Demographics and FDI: Lessons from China's One-Child Policy*

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Abstract

Following the introduction of the one-child policy in China, the capital-labor ratio of China increased relative to that of India, while FDI/GDP inflows to China vs India simultaneously declined. These observations are explained in the context of a simple neoclassical OLG paradigm. The adjustment mechanism works as follows: the reduction in the growth rate of the (urban) labor force due to the one-child policy increases the *capital per worker* inherited from the previous generation. The resulting increase in China's *domestic* capital-labor ratio thus 'crowds out' the need for FDI in China relative to India. Our paper is a contribution to the nascent literature exploring demographic transitions and their effects on FDI flows.

Keywords: open economy, OLG model, population aging, capital-labor ratio, FDI-intensity, one-child policy

JEL classification: F11, F21, J11, O11, E13

1. Introduction

A central tenant of neoclassical growth theory asserts that the marginal product of capital is high when the capital-labor ratio is low. This led Lucas (1990) to ask the question: "Why doesn't capital flow from developed to developing countries?," the implicit assumption being that developed and developing countries are characterized by high and low capital-labor ratios respectively. In this paper we explore one mechanism by which cross-country differences in population growth rates can dominate these capital flows.

The mechanism arises through an exogenous steady decline in the working age population in one of the countries under study. In every generation, the capital accumulation (savings) of the old age cohorts in the country with a declining population accrues to a significantly smaller generation of workers. The resulting endogenous increase in capital per worker has the consequence of reducing the relative foreign direct investment (FDI) flows into the country experiencing the population decline. We illustrate this mechanism in a simple two country and the 'Rest of the World' model that generates closed-form steady-state characterizations which conveniently highlight the relative FDI/GDP consequences for the country with a declining population.[1] Two institutional assumptions are key to the model's results:

1. Home bias in investment financing: in either country, investment financing needs are first satisfied using domestically generated savings with FDI covering any shortfall. Emerging markets economies are typically characterized by a shortfall of domestic investment capital with FDI serving as a supplement.

2. Household savings rates are undiminished by reduced fertility: the requirement that any bequests be distributed over fewer progeny does not diminish aggregate household wealth

[1] A recent study examining the impact of population aging on economic performance through savings channels is Eggertsson et al. (2019).

accumulation. Indeed, the literature identifies an enormous increase in China's savings rate following the one-child policy implementation.[2]

For the model's empirical exercise, we exploit a natural policy experiment, the 1982 introduction of the one-child policy in China. We contrast the pattern of FDI flows into China with those of India which had in place, a largely unsuccessful two-child policy initiative of its own. Except for labor force growth differences, both countries experienced similar growth in all major macroeconomic aggregates, most critically output and total factor productivity. Due to the one-child policy intervention, the population (and labor force) growth rate of China declined substantially relative to that of India. These collective events are observed simultaneously with a significant decline in relative FDI intensity (FDI/GDP), China versus India, illustrating the proposed mechanism.

Any strong motivation for increased domestic savings in company with the indicated demographic intervention has the potential to diminish the relative significance of the mechanism emphasized in this paper, and perhaps to overwhelm it. One such motivation is increased life expectancy, while another is a longer retirement period. As we will show, neither of these generalizations reverses our results: the macroeconomic consequences contingent on reduced population growth dominate the consequences of either of the above phenomena. Indeed, even postulating the greatest permanent increase in China's savings rate for which there is empirical support, the model confirms that the demographic effects we detail have greater consequences for changes in China's long-run relative FDI/GDP ratio.[3]

As background to our relative FDI/GDP analysis we describe the steady-state evolution

[2] Various papers offer different explanations for this savings increase, all of a social nature. See Appendix G for a full discussion.

[3] From the empirical side, the increase in life expectancy in India and China during the period under study was essentially the same. Furthermore, there is little evidence that the bequest motive is a dominant social force in China (see Horioka, 2014). These observations suggest that neither phenomenon has been a significant determinant of relative FDI flows into China vs India.

of the economy's consumption, investment, capital stock and labor supplied. We are also able to detail the equilibrium fraction of the economy's aggregate capital stock that is domestically vs. foreign owned, and how these various aforementioned quantities are affected by the level of the world rate of interest.

In summary, we argue that population dynamics can play a dominant role in determining cross country relative FDI/GDP flows and that these effects can well dominate the consequences of changes in savings rates, whatever their origin. As with Lucas (1990), McGrattan and Prescott (2009, 2010) and Holmes et al. (2015), our analysis relies only the standard neoclassical framework.[4]

An outline of the paper is as follows. Section 2 documents the relative population dynamics and FDI flows for India and China post China's implementation of the one child policy. Sections 3 and 4 present a parsimonious neoclassical international investment model, the implications of which are shown to replicate the patterns found in the data. Section 5 concludes.

2. Comparative population policies and macroeconomic dynamics in China and India: data

2.1 Comparative population policies and dynamics

Both China and India initiated public policies to control population growth. India's two-child policy was voluntary and largely ineffective. In contrast, China's one-child policy was mandatory and highly effective.

[4] These papers require identical population growth rates across countries, which excludes the very phenomenon we propose to explore. Backus et al. (2014) and Cooley and Henriksen (2018) are two additional citations with a demographic emphasis. In the former, the authors directly explore the implications of differing population dynamics (life expectancies, population age distributions) for capital flows between countries. In the latter the focus is on the implications of population dynamics for economic growth rates within countries, particularly Japan and the US. The mechanism we have emphasized, however, is not showcased in either of these papers.

