_{H,1}=\tau^b_{L,1}=2000$
Mean V^H_{SSP}	1.1	0.9	1.1	0.9
Std of V^H_{SSP}	0.8	0.8	0.8	0.9
Mean V^L_{SSP}	2.2	1.9	2.3	1.9
Std of V^L_{SSP}	1.3	1.8	1.2	1.8
	Stationary prob. distribution: State 1: .18546, State 2: .15518 State 3: .26147, State 4: .29498 State 5: .10292		Stationary prob. distribution: State 1: .21993, State 2: .12453 State 3: .22620, State 4: .32600 State 5: .10334	

[(i)] In all cases, $\gamma = 4, \beta = 0.44, h = 0.45$; $(w_0, w_2, w_L, w_{H,1}(j), y(j))$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2, 3, 4$ and $A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.

value of the put depends upon the stationary distribution of security prices, which is largely invariant across all the cases. Note that the two estimates of the put value use the marginal rate of substitution (MRS) of the respective income groups. If all the calculations use the MRS of the high-income middle-aged, then V^H_{SSP} and V^L_{SSP} coincide in every state.

These figures suggest a 95 percent upper bound probability that $V^H_{SSP} \leq 3$ percent and $V^L_{SSP} \leq 6$ percent. The former figure is more realistic as any shortfall is more likely to be made up with higher taxes on the middle-aged upper income population cohort. A wide class of comparative dynamics exercises support these assertions.

We argue that these put valuations provide a reasonable upper bound for its 'true value'. Three justifications underlie this assertion. First, our 'disaster calibration' is extreme in the sense that the assumed percentage reduction in output exceeds that observed for the U.S. during the period of the Great Depression of the 1930s. Secondly, the assumed coefficient of relative risk aversion falls in the upper range of generally accepted values. Lastly, the MRS calculation is overstated it the sense of ignoring both the positive payments the put provides to the holder, and the taxes levied to discharge obligations under the previous period's put. These effects reinforce one another with respect to put valuation.

The value of the put is drastically reduced if the old generation is guaranteed only 90 percent of the mandated benefits rather than 100 percent of the benefits. The mean value of the put decreases to a maximum (across all probability structures) of .028 percent, from the perspective of the high-income middle-aged. If we take the viewpoint of the low-income middle-aged consumer, the put declines to roughly 1/10th of its full benefits value.

These figures ignore the fact that in states of high security prices the fund would be able to provide benefits in excess of the mandated minimums, a surplus surely available to be carried over to future periods at least in part.[22] This would reduce future put costs accordingly. At 10 percent, our stationary probability of disaster is also high; if this probability is reduced to 5 percent, the corresponding mean values of the puts decrease by one half.

When the SSTF revenues are invested in bonds, the mean value of the put for the high-income middle-aged declines relative to equity investment, partly because bonds are less risky. However, for the low-income middle-aged, the opposite is true. This observation reflects the fact that the shift to bond investment results in a lower return (see Table 2A). The MRS of the low-income middle-aged is, on average, higher since their old age income is, on average, less than under equity investing. For the low-income middle-aged, this latter effect (which increases put values) dominates the former one (less risky investment vehicle) to create significant increases in V^L_{SSP}. For the high-income middle-aged, however, the same effect is much less strong because they have other investing alternatives (in particular, they hold all the equity) for maintaining the smoothness of their income stream.

5.1.2 Equilibrium security returns

In Table 2, we present a statistical summary of the effects of Plan 1 on equilibrium security returns. The statistics are reported on an annualized basis following a procedure we now describe. For all securities, the mean return is defined as $100\times[\{$mean of the 20-year holding period return$\}^{1/20}-1]$. The standard deviation of the (equity, bond or consol) return is defined as $100\times$[std $\{$(20-year holding period return) $^{1/20}\}$]. The mean equity premium return over the bond return is

[22] This surplus could be priced as the value of the corresponding call with the same exercise priced as the put, $M_H - \tau_0 - \tau_{H,1}$ in the case of the high-income middle-aged consumer. We cannot explicitly account for this carry-over, however, as we do not admit a storage technology.

Table 2. Equilibrium security returns$^{(\mathrm{i}),(\mathrm{ii})}$

Panel A

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1						
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)		
	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$
Mean r^e	11.5	8.5	9.2	11.5	8.6	9.3
Std of r^e	9.2	5.8	6.2	9.1	5.7	6.1
Mean r^b	4.6	3.2	2.7	4.6	3.2	2.7
Std of r^b	7.8	4.4	3.8	7.8	4.4	3.8
Mean r^f	0.8	0.0	−0.6	0.8	0.0	−0.6
Std of r^f	7.4	4.4	3.5	7.4	4.4	3.5
Mean r^p	10.6	8.5	9.8	10.7	8.5	9.9
Std of r^p	6.1	5.8	5.6	6.0	5.8	5.6