Figure 1 illustrates these exogenous demographic policy interventions. It depicts various population growth scenarios for both countries with confidence intervals obtained through a Bayesian averaging method.[5]

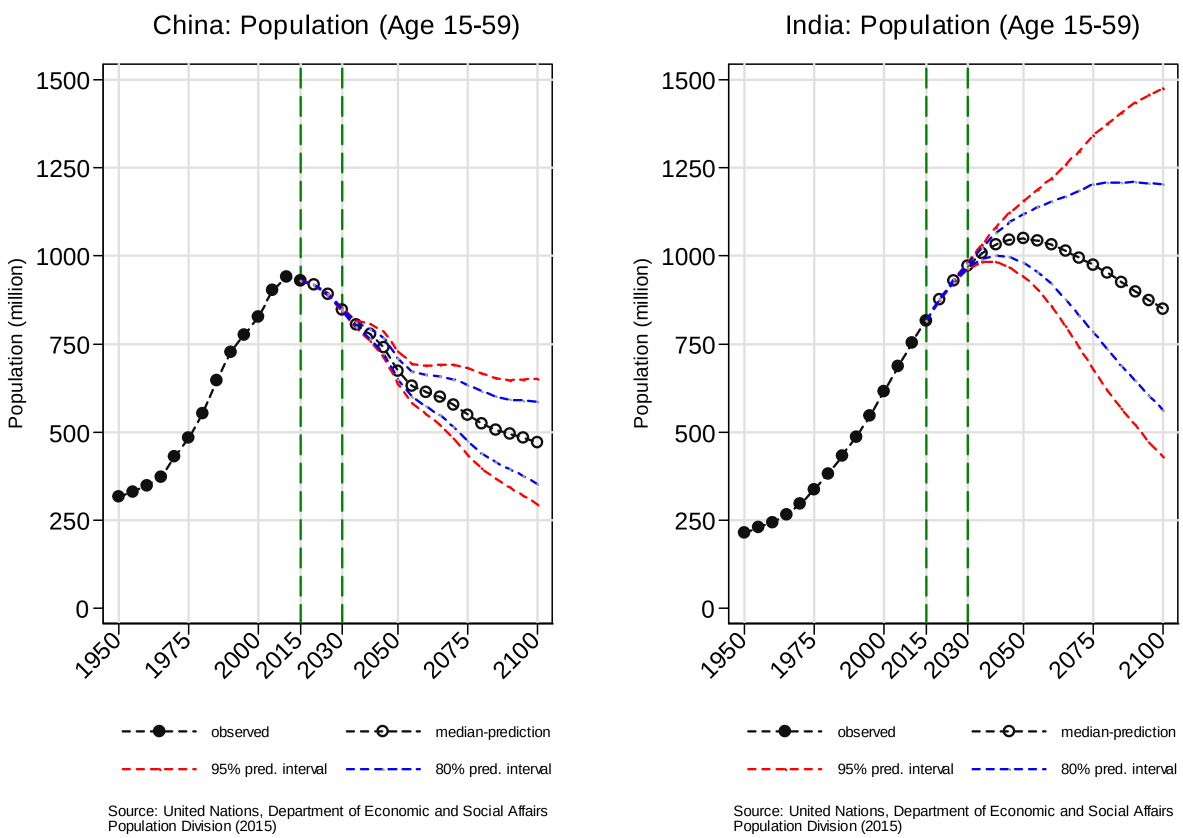

Figure 1 – Working-age population dynamics in China and India. The two vertical dashed lines indicate that, until 2030, the assumed working population dynamics are robust to any realistic population-growth scenario.

Three key observations result from Figure 1:

1. In China, an absolute decline in the working-age population (aged 15-59) began in 2010 and will continue under all reasonable scenarios. The increase

[5] Both data and population projection scenarios portrayed in Figure 1 are obtained from the United Nations Population division. Computations are done using an open source package described in Raftery et al. (2012) and Gerland et al. (2014).

in China's working-age population for roughly 18 years following the one-child policy implementation reflects the delayed reaction due to schooling and other work preparation activities until at least the age of 16.[6]

2. With a high degree of confidence, the working-age population of India is projected to continue increasing at least until 2030.

3. After 2025, the working-age population of India is projected to exceed that of China under all realistic scenarios.[7]

Figure 1 clearly demonstrates that, in contrast to India, China's policy intervention was not only effective shortly after implementation, but also that its effects on population dynamics are expected to persist beyond one generation.[8] The anticipation of these persistent policy effects is crucial for investment decisions because investors are forward-looking and major investments are typically long-lived.

2.2 Comparative macroeconomic performance

Table 1 presents comparative productivity and GDP growth rates. These were similar in China and India before and, significantly, after the exogenous demographic intervention, which allows us to plausibly attribute FDI trend differences between China and India principally to China's exogenous demographic intervention.

Both China and India experienced similar rapid real GDP growth in the post implementation (1982-2014) period (see the two columns under "g_Y" in Table 1).[9] Note that labor

[6] This delayed reaction is also due to a gradual increase in policy effectiveness and the gradual elimination of rural exemptions. The model to be proposed captures this decline as occurring in a single 25 year period, which is an artifact of the model's parsimony and the choice of a time interval equivalent to 25 years.
[7] In early 2023, the total population of India exceeded that of China.
[8] The recently introduced (2017) two-child policy in China may alter the anticipated population dynamics in China, depicted in the left panel of Figure 1, after 2030. Nevertheless, population dynamics 15 years ahead will not be affected: see the time interval bracketed by the vertical dashed lines. As of January 2020, there has been no uptick in Chinese fertility. See also Zhao (2017).
[9]

productivity growth, g_A, was also similar in China and India both in Period 1, and especially in Period 2 while increasing in both.[10] Real capital stock grew slightly more rapidly in China in the latter period, while the dramatic labor force growth slowdown in China is clearly evident in the "g_L" column (L denotes aggregate hours worked).

Table 1 Growth rates of macro aggregates. Annual rates (%).

	g_L growth rate of labor		g_K growth rate of capital		g_Y growth rate of GDP		g_A labor productivity growth	
	China	India	China	India	China	India	China	India
Period 1 (1960-1981)	2.05	2.27	7.89	3.52	5.11	4.14	1.69	2.17
Period 2 (1982-2014)	0.82	1.99	13.97	12.42	9.14	9.28	5.94	5.74

Source: Penn World Tables and United Nations. Data from the 1960s and 1970s is presented for comparison purposes only. Both China and India instituted market economy reforms in 1992. Our comparative model, to be detailed in Sections 3 and 4, thus provides insights only for the post 1992 period.

We define $\Delta g_{x,t} = g_{x,C,t} - g_{x,I,t}$ as the growth rate differential between China and India for any variable x. Figure 2 plots $\Delta g_{L,t}$ and $\Delta g_{K,t}$, prior and post 1982, when the one-child policy was first implemented. Solid lines represent the Hodrick-Prescott filtered series using the smoothing parameter $\lambda = 6.25$. Shortly thereafter, $\Delta g_{L,t}$ assumes negative values

For the calculations in Table 1 and the subsequent theoretical analysis, we employ a Cobb-Douglas production function given by $Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}$, where Y is GDP, K is capital, L is labor, and A is labor productivity (K is measured as the value of the capital stock and L as total hours worked). The two columns of Table 1 under "g_A", labor productivity growth, have been calculated using the formula $g_A = (g_Y - \alpha g_K) / (1-\alpha) - g_L$, where we have assumed that the capital intensity parameter, $\alpha = 1/3$ for both China and India.

[10] The similarities in productivity differences between China and India are also supported by Hsieh and Klenow (2009), and Bollard, Klenow and Sharma (2013).

which persist (right axis in Figure 2), capturing the long-term impact of the strictly enforced policy directive in China relative to India. A key feature of Figure 2 is the simultaneous reversal of the $\Delta g_{L,t}$ and $\Delta g_{K,t}$ trajectories, implying a causal link between the demographic intervention and the differential capital-accumulation dynamics in the two countries post 1982.

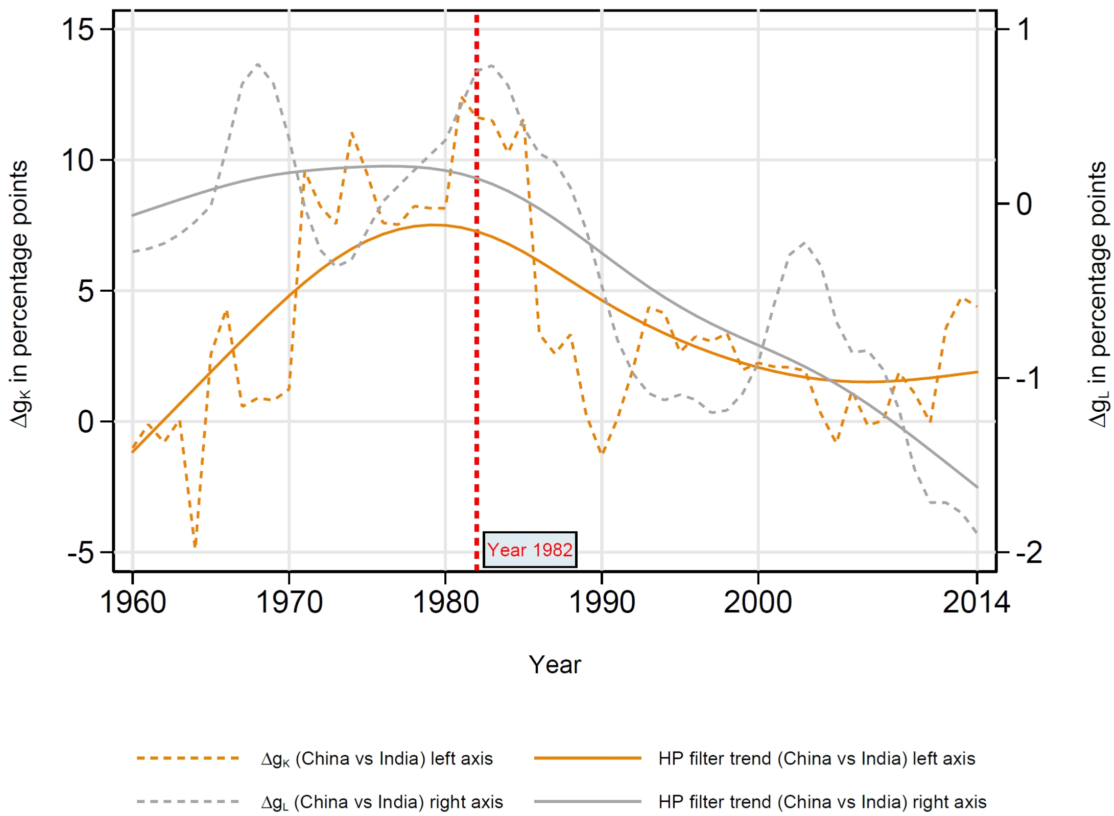

Figure 2 - Differential growth rates of capital and labor: China vs India.

As Table 1 indicates, Δg_A rose from -0.48% pre 1982 to 0.20%. This increase was, however, not strong enough to compensate for the impact of differential population growth on capital growth: $\Delta g_{K,t}$, while positive, is in general decline after 1982.