Panel B

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.8						
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)		
	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$
Mean r^e	10.9	8.0	8.6	11.0	8.0	8.7
Std of r^e	9.4	5.9	6.4	9.3	5.8	6.2
Mean r^b	4.6	3.2	2.7	4.6	3.2	2.7
Std of r^b	7.6	4.2	3.6	7.7	4.2	3.6
Mean r^f	0.9	0.0	−0.6	0.9	0.0	−0.6
Std of r^f	7.4	4.4	3.4	7.4	4.4	3.5
Mean r^p	10.0	7.9	9.2	10.1	8.0	9.3
Std of r^p	6.3	6.0	5.8	6.2	5.9	5.7

(i) In all cases, $\gamma=4, \beta=0.44, h=0.45; w_0, w_2, w_L, w_{H,1}(j), y(j)$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with $\eta_1=0.10, \eta_i=0.09, i=2,3,4$ and $A_1=A_3=0.05, A_2=A_4=0.35$.
The mean security return is defined as $100\times[\{\text{mean of the 20-year holding period return}\}^{1/20}-1]$.
The standard deviation of the return is defined as $100\times[\text{std }\{(\text{20-year holding period return})^{1/20}\}]$.
(ii) Stationary probability distributions are as in Table 2.

Table 3. Expected consumption levels: various social security financing schemes, plan 1[(i)(ii)]

	Correlation $(\tilde{y}, \tilde{w}_{H,1}) = 0.1$						Correlation $(\tilde{y}, \tilde{w}_{H,1}) = 0.8$					
	ϕ=.5298			ϕ=.8393			ϕ=.5496			ϕ=.8996		
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}(0.1)$			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}(0.8)$			Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}(0.1)$			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{\mathrm{H},1}(0.8)$		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
Econsol	10,000	24,282	12,753	10,000	23,348	12,671	10,000	24,138	12,768	10,000	23,059	12,682
Econsml	26,000	26,000	26,000	26,000	26,000	26,000	26,000	26,000	26,000	26,000	26,000	26,000
Econsoh	116,580	99,544	113,550	106,790	90,893	103,860	116,440	99,588	113,390	107,190	91,658	104,250
Econsmh	53,183	52,767	52,852	53,074	52,655	52,741	53,226	52,801	52,989	53,141	52,710	52,806

(a) Pay-as-you-go
(b) $\tau^e_{H,1} = \tau^e_{L,1} = 2000$
(c) $\tau^b_{H,1} = \tau^b_{L,1} = 2000$
(i) The following abbreviations are used: ECONSOL(ECONSOH) denotes the mean(unconditional) consumption for a low(high) income old agent . Analogously, ECONSML(ECONSMH) denote, respectively, the mean(unconditional) consumption levels for the low and high income middle aged agents. Notice that the introduction of SSTF investing in equities or bonds reduces the average consumption of the high income middle aged and old cohorts. We return to this fact in Section V.4 of the paper.
(ii) The stationary probability distributions and parameter values in these cases are, respectively, the same as in Tables 1 and 2.

defined as the difference between the mean return on equity and the mean return on the bond while the standard deviation of the premium of equity return over the bond return is defined as $100\times$[sample std $\{$(20-year equity return)$^{1/20}$ - (20-year bond return)$^{1/20}\}$]. Plan 1 increases prices and substantially reduces expected returns and return standard deviations relative to the full pay-as-you-go benchmark. The effect is greatest on equity returns when the SSTF revenues are invested in equity ($\tau^e_H = \tau^e_L = 2000$) and greatest on bond returns, when the SSTF revenues are invested in bonds ($\tau^b_H = \tau^b_L = 2000$). When the SSTF invests exclusively in equity, in particular, mean equity returns decline by about 3 percent (e.g., from 11.5 to 8.5 percent), long term bond and risk free returns simultaneously decline by 1.5 percent and .8 percent, respectively. When the SSTF exclusively invests in bonds, the mean risk free rate declines by about 1.4 percent, the mean long-term bond rate by 1.75 percent and the mean equity rate by 2.3 percent. In fact, for the reported parameterization, the real risk free rate becomes negative. In all cases, the premium also declines compared to the pay-as-you-go system. The fact that the qualitative impact of the SSTF's forays into the market is similar irrespective of the investment vehicle chosen reflects the considerable substitutability of equity and bonds. While the magnitude of Trust Fund participation in the financial markets is not large when measured as a percentage of national income, this simple model suggests that its influence on equilibrium returns may be substantial. The counterfactually high level of mean security returns is due largely to the extreme income, dividend, and wage uncertainty we have imposed upon the model.

It represents an unattractive consequence of seeking upper bound estimates for put values.

5.1.3 Old-Aged Income Inequality

The results obtained above are due almost exclusively to the securities market participation by the Fund on behalf of low-income consumers who would otherwise not hold stock or long-term debt. Low-income consumers benefit from these purchases in that, on an expected basis, they are better off as old persons relative to their situation under pure lump sum taxation. This assertion is confirmed in Table 3 where we present the lifetime expected consumption profiles for both agents under a variety of SSTF financing alternatives for the same parameterizations as underlie Tables 1 and 2. For the benchmark $\phi = .5298$ case, under pure lump sum taxation, the consumption of the low-income old is \$10,000; if $\tau^e_{L,1} = 2000$, its unconditional expected value is roughly \$24,000. In either case the lower income middle aged enjoy a consumption level of \$26,000 (their aggregate tax payments do not change), implying a much smoother consumption profile under privatization.