2.3 Comparative K/L and FDI dynamics

Figure 3 presents the post-1982 time path of $\log((FDI/GDP)_{China}/(FDI/GDP)_{India})$ and $\log((K/L)_{China}/(K/L)_{India})$.[11] It highlights two insights. First, China's K/L ratio outpaced India's following the 1982 policy intervention. Second, during the same period, FDI intensity (FDI as a share of GDP) grew faster in India than in China. In 1990, China's FDI/GDP ratio was about 30 times larger than that of India, but by 2014, it had declined to less than twice that of India.[12]

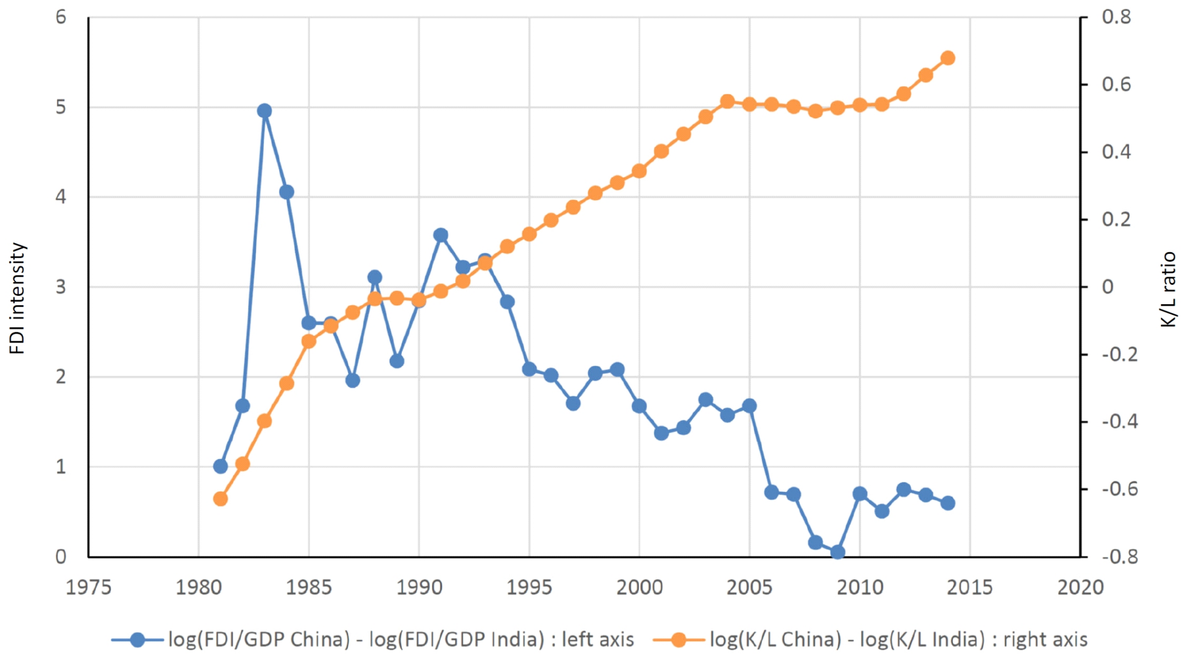

Figure 3 - Differential growth rates of FDI/GDP and K/L: China vs India.

In the what follows, we propose a model to explain these empirical observations.

[11]The data underlying Figure 3 is found in Table A.1 of Appendix E, available online.
[12]In Appendix H we document the data used in Figure 3 and offer a robustness check focusing on K/L trends of the non-agricultural workforce in both countries (see Figure H.6 and Table H.2 in Appendix H). It is important to note that FDI in China and India during the period examined did not represent the purchase of existing domestic capital by foreign entities; observed FDI data predominantly describes the formation of new capital. This experience contrasts with that of the US where the vast majority of FDI is for the purchase of claims to already existing capital stock.

3. The Model

3.1 Production

Aggregate domestic production in country $i \in \{1,2\}$ in period t is characterized by the production technology,[13]

$$\bar{Y}_{i,t} = Y_{i,t} + Y^r_{i,t} \ , \tag{1}$$

where,

$$Y_{i,t} = (K_{i,t})^{\alpha_i} \left(\bar{A}_{i,t} L_{i,t}\right)^{1-\alpha_i} \ , \qquad \alpha_i \in (0,1) \tag{2}$$

and

$$Y^r_{i,t} = (FDI_{i,t})^{\alpha_i} \left(\bar{A}_{i,t} L^r_{i,t}\right)^{1-\alpha_i} \ . \tag{3}$$

Superscript "r" denotes capital from the ROW, while the location of production is country i. Specifically, $K_{i,t}$ is the period t capital of country i invested by domestic firms, while $FDI_{i,t}$ is the accumulated stock of FDI capital invested by ROW firms in country i. $L_{i,t}$ is the workforce of country i working in firms using capital internally financed by country i, while $L^r_{i,t}$ denotes workers of country i that work for ROW companies using FDI. Variables with a bar denote country aggregates (see, for example $\bar{Y}_{i,t}$ in equation (1)). The common depreciation rate for capital $K_{i,t}$ and $FDI_{i,t}$ is $\delta \in (0,1]$, for $i \in \{1,2\}$, while $\bar{A}_{i,t}$ is the period-t level of labor productivity, common to both sectors in country i.[14] In each country i, we postulate a large number of identical firms operating the technologies described by equations (2) and (3).

Based on the assumption of no cross country labor force mobility, and assuming full

[13] Our production structure is a simplified version of the one in McGrattan and Prescott (2009, 2010) and Holmes et al. (2015).

[14] While labor productivity may be firm-specific, we lack any data on productivity growth in foreign-owned vs domestically-owned firms in either China or India; hence the simplifying assumption. Assuming $A_{i,t} \neq A^r_{i,t}$ leads to the same conclusions as the present formulation.

employment in each country,

$$\bar{L}_{i,t} = L_{i,t} + L^r_{i,t} \ , \tag{4}$$

where $\bar{L}_{i,t}$ is the total workforce (population) in country $i \in \{1,2\}$. We maintain our assumption that population growth and productivity growth in country $i \in \{1,2\}$ are both constant over time, i.e.,[15]

$$\frac{\bar{L}_{i,t+1}}{\bar{L}_{i,t}} = e^{g_{\bar{L},i}} \ , \ \ \frac{\bar{A}_{i,t+1}}{\bar{A}_{i,t}} = e^{g_{\bar{A},i}} \ . \tag{5}$$

Output in country $i \in \{1,2\}$ is given by,[16]

$$\bar{Y}_{i,t} = \bar{K}^{\alpha_i}_{i,t}\left(\bar{A}_{i,t}\bar{L}_{i,t}\right)^{1-\alpha_i} = (K_{i,t} + FDI_{i,t})^{\alpha_i}\left(\bar{A}_{i,t}\bar{L}_{i,t}\right)^{1-\alpha_i} \ . \tag{6}$$

3.2 Efficient factor allocation

Competitive-equilibrium factor inputs $\left(K_{i,t}, FDI_{i,t}, L_{i,t}, L^r_{i,t}\right)$ are efficiently allocated within each country to maximize domestic production. Profit-maximizing firms, located in country $i \in \{1,2\}$, domestic or foreign, thus equate marginal products to factor prices. The intra-temporal conditions for the efficient allocation of these factor inputs are,

$$MPK_{i,t} = MPK^r_{i,t} \quad \text{and} \quad MPL_{i,t} = MPL^r_{i,t} \ , \tag{7}$$

where "MPK" and "MPL" signify the marginal products of capital and marginal product of labor respectively.

Let r^* denote the prevailing world rate of interest and w_t the period t wage rate common to both domestic and foreign firms within a country. Equation (6) yields a key implication:[17]

[15] These growth rates need not be identical across countries as our notation allows. Indeed, the one-child policy will manifest itself as a structural change in the constant population growth rate in one country.
[16] See Online Appendix A for the derivation of expression (6).

$$r^* + \delta = \frac{\partial \bar{Y}_{i,t}}{\partial \bar{K}_{i,t}} \equiv \overline{MPK}_{i,t} = MPK_{i,t} = MPK^r_{i,t} \ , \ \ i \in \{1, 2\} \ . \tag{8}$$

From equation (6) we obtain,

$$w_t = (1 - \alpha) \left(\frac{\bar{K}_t}{\bar{A}_t \bar{L}_t} \right)^{\alpha} \bar{A}_t \ . \tag{9}$$

From (6) we also obtain,

$$r^* + \delta = \alpha \left(\frac{\bar{K}_t}{\bar{A}_t \bar{L}_t} \right)^{\alpha - 1} \ , \tag{10}$$

which implies,

$$\frac{\bar{K}_t}{\bar{A}_t \bar{L}_t} = \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{1}{1-\alpha}} \ . \tag{11}$$

Combining (11) and (9), we obtain,

$$w_t = (1 - \alpha) \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \ . \tag{12}$$

3.3 Households, Domestic Savings, National Capital and Equilibrium

3.3.1 Households

We use a variant of Diamond's (1965) overlapping generations model. All agents live for T periods so that at any time period t, there are T representative agents alive, one from each generation. At the end of period T_R, where $0 < T_R \leq T$, individuals retire and earn no labor income in their retirement periods $T_R + 1, T_R + 2, ..., T$. During their working periods, agents save and accumulate capital from which they consume in retirement. At the close of their lives, agents may leave bequests paid out in period $T + 1$ which provides them with utility ex ante.

[17]Note that the capital return in both countries is fixed by the world interest rate r^* plus the common depreciation rate. This is a consequence of the competitive assumption and the free flow of capital internationally. Returns, particularly in China, have historically shown wide variation, however. See Appendix G for a discussion of the related literature.

The following notation applies going forward:

$c_{i,b,j} \equiv$ consumption of an agent of country i, born in period b in her j-th period of life;

$L_{i,t} \equiv$ total labor force in country i, at time period t;

$w_{i,t} \equiv$ the period-t competitive wage for country i paid to all agents at work in country i;

$a_{i,b,j} \equiv$ wealth of an agent of country i, born in period b in her j-th period of life;

$r^* \equiv$ the world rate of interest assumed to be constant across all periods.

Let us for the moment suppress the index i, since the structure of both economies is the same except for differences in the assumed labor-force growth rates. Agents who work in a period, work for the entire period; i.e., we abstract from any labor-leisure tradeoff.

Assuming no bequests, the benchmark problem confronting a representative agent born in period t is as follows:

$$\max_{\{c_{b,j}\}_{j=1}^{T}, a_{t,T+1}} \sum_{j=1}^{T} \beta^{j-1} \frac{c_{b,j}^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \tag{13}$$

$$\text{s.t.} \qquad a_{b,j+1} = (1+r^*)\, a_{b,j} + w_{b+j-1} - c_{b,j} \ , \qquad j = 1, 2, ..., T. \tag{14}$$

$$a_{b,T+1} \geq 0 \ , \tag{15}$$

given that,

$$a_{b,1} = 0 \ . \tag{16}$$

Equations (15) and (16) confirm the absence of bequests: agents start with zero wealth and leave zero wealth in the last period of their lives (observe that, given (15), $a_{b,T+1} = 0$ is a necessary condition for an optimum).

Proposition 3.3.1 The solution to problem (13)-(16), is given by,

a.

$$c_{b,j} = \left[\beta\left(1+r^*\right)\right]^{\eta(j-1)} c_{b,1} \ , \tag{17}$$

where if $\beta(1+r^*) < 1$, then $c_{b,j} < c_{b,1}$ and is declining with j, if $\beta(1+r^*) = 1$, then $c_{b,j} = c_{b,1}$, for all j, while $c_{b,j}$ increases with j if $\beta(1+r^*) > 1$.

b.

$$c_{b,1} = \frac{\sum_{j=1}^{T} \frac{w_{b+j}}{(1+r^*)^{j-1}}}{1 + \sum_{j=2}^{T} \left[\beta^{\eta}\left(1+r^*\right)\right]^{j-1}} \ . \tag{18}$$

Ceteris paribus, a higher r^* reduces $c_{b,1}$, but causes the rate of growth of consumption thereafter to increase. Since

$$w_{b+j} = e^{g_{\bar{A}}(j-1)} w_b \ , \quad j = 1, ..., T, \tag{19}$$

$c_{b,1}$ may be rewritten as,

c.