The effects of privatization on the high income cohort are very different, however. Average consumption for both the high income old and the high income middle-aged is significantly reduced under privatization. Let us focus on SSTF equity investing and and compare columns (a) and (b) for the high income groups. The explanation for these declines lies in the fact that SSTF investing, in general equilibrium, must inevitably crowd out private investing by the high income cohort. Not only are the equilibrium holdings of securities by the old and thus the aggregate dividends they receive reduced relative to a pay-as-you-go system, but their consumption as middle aged is also diminished as a consequence of the higher security prices they must pay. For the old, the higher prices are generally insufficient to offset the lower dividends. For all non disaster states, the consumption under SSTF equity investing is reduced vis-à-vis the pay-as-you-go system for both the high income old and middle aged agents. In the disaster state, a similar result generally holds, but not uniformly, due to the dramatically lower security prices in that state (anticipating a steeply upward sloping consumption profile).

To give a simple measure of these differential effects, compare the ratio of the expected consumption of the high income old to that of the low income old. Before privatization this ratio is roughly 11; after privatization (partial equity purchases), the ratio falls to approximately four (c.f. Table 3). These ratios are robust, furthermore, across all the probability structures. It is on this basis that we argue for SSTF stock market participation as a mechanism for reducing old age income inequalities.

Since low income middle aged consumption is unaffected, a similar reduction in middle aged income inequality is born out in equilibrium as well. While SSTF equity investing is frequently advocated by high income groups and disparaged by those representing the interests of lower income individuals, our results suggest that the self interest of the relevant parties should dictate that exactly the opposite pattern of advocacy be observed. Our results are only strengthened if the put is active.

5.1.4 Comparative dynamics

Mention should be made of the sensitivity of our results to alternative parameter specifications. First, we consider changes in the CRRA and the results across all the probability structures are essentially the same. In Table 4, we report results for the extremes of ϕ = .5298 and ϕ = .8996. If the RRA coefficient increases from γ=4 to γ=6, the expected value of the put to the high-income middle-aged, EV^H_{SSP}, approximately doubles to two percent of national income; the standard deviation approximately doubles as well. For the low-income middle-aged, the increase is much larger and exceeds ten. The general explanation for these results is straightforward: as consumers become more risk averse, the income insurance represented by the put becomes more valuable to them. The fact that the change is so much greater for the low-income middle-aged is attributable to the higher MRS attendant to their much greater inter-temporal income inequality.

Significant changes are also manifest in the pattern of security returns. In the presence of greater risk aversion, all security returns decline and the risk free rate

Table 4. Value of put and security returns for different risk aversion$^{(\mathrm{i}),(\mathrm{ii})}$ (all valuations computed as a percentage of average national income)

	Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1											
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)						High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)					
	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$		$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$		$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$		$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$		$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$		$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	
	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$		$\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$		$\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$		$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$		$\tau^e_{H,1}=2000$ $\tau^e_{L,1}=2000$		$\tau^b_{H,1}=2000$ $\tau^b_{L,1}=2000$	
	γ = 4	γ = 6	γ = 4	γ = 6	γ = 4	γ = 6	γ = 4	γ = 6	γ = 4	γ = 6	γ = 4	γ = 6
Mean V^H_{SSP}	0	0	1.0	2.3	0.9	2.2	0	0	1.1	2.4	0.9	2.3
Std of V^H_{SSP}	0	0	0.7	1.4	0.8	1.4	0	0	0.8	1.5	0.9	1.4
Mean V^L_{SSP}	0	0	2.2	24.4	2.4	36.5	0	0	2.3	25.5	1.9	27.3
Std of V^L_{SSP}	0	0	1.3	13.5	2.3	27.5	0	0	1.2	13.9	1.8	17.3
Mean r^e	11.5	11.4	8.5	7.9	9.2	8.7	11.0	10.9	8.0	7.4	8.7	8.2
Std of r^e	9.2	11.1	5.8	5.9	6.2	6.3	9.3	1.1	5.8	5.7	6.2	6.2
Mean r^b	4.6	4.5	3.2	2.7	2.7	2.2	4.6	4.5	3.2	2.7	2.7	2.2
Std of r^b	7.8	10.2	4.4	5.3	3.8	4.4	7.7	10.1	4.2	5.1	3.6	4.1
Mean r^f	0.8	−1.3	0.0	−2.4	−0.6	−2.8	0.9	−1.3	0.0	−2.4	−0.6	−2.8
Std of r^f	7.4	9.6	4.4	5.2	3.5	3.9	7.4	9.6	4.4	5.2	5.7	3.9
Mean r^p	10.6	12.7	8.5	10.3	9.8	11.4	10.1	12.1	8.0	9.8	9.3	10.9
Std of r^p	6.1	6.0	5.8	5.9	5.6	5.6	6.2	5.7	5.9	5.7	5.7	5.3

(i) In all cases, $\beta = 0.44, h = 0.45$; $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2, 3, 4$ and $A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.
The mean security return is defined as $100 \times [\{$mean of the 20-year holding period return$\}^{1/20} - 1]$.The standard deviation of the return is defined as $100 \times [$std $\{$(20-year holding period return) $^{1/20}\}]$.
(ii) Stationary probability distributions as in Table 1, where applicable.

Table 5. Security returns and participation (h): plan 1$^{(i),(ii)}$ (all valuations computed as a percentage of average national income)

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1 Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)			
$\tau^b_{H,1} = \tau^b_{L,1} = 0$ and $\tau^e_{H,1} = \tau^e_{L,1} = 2000$			
	h = 0.3	h = 0.45	h = 0.5
Mean V^H_{SSP}	0.7	1.0	1.1
Std of V^H_{SSP}	0.5	0.7	0.8
Mean V^L_{SSP}	1.5	2.2	2.3
Std of V^L_{SSP}	1.0	1.3	1.4
Mean r^e	11.7	8.5	7.6
Std of r^e	5.8	5.8	5.6
Mean r^b	4.0	3.2	2.9
Std of r^b	4.4	4.4	4.5
Mean r^f	1.2	0.0	−0.3
Std of r^f	4.7	4.4	4.3
Mean r^p	10.5	8.5	7.9
Std of r^p	6.8	5.8	5.8

$^{(i)}$ In all cases $\gamma = 4, \beta = 0.44. \{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2, 3, 4$ and $A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.
The mean security return is defined as $100 \times [\{\text{mean of the 20-year holding period return}\}^{1/20} - 1]$.The standard deviation of the return is defined as $100 \times [\text{std } \{(\text{20-year holding period return})^{1/20}\}]$.
$^{(ii)}$ For all cases presented here the stationary probability distribution corresponds to that of the left case of Panel A, Table 1.

becomes negative. Consumers who are more risk-averse desire smoother inter-temporal consumption profiles and the only way to accomplish this objective is to demand more securities of both types (their returns being less than perfectly positively correlated allows for some diversification). Therefore, prices increase, but do so differentially, and the equity premium increases. These remarks and those of the preceding paragraph above represent a brief summary of the results presented in Table 4.

Of additional interest is the effect on put values and equilibrium return distributions of changes in the extent of privately held stockholdings (the parameter h). These are presented in Table 5, in this case of a single representative probability structure. It is seen that as the fraction of the population participating in the securities markets rises, equilibrium returns decline sharply. Simultaneously, the expected value and standard deviation of the put rise. The former result is due, in part, to the enhanced volatility of equilibrium security (underlying asset) returns and, in part, to the lower mean returns which afford less opportunity for consumption smoothing and thus create a higher average MRS for consumers of both income categories.

5.2 Social Security Plan 2

Recall that the idea underlying this plan is for the SSTF to take advantage of the potentially higher returns afforded by stocks (if history repeats itself), by lowering

Social Security taxes rather than enhancing benefits. Compared with a pay-as-you-go system (with an implicit zero rate of return), less needs to be set aside today in order to create the required level of expected benefits in the future.[23] The portion of the Social Security tax devoted to security purchases must have a value equal not to the associated benefits but to their present value. The issue is only the *rate* at which the benefits are to be discounted, recognizing that the very institution of such a policy will have equilibrium effects on the rates themselves.

For the purposes of our calculation, we set the discount rate equal to the prevailing (in the model) rate under the pay-as-you-go system. We advocate this choice as a realistic model counterpart based on the following two arguments. First, it reflects 'historical' experience and can be argued on that basis, and (at least in our context) justifies the greatest tax relief. Second, general equilibrium effects are almost surely too subtle to be computed with any confidence in a real world context. By adopting this convention, we maintain our objective of seeking reasonable upper bound estimates for the put values. That we would obtain an upper bound follows from the fact that the public purchase of securities on behalf of the Trust Fund in general serves to lower equilibrium returns vis-à-vis their pay-as-you go levels. The present value of future benefits should thus be higher than we assume, thereby increasing the likelihood that the put will be in the money at its expiration.

5.2.1 Put valuation

In Table 6, we present the results of this exercise when we eliminate the disaster state (i.e., $\eta_i \equiv 0 \forall_i$, and $A_1 = .5$, $A_2 = .5$, $A_3 = 0$, and $A_4 = 0$) for a representative pair of cases. Under Plan 2, there is no longer a need to retain the possibility of disaster to guarantee that the put has value since the plan at best guarantees only the expected level of benefits and not their level state by state. Subsequently we reintroduce the disaster state. For economy of presentation, only the results for the probability structure corresponding to ϕ = .5298 and ϕ = .8996 are presented; the results are similar across the other probability structures.

Both put values are very small—at least from the perspective of the high-income middle-aged. In this case, the value of the put is less than .5 percent of National Income. If it were guaranteed by the high-income middle-aged (the most likely scenario), this same estimate would also apply to the put insurance of the low-income elderly. As before, the substantially higher values of V^L_{SSP} are due to the much greater average MRS attendant to their less smooth inter-temporal income profiles under this plan.[24] Although we do not report them, the corresponding values are almost insignificant if the guarantee is again reduced to 90 percent of currently mandated benefits.

[23] For example, Feldstein and Samwick [12] argue that an 18.75 percent payroll tax rate necessary to sustain promised benefits as the United States population ages could be replaced with a 2 percent tax rate in the long run under all equity investing.