$$c_{b,1} = \frac{1-\psi}{1-\psi^T} \frac{1-\xi^{T_R}}{1-\xi} \cdot w_b \ , \tag{20}$$

where $\psi \equiv \beta^{\eta}(1+r^*)^{\eta-1}$ and $\xi \equiv e^{g_{\bar{A}}} / (1+r^*)$.

d. The optimal evolution of wealth for an agent born at time period t satisfies

$$a_{b,j} = \begin{cases} (1+r^*)^{j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(\frac{1-\xi^{j-1}}{1-\xi^{T_R}} - \frac{1-\psi^{j-1}}{1-\psi^T} \right) w_b & , \quad j = 1, ..., T_R + 1 \\ (1+r^*)^{j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{j-1}}{1-\psi^T} \right) w_b & , \quad j = T_R + 2, ..., T \end{cases} . \tag{21}$$

Proof See Online Appendix B.

At this juncture several implications can be drawn. They are the subject of the following two corollaries.

Corollary 3.3.1 Later retirement (a larger T_R) increases consumption in all periods of life, reduces wealth in all periods up to retirement, and increases accumulated wealth after retirement.

Proof The corollary rests on the behavior of $c_{b,1}$, as T_R increases. Consider expression (20) for $c_{b,1}$, and notice that the term $(1-\psi)/\left(1-\psi^T\right)$ is unambiguously strictly positive for any $\psi \neq 1$, and any $\beta, r^*, \eta > 0$. Turning to the term $\left(1-\xi^{T_R}\right)/(1-\xi)$, provided $\xi \neq 1$, the derivative of this term with respect to T_R is always strictly positive; accordingly, the same is true for $c_{b,1}$ and, by (17), for all $c_{b,j}$. Coming to the behavior of wealth before retirement, we first focus on the top branch of the right-hand side of equation (21), which can be expressed as

$$a_{b,j} = (1+r^*)^{j-2}\left(\frac{1-\xi^{j-1}}{1-\xi} - \frac{1-\psi^{j-1}}{1-\psi^T}\frac{1-\xi^{T_R}}{1-\xi}\right) w_b\ ,\ \ j = 1, ..., T_R + 1. \tag{22}$$

Observe that, for $\psi \neq 1$, the term $\left(1-\psi^{j-1}\right)/\left(1-\psi^T\right)$ in (22) is strictly positive, and the term $\left(1-\xi^{T_R}\right)/(1-\xi)$ in (22) has a positive first derivative with respect to T_R as long as $\xi \neq 1$, proving that $\partial a_{b,j}/\partial T_R < 0$ for $j = 2, ..., T_R+1$. Regarding wealth after retirement, we focus on the bottom branch of the right-hand side of equation (21), which can be expressed as

$$a_{b,j} = (1+r^*)^{j-2}\frac{1-\xi^{T_R}}{1-\xi}\left(1 - \frac{1-\psi^{j-1}}{1-\psi^T}\right) w_b\ ,\ \ j = T_R + 2, ..., T. \tag{23}$$

For all $j < T$, the term $1 - \left(1-\psi^{j-1}\right)/\left(1-\psi^T\right) > 0$, in (23), and since the term $\left(1-\xi^{T_R}\right)/(1-\xi)$ in (23) has a positive first derivative with respect to T_R as long as $\xi \neq 1$, $\partial a_{b,j}/\partial T_R > 0$ for $j = T_R + 2, ..., T$. Q.E.D.

Corollary 3.3.2 For a fixed T_R, a longer lifespan T reduces consumption and increases wealth in all periods life .

Proof Once again the focus is on expression (20). For $\xi \neq 1$, the term $\left(1-\xi^{T_R}\right)/(1-\xi)$ is strictly positive, and the term $(1-\psi)/\left(1-\psi^{T}\right)$ depends negatively on T for all $\psi \neq 1$. Therefore, $\partial c_{b,1}/\partial T < 0$ and, by (17), for all $c_{b,j}$, $j=1,...,T$. Similarly, from (21), we can see that both branches of its right-hand side depend positively on T. To see this, observe that in both (22) and (23), the term $\left(1-\xi^{T_R}\right)/(1-\xi)$ is strictly positive for $\xi \neq 1$, while the term $(1-\psi)/\left(1-\psi^{T}\right)$ depends negatively on T for all $\psi \neq 1$, establishing that $\partial a_{b,j}/\partial T > 0$ for $j=2,...,T$. Q.E.D.

3.4 Aggregate relationships

Let us now return to identifying quantities by the relevant country i. Since each cohort (generation) lives for T periods, there are T distinct cohorts alive at any time t. It follows that aggregate consumption at time t for country i satisfies:

$$C_{i,t} = \sum_{j=0}^{T-1} c_{i,t-j,j+1} L_{i,t-j,j+1} \tag{24}$$

Identity (24) simply states that time t aggregate consumption in country i is the sum of the consumptions of each generation then alive from the generation just born ($j=0$) to the generation born $T-1$ periods ago and in its final year of life. Since the only asset in the economy by which workers in country i may accumulate wealth is capital stock accumulation, private wealth aggregated across all living generations and the aggregate capital stock must coincide. This is the substance of identity (25)

$$K_{i,t} = \sum_{j=0}^{T-1} a_{i,t-j,j+1} L_{i,t-j,j+1} \ , \tag{25}$$

where $K_{i,t}$ is country i's aggregate domestic capital.