[24] In particular, for the states in which the security returns are negative, the MRS of the low-income middle-aged consumers is much greater than under the pure pay-as-you-go system.

Table 6. Value of put and security returns: plan 2[(i)] (all valuations computed as a percentage of average national income)

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1						
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)		
	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=270.5$ $\tau^e_{L,1}=270.5^{(a)}$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=633$ $\tau^b_{L,1}=633^{(b)}$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=210$ $\tau^e_{L,1}=210^{(c)}$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=352$ $\tau^b_{L,1}=352^{(d)}$
Mean V^H_{SSP}	0.0	0.4	0.2	0.0	0.2	0.1
Std of V^H_{SSP}	0.0	0.3	0.2	0.0	0.2	0.2
Mean V^L_{SSP}	0.0	20.0	12.5	0.0	19.0	8.4
Std of V^L_{SSP}	0.0	10.0	14.2	0.0	18.0	13.3
Mean r^e	10.5	9.9	9.7	11.9	11.2	11.1
Std of r^e	5.0	4.8	4.7	4.4	4.2	4.2
Mean r^b	5.9	5.4	5.2	9.0	8.4	8.2
Std of r^b	3.0	2.8	2.6	2.0	1.9	1.8
Mean r^f	6.4	5.9	5.7	9.4	8.8	8.6
Std of r^f	2.6	2.3	2.2	2.1	2.0	1.9
Mean r^p	4.2	4.0	4.0	2.5	2.4	2.5
Std of r^p	4.5	4.4	4.3	2.9	2.8	2.9
	Stationary prob. distribution: State 1: .28476, State 2: .22590 State 3: .22580 State 4:.27355 State 5: .0000			Stationary prob. distribution: State 1: .44799, State 2: .04890 State 3: .05166, State 4: .45145 State 5: .0000		

[(i)]In all cases $\gamma = 4, \beta = 0.44, h = 0.45; \{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in (4.3).The transition matrix is as per (4.4) . In both cases $\sigma(\tilde{y})/E(\tilde{y}) = 0.25, \sigma(\tilde{w}_{H,1})/E(\tilde{w}_{H,1}) = 0.10$.

[(a)]$270.5 \approx \frac{2000}{(1.1052)^{20}}$

[(b)]$633 \approx \frac{2000}{(1.0592)^{20}}$

[(c)]$210.3 \approx \frac{2000}{(1.1192)^{20}}$

[(d)]$352.3 \approx \frac{2000}{(1.0907)^{20}}$

5.2.2 Equilibrium security returns

Security returns reflect the by-now expected pattern: Trust Fund purchases increase net demand and prices, and lower returns. That our estimates are not too far off the corresponding rational expectations values (that is, where the rates assumed in determining the $\tau^e_{H,1}, \tau^e_{L,1}, \tau^b_{H,1}, \tau^b_{L,1}$ values coincide with the actual prevailing rates) is confirmed by the relatively modest decline in equilibrium rates. The fact that rates are in general lower than in Tables 1–3 reflects the reduced income and dividend uncertainty in the absence of a disaster state.

In Table 7, we present the corresponding results when the disaster state is reintroduced. The natural comparison of Plan 1 and Plan 2 can be found by matching panels A and B of Tables 1 and 2 respectively, with the corresponding panels in Table 7. Under Plan 2, the V^H_{SSP} rises as high as 2.5 percent of average national income; for V^L_{SSP} it is as high as 30 percent. This is not entirely surprising: under Plan 2, the likelihood of a Social Security shortfall, and, should it occur, the magnitude of the shortfall, are both greater than under Plan 1 because the amounts invested are much lower (227.6 vs. 2000 in the case of equity, 818.25 vs. 2000 in the value of long term bonds for the Panel A scenario). The huge increase in the V^L_{SSP} is due again to the high average MRS value for the low-income middle-aged; their consumption here is much less effectively smoothed than in Tables 1 and 2. There is also a difference in the pattern of equilibrium returns across the various financing options. Under Plan 2, all rates decline monotonically as we pass from a full pay-as-you-go to partial equity to partial long term debt financing.

In our search for reasonable upper bounds on the put, there is one more experiment open to us: use the rates for a non-disaster economy to determine the level of Plan 2 investments in a disaster state economy. The idea here is to examine the consequences of the Trust Fund's ignoring the possibility of a disaster state.

These results are presented in Table 8 for a representative case. Under partial equity financing, V^H_{SSP} achieves a value of 2.44 percent of average National Income, which is less than the corresponding figure under disaster anticipation (2.49 percent). If long-term bond financing is used, however, V^L_{SSP} rises to 33.51 percent of average income, thereby vastly exceeding the 23.65 percent under disaster anticipation (Table 7). Why the conflicting results? Let us consider V^L_{SSP} under bond financing first, as its explanation is most straightforward. There are two effects. First, the equilibrium risk free rate under the disaster scenario (3.17 percent) is lower than under the no disaster one (5.92 percent) since risk free securities are more desirable in the latter environment. The Fund thus anticipates earning a higher return on its investments than actually turns out to be the case. In addition, by anticipating higher rates, the Fund invests less (633.1) than a proper equilibrium analysis would presume (818.5). These two effects reinforce one another to reduce the value of Trust Fund assets and substantially increase the value of the put.

Under partial equity financing, the analysis is somewhat more complex. Since the amount invested is determined under a no disaster scenario, the assumed rate (10.52 percent) is lower than in the corresponding disaster (11.48 percent) scenario and more is invested (270 vs. 227). Ceteris paribus, this should reduce the value of the put. Working in the opposite direction is the rate effect: the higher equity purchases on behalf of the low-income middle-aged lowers equilibrium returns (10.03 percent) below the disaster level. Thus, rates are not as high, an effect that, ceteris paribus, should increase the value of the put. For this set of cases, the latter effect dominates the former to reduce V^H_{SSP} to 2.44 percent of output. The general pattern of security returns does not dramatically differ from what has been presented earlier.

Table 7. Value of put and security returns—plan 2 and anticipated disaster state(i) (all valuations computed as a percentage of average national income)

	Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1					
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)		
	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=228$ $\tau^e_{L,1}=228^{(a)}$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=818$ $\tau^b_{L,1}=818^{(b)}$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$	$\tau^b_{H,1}=0$ $\tau^b_{L,1}=0$ $\tau^e_{H,1}=249$ $\tau^e_{L,1}=249^{(c)}$	$\tau^e_{H,1}=0$ $\tau^e_{L,1}=0$ $\tau^b_{H,1}=818$ $\tau^b_{L,1}=810^{(d)}$
Mean V^H_{SSP}	0.0	2.5	1.9	0.0	2.6	2.0
Std of V^H_{SSP}	0.0	1.3	1.1	0.0	1.4	1.2
Mean V^L_{SSP}	0.0	29.4	23.7	0.0	30.6	24.1
Std of V^L_{SSP}	0.0	13.0	15.9	0.0	23.3	12.0
Mean r^e	11.5	10.1	9.7	11.0	9.5	9.2
Std of r^e	9.2	7.4	6.8	9.3	7.3	6.8
Mean r^b	4.6	3.5	3.1	4.6	3.5	3.1
Std of r^b	7.8	5.9	4.7	7.7	5.6	4.5
Mean r^f	0.9	−0.5	−0.6	0.9	−0.4	−0.6
Std of r^f	7.4	5.6	4.3	7.4	5.5	4.4
mean r^p	10.6	10.6	10.4	10.1	10.0	9.9
Std of r^p	6.1	5.9	5.8	6.2	6.0	5.9
	Stationary prob. distribution: State 1: .21635, State 2: .20007 State 3: .23103. State 4: .24926 State 5: .10329			Stationary prob. distribution: State 1: .21933, State 2: .12453 State 3: .22620, State 4: .32600 State 5: .10334		

(i) In all cases $\gamma = 4, \beta = 0.44, h = 0.45$; $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2,3,4$ and $A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.
In both cases $\sigma(\tilde{y})/E(\tilde{y}) = 0.25, \sigma(\tilde{w}_{H,1})/E(\tilde{w}_{H,1}) = 0.10$.
(a) $227.6 \approx \frac{2000}{(1.1148)^{20}}$
(b) $818.3 \approx \frac{2000}{(1.0457)^{20}}$
(c) $249.0 \approx \frac{2000}{(1.1098)^{20}}$
(d) $810.5 \approx \frac{2000}{(1.0462)^{20}}$

5.2.3 – 5.2.4 Old age income equality and comparative dynamics

The comparative dynamics results reported earlier carry over qualitatively to the case of Plan 2 without exception. As agents become more risk averse, for example, steady state put prices also rise and equilibrium security prices decline.

With regard to relative income differentials, however, the results diverge in certain respects from what was presented earlier. Table 9 presents the expected lifetime consumption profiles for high and low income agents. For all cases, SSTF equity investing, per se, continues to reduce middle aged income inequality: the lower taxation attendant to Plan 2 increases the consumption of both the middle aged high

Table 8. Value of put and security returns—plan 2 and unanticipated disaster state$^{(i),(ii)}$ (all valuations computed as a percentage of average national income)

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1 Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)$^{(c)}$			
	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=0$	$\tau^b_{H,1}=\tau^b_{L,1}=0$ $\tau^e_{H,1}=\tau^e_{L,1}=270.5^{(a)}$	$\tau^e_{H,1}=\tau^e_{L,1}=0$ $\tau^b_{H,1}=\tau^b_{L,1}=633^{(b)}$
Mean V^H_{SSP}	0.0	2.4	2.1
Std of V^H_{SSP}	0.0	1.3	1.2
Mean V^L_{SSP}	0.0	26.1	33.5
Std of V^L_{SSP}	0.0	12.5	18.6
Mean r^e	11.5	10.0	9.9
Std of r^e	9.2	7.3	7.0
Mean r^b	4.6	3.5	3.2
Std of r^b	7.8	5.8	5.0
Mean r^f	0.9	−0.5	−0.6
Std of r^f	7.4	5.5	4.6
Mean r^p	10.6	10.5	10.5
Std of r^p	6.0	5.9	5.8

$^{(i)}$ In all cases, $\gamma=4, \beta=0.44, h=0.45$; $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1=0.10, \eta_i=0.09, i=2,3,4$ and $A_1=A_3=0.05, A_2=A_4=0.35$.
In both cases $\sigma(\tilde{y})/E(\tilde{y})=0.25, \sigma(\tilde{w}_{H,1})/E(\tilde{w}_{H,1})=0.10$.
$^{(a)}270.5\approx\frac{2000}{(1.1052)^{20}}$
$^{(b)}633\approx\frac{2000}{(1.0592)^{20}}$
$^{(ii)}$The stationary probability distribution is as in Table 7, left panel.