In order to calculate aggregate domestic savings, we first consider the household budget constraint given by (14). By re-arranging (14) we obtain,

$$a_{i,t,j+1} - a_{i,t,j} = r^* a_{i,t,j} + w_{i,t+j-1} - c_{i,t,j} \ , \quad j = 1, 2, ..., T \ . \tag{26}$$

Household savings of generation t in period $j \in \{1, ..., T\}$, denoted by $s_{i.t,j}$, is thus defined by,

$$s_{i,t,j} = a_{i,t,j+1} - (1-\delta) \, a_{i,t,j} = (r^* + \delta) \, a_{i,t,j} + w_{i,t+j-1} - c_{i,t,j} \ , \quad j = 1, 2, ..., T \ . \tag{27}$$

Aggregate domestic savings is thus given by,

$$S_{i,t} = \sum_{j=0}^{T-1} s_{i,t-j,j+1} L_{i,t-j,j+1} \ . \tag{28}$$

Summing up across cohorts in equation (27), and using the definitions given by (24), (25), and (28), we obtain,

$$S_{i,t} - \delta K_{i,t} = K_{i,t+1} - K_{i,t} = r^* K_{i,t} + w_{i,t} \bar{L}_{i,t} - C_{i,t} \ . \tag{29}$$

Notice that in obtaining equation (29) we have used the fact that, in equilibrium, every cohort both starts and ends its life with zero wealth. Moreover (29) emphasizes that households in country i receive labor income from working in both the domestic production sector that produces aggregate income $Y_{i,t}$, and in the FDI sector that produces aggregate income, $Y_{i,t}^r$, with $\bar{Y}_{i,t} = Y_{i,t} + Y_{i,t}^r$. From equation (6) and (8) we know that,

$$\bar{Y}_{i,t} = (K_{i,t} + FDI_{i,t}) \, (r^* + \delta) + w_{i,t} \bar{L}_{i,t} \ . \tag{30}$$

Combining (30) and (29), we find,

$$S_{i,t} = \bar{Y}_{i,t} - FDI_{i,t} \, (r^* + \delta) - C_{i,t} \ = I_{i,t} \ , \tag{31}$$

where $I_{i,t}$ is domestic aggregate investment. Finally, from (29) we can see that,

$$K_{i,t+1} = S_{i,t} + (1-\delta) K_{i,t} \ , \tag{32}$$

while (31) and (32) reconfirm the aggregate-domestic-capital-accumulation identity:

$$K_{i,t+1} = I_{i,t} + (1-\delta) K_{i,t} \ . \tag{33}$$

We next consider time-invariant relationships that will ultimately allow us to identify $S_{i,t}/\bar{Y}_{i,t}$, $FDI_{i,t}/\bar{Y}_{i,t}$, and $FDI_{i,t}/S_{i,t}$ along the economy's steady-state growth path. This is accomplished by the Proposition 3.4.1.

Proposition 3.4.1 In a steady state, when the population growth rate, $g_{\bar{L}}$, is constant, the FDI/GDP ratio of country i in period t is given by,

$$\frac{FDI_{i,t}}{\bar{Y}_{i,t}} = \frac{\alpha}{r^* + \delta} - \nu \ , \tag{34}$$

where

$$\nu \equiv \frac{1-\alpha}{1+r^*} \frac{1-\chi\xi}{1-(\chi\xi)^T} \frac{1-\xi^{T_R}}{1-\xi} \left\{ \frac{1}{1-\xi^{T_R}} \left[\frac{1-\chi^{T_R+1}}{1-\chi} - \frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi} \right] + \right.$$
$$\left. + \frac{1}{1-\psi^T} \left[\frac{1-(\chi\psi)^T}{1-\chi\psi} - \frac{1-\chi^T}{1-\chi} \right] + \frac{\chi^{T_R+1}-\chi^T}{1-\chi} \right\} \ , \tag{35}$$

$\psi = \beta^\eta (1+r^*)^{\eta-1}$, $\xi = e^{g_{\bar{A}}} (1+r^*)$, and

$$\chi \equiv \frac{1+r^*}{e^{g_{\bar{A}}+g_{\bar{L}}}} \ . \tag{36}$$

The savings to GDP ratio of country i in period t is given by,

$$\frac{S_{i,t}}{\bar{Y}_{i,t}} = \left(e^{g_{\bar{A}}+g_{\bar{L}}} - 1 + \delta \right) \nu \ . \tag{37}$$

Proof See Online Appendix C.

Based on Proposition 3.4.1, Corollary 3.4.1 specifies the determinants of the relationship between FDI and the stock of aggregate domestic capital, i.e., the ratio FDI_t/K_t.

Corollary 3.4.1 The ratio between FDI and the domestic stock of capital is given by,

$$\frac{FDI_t}{K_t} = \frac{\alpha}{\nu\left(r^* + \delta\right)} - 1 \ . \tag{38}$$

Proof From equation (6), $r^* + \delta = \alpha \bar{Y}_t / \bar{K}_t$, which implies

$$\frac{\bar{K}_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ . \tag{39}$$

Given that $\bar{K}_t = FDI_t + K_t$, equation (39) becomes,

$$\frac{FDI_t}{\bar{Y}_t} + \frac{K_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ . \tag{40}$$

Equations (34) and (40) imply,

$$\frac{K_t}{\bar{Y}_t} = \nu \ . \tag{41}$$

Since $FDI_t/K_t = \left(FDI_t/\bar{Y}_t\right) / \left(K_t/\bar{Y}_t\right)$, using (41) and (34), we prove equation (38). Q.E.D.

3.5 Dependence of the steady-state FDI/GDP ratio on population growth rate and interest rate

Proposition 3.4.2 examines how a decrease in the population growth rate, $g_{\bar{L}}$, i.e., an exogenous demographic intervention such as the one-child policy, will affect the FDI/GDP ratio.

This question is central to this paper, since we claim that the one-child policy in China substantially contributed to a decrease in its FDI/GDP ratio, relative to India's. Proposition 3.4.2 focuses on examining cases in which the calibrating parameters have values that reflect empirical observations. Specifically, we focus on cases where households accumulate positive wealth until retirement and then gradually deplete it, i.e., cases where average household savings are always positive, as empirically observed.