Table 9. Expected consumption levels: various social security financing schemes, plan 2

	Correlation $(\tilde{y}, \tilde{w}_{H,1})=0.1$			Correlation $(\tilde{y}, \tilde{w}_{H,1})=0.8$		
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}(0.1)$			High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}(.8)$		
	pay-as-you-go	$\tau^e_{H,1}=\tau^e_{L,1}=270.5$	$\tau^b_{H,1}=\tau^b_{L,1}=633$	pay-as-you-go	$\tau^e_{H,1}=\tau^e_{L,1}=210.3$	$\tau^b_{H,1}=\tau^b_{L,1}=352.3$
ECONSOL$^{(i)}$	10,000	10,496	9,942	10,000	10,288	9,773
ECONSML	26,000	27,730	27,367	26,000	27,790	27,648
ECONSOH	124,440	121,134	122,680	118,894	115,820	116,760
ECONSMH	65,021	65,361	65,135	69,984	70,570	70,428

(i) ECONSOL, ECONSML, ECONSOH, ECONSMH as in Table 4; parameters as in Table 8; so also for the respective stationary probability distributions on which these cases are based.

and low income cohorts and proportionately more so for the low income group. This reduces middle aged income inequality.

As for the old aged cohort, SSTF equity investing reduces the consumption of the high income old across the board by the same crowding out effect as in Plan 1 (the degree of the reduction is much lower, however). In contrast, the pattern for the low income old is mixed, a reflection of the fact that SSTF purchases are now tailored to yield only the expected pay-as-you-go benefit levels. In some states the low income old enjoy greater than the pay-as-you-go levels and in others, less. A narrowing of average old age income differentials is thus not necessarily observed under Plan 2.

5.3 Extending security investing to the young

A slight generalization of our model can be effected if we admit Social Security investments in stock and risky debt financed by taxes imposed on the young generation as well as the middle-aged. Given the high average returns on these securities, it is natural for the Fund to wish to invest as much as possible in these vehicles. To do so breaks the direct connection, however, between a middle-aged consumer's Social Security investments in risky securities and their own level of retirement benefits that is present in the current formulation. Rather, under this formulation, both young and old contribute to a general pool of securities, the aggregate value of which in part determines the welfare of the generation that will be old in the subsequent period. Therefore, the young do not directly benefit from their *own* Social Security purchases when they themselves are old. In this sense, we no longer have generation specific accounts.

The results of this exercise are contained in Tables 10 and 11, which are, respectively, the direct analogues of Tables 1 and 2, except that the total Social Security investment in stock and/or debt is split evenly between the young and middle-aged ($1000 to each) holding each generation's (and each middle-aged income category's) aggregate tax payment constant. A quick comparison of the results in Tables 1 and 10 reveals that this modification produces virtually no change in the value of the Social Security put, across the various probability transition matrices. A comparison of Tables 2 and 11 leads to an even stronger result for security returns: they are identical vis-à-vis their statistical summaries. Demand is unaffected on a state-by-state basis, leaving security return patterns unchanged.

5.4 Comparison with earlier literature

We compare our results with those of Smetters [25] and Feldstein and Samwick [12]. Smetters [25] considers the case of complete privatization and fixes the Social Security tax rate such that the assets of the Fund fully cover, on an expected basis, the ex ante promised benefits (the present value of the expected shortfall is zero). When the value of the put is added, however, the reduction in unfunded liabilities is only 21.1 percent (see Smetters [25], Table 3.1, $\Psi = \chi = 1$, $\bar{e} = .07$). Since the current level of unfunded benefits is approximately $8 trillion, or 80 percent of GDP, this calculation implicitly values the put at 63 percent of GDP.

Our earlier estimate of the value of the put is significantly smaller than that of Smetters (2001) partly because we do not consider complete privatization. We estimate the value of the put under complete privatization by setting parameter values $\tau_0 = \tau_{L,1} = \tau_{H,1} = 0$, $\tau^e_{H,1} = (1.1052)^{-20} 12,000 = 1623$, $\tau^e_{L,1} = (1.1052)^{-20}$

Table 10. Value of put when the young are taxed: plan1 [(i),(ii)] (all valuations computed as a percentage of average national income)

Panel A				
Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1				
	Low serial autocorr. of and $\tilde{y}$ of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$
Mean V_{SSP}^H	1.02	0.86	1.04	0.86
Std of V_{SSP}^H	0.74	0.76	0.76	0.77
Mean V_{SSP}^L	2.15	2.42	2.18	2.37
Std of V_{SSP}^L	1.32	2.25	1.34	2.24
Panel B				
Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.8				
	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$
Mean V_{SSP}^H	1.05	0.87	1.07	0.88
Std of V_{SSP}^H	0.75	0.83	0.77	0.85
Mean V_{SSP}^L	2.24	1.89	2.26	1.86
Std of V_{SSP}^L	1.27	1.78	1.24	1.78

(i) In all cases $\gamma = 4, \beta = 0.44, h = 0.45$; $\{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii).
The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2, 3, 4 and A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.
(ii) Stationary probability distribution as in Table 1.

$10,000 = 1353$, and $\phi = 0.5298$ in our Plan 2. For the parameterization otherwise as in Table 7, this calculation yields $EV_{SSP}^H \approx .03E[y]$, which is still small. Other values of ϕ produce similar results. This reasonably approximates the Smetters [25] estimate, however, when we adjust for the model's twenty year time period (ignoring discounting). These calculations also neglect the fact that there are two income groups in our society. If we compute the overall value of the put as the weighted average (relative population weights) of the put values assigned by the high and low income middle aged, we obtain values in excess of the Smetters [25] standard.

It is also of interest to compare our results with the "two percent rule" propounded in Feldstein and Samwick [12]. These authors demonstrate that a two percent contribution level, when invested in equities, would be able to replace fully

Table 11. Security returns when the young are taxed: plan 1$^{(\mathrm{i}),(\mathrm{ii})}$

Panel A

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.1

	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b =$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e =$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b =$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e =$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$
Mean r^e	8.5	9.2	8.6	9.3
Std of r^e	5.8	6.2	5.7	6.1
Mean r^b	3.2	2.7	3.2	2.7
Std of r^b	4.4	3.8	4.4	3.8
Mean r^f	0.0	−0.6	−0.0	−0.6
Std of r^f	4.4	3.5	4.4	3.5
Mean r^p	8.5	9.8	8.6	9.9
Std of r^p	5.8	5.6	5.8	5.6

Panel B

Correlation $(\tilde{y}, \tilde{w}_{H,1})$ = 0.8

	Low serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.1)		High serial autocorr. of $\tilde{y}$ and of $\tilde{w}_{H,1}$ (0.8)	
	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$	$\tau_0^b = \tau_{H,1}^b = 0$ $\tau_{L,1}^b = 0$ $\tau_0^e = \tau_{H,1}^e = 1000$ $\tau_{L,1}^e = 1000$	$\tau_0^e = \tau_{H,1}^e = 0$ $\tau_{L,1}^e = 0$ $\tau_0^b = \tau_{H,1}^b = 1000$ $\tau_{L,1}^b = 1000$
Mean r^e	8.0	8.6	8.0	8.7
Std of r^e	5.9	6.4	5.8	6.2
Mean r^b	3.2	2.7	3.2	2.7
Std of r^b	4.2	3.6	4.2	3.6
Mean r^f	0.0	−0.6	0.0	0.6
Std of r^f	4.4	3.4	4.4	3.5
Mean r^p	7.9	9.2	8.0	9.3
Std of r^p	6.0	5.8	5.9	5.7

(i) In all cases, $\gamma = 4, \beta = 0.44, h = 0.45. \{w_0, w_2, w_L, w_{H,1}(j), y(j)\}$ are defined in the main text of the paper in 4.3 (i), (ii). The transition matrix is per (4.5) with $\eta_1 = 0.10, \eta_i = 0.09, i = 2, 3, 4$ and $A_1 = A_3 = 0.05, A_2 = A_4 = 0.35$.
(ii) Stationary probability distributions as in Table 1.

the current pay-as-you-go system. In Table 6 (Plan 2) mean return levels are such as to suggest that the corresponding tax levels imposed on the high and low-income middle-aged, respectively, under full equity investing (no-pay-as-you-go or bond components) would be 1623 and 1353 (see the calculation above). As a proportion of the average underlying wage for that same case, these figures represent an average tax rate of 2.28 percent, which is broadly consistent with Feldstein and Samwick [12].

6 Concluding remarks

If a large fraction of Social Security taxes is invested in equities, there is a distinct possibility that Social Security funds decline in value to a level such that they are inadequate to provide a subsistence level of benefits to the old generation. Since Social Security is a form of social insurance that implicitly provides a guarantee on a minimum consumption level of the older generation, the government may be compelled to remedy a shortfall by raising taxes on the younger working generations. We argue that any time-consistent discussion on privatizing Social Security must take into account this *de facto* role of the younger working generations as insurers of last resort. We price this implicit insurance provided by the younger working generations as a put option on the value of the Social Security Trust Fund with strike price equal to the implicit guaranteed level.

From the perspective of the high-income middle-aged—the group most likely to have to cover shortfalls of the Social Security Trust Fund—the value of the put is estimated to be slightly in excess of one percent of GDP under our most realistic scenario (Plan 1). This corresponds to $100 billion, or at most a 25 percent increase in Social Security taxes, if it were to be honored. A put that guarantees only 90 percent of the currently mandated benefits is priced at roughly .03 percent of GDP, which is proportionately much less. We do not regard the cost of even the 100 percent guarantee as insurmountably large. Instituting such privatization policies is further seen to give rise to a substantial increase in security prices with the consequent reduction in returns. It is also seen to substantially reduce income inequality across the old consumers. This latter consequence may ultimately be the greatest argument in its favor.

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