Proposition 3.4.2 If r^*, $g_{\bar{A}}$, $g_{\bar{L}}$, β, η, are such that $\phi(j) > 0$, $j \geq 1$, where,

$$\phi(j) \equiv \begin{cases} \frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T} & , \quad j = 0, ..., T_R \\ 1 - \frac{1-\psi^j}{1-\psi^T} & , \quad j = T_R + 1, ..., T-1 \end{cases} , \tag{42}$$

and if $T = 2$, i.e., there are only two overlapping generations, then a decrease in the population growth rate, $g_{\bar{L}}$, leads to a lower FDI/GDP ratio, for any initial value $g_{\bar{L}} \neq 0$. For $T > 2$, as long as $\phi(j) > 0$, $j \geq 1$,

$$\frac{\partial\left(\frac{FDI_t}{\bar{Y}_t}\right)}{\partial g_{\bar{L}}} > 0 \Leftrightarrow \left(\frac{1}{e^{g_{\bar{L}}} - 1} - \frac{T}{e^{g_{\bar{L}}T} - 1}\right) \frac{\sum_{j=0}^{T-1} \chi^j \phi(j)}{\sum_{j=0}^{T-1} j\chi^j \phi(j)} < 1 \; . \tag{43}$$

Proof See Online Appendix D.

The general analytical result conveyed by Proposition 3.4.2, serves as a guide for all calibration exercises that follow. Yet, Proposition 3.4.3 shows the dependence of the FDI/GDP ratio on the world interest rate, r^*.

Proposition 3.4.3 For characterizing the dependence of the FDI/GDP ratio on the world interest rate, r^*, parameters α, δ, η, β, $g_{\bar{L}}$, $g_{\bar{A}}$, T, T_R, and r^* must be guided by,

$$\frac{\partial\left(\frac{FDI_t}{\bar{Y}_t}\right)}{\partial r^*}<0 \Leftrightarrow \frac{\alpha}{\left(r^*+\delta\right)^2}+\nu\cdot\left[\frac{1}{1+r^*}\left(\frac{T_R\xi^{T_R}}{1-\xi^{T_R}}-\frac{T\xi^T}{1-\xi^T}-1\right)+\frac{m'\left(r^*\right)}{m\left(r^*\right)}\right]>0\ , \tag{44}$$

where,

$$m\left(r^*\right)=\frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-\left(\chi\xi\right)^{T_R+1}}{1-\chi\xi}\right]+ \\ +\frac{1}{1-\psi^T}\left[\frac{1-\left(\chi\psi\right)^T}{1-\chi\psi}-\frac{1-\chi^T}{1-\chi}\right]+\frac{\chi^{T_R+1}-\chi^T}{1-\chi}\ . \tag{45}$$

Proof See Online Appendix D.

Holston et al. (2017) estimate r^*, reporting that, in the past few decades, r^* has decreased substantially in different major economic regions of the world, suggesting that there are global factors behind this decline. Proposition 3.4.3 offers verifiable parameter conditions dictating how the FDI/GDP could be affected by such a decrease in r^*. In all our calibration exercises outlined below, we have robustly found a negative dependence, i.e., $\partial\left(FDI_t/\bar{Y}_t\right)/\partial r^*<0$.[18] According to the present model, this negative dependence implies that the recent decline in r^* should be pushing FDI/GDP ratio upwards. Such an effect may be common to both China and India, but it tends towards the opposite direction to the demographic effect of the one-child policy in China for the same calibrating parameters, based on Proposition 3.4.2.[19]

[18] A quick way to numerically verify the sign of the term $m'\left(r^*\right)/m\left(r^*\right)$ is to compute the difference in $\ln\left[m\left(r^*\right)\right]$ for small changes in r^*.

[19] For details, see our calibration section below and our robustness checks in Online Appendix F.

3.6 A Permanent Exogenous Demographic Intervention: Transition Dynamics

The one-child policy in China represents a long-term exogenous demographic intervention. To approximate the conditions implied by China's one-child policy, we next examine the consequences of a permanent exogenous change in the population growth rate. Specifically, assume that in some period $\hat{t} > 0$, a permanent change from population growth rate $g_{L,1}$ (for all $t \in \{0, 1, ..., \hat{t}\}$) to population growth rate $g_{L,2}$ (for all $t \in \{\hat{t}+1, \hat{t}+2, ...\}$) occurs as an unforeseen event. All cohorts that are alive in period $\hat{t}$, must accordingly adapt their savings plans from period $\hat{t}$ and on.

In order to see the impact of this permanent change in g_L on the FDI/GDP ratio, our principal quantity of interest, we return to equation (40), an aggregate relationship that holds no matter if the economy is in a steady state or in a transition between steady states:

$$\frac{FDI_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} - \frac{K_t}{\bar{Y}_t} \ . \tag{46}$$

By equation (46), in order to understand the dynamics of the FDI/GDP ratio, it suffices to analyze the dynamics of $K_t/\bar{Y}_t$. More specifically, equations (11) and (6) imply,

$$\bar{Y}_t = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \bar{L}_t \ . \tag{47}$$

Inserting (46) into (47) gives,

$$\frac{FDI_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} - \left(\frac{\alpha}{r^* + \delta}\right)^{-\frac{\alpha}{1-\alpha}} \frac{K_t}{\bar{A}_t \bar{L}_t} \ . \tag{48}$$

In (48), the only variables that are affected by the change in g_L are K_t and $\bar{L}_t$. The impact of the change in g_L on $\bar{L}_t$ is direct. The impact of the change in g_L on K_t is based on equation (25), and requires an understanding of how the change in g_L affects individual savings.

According to equations (21) and (42),

$$a_{t-j,j+1} = (1 + r^*)^{j-1} \frac{1 - \xi^{T_R}}{1 - \xi} \phi(j) \, w_{t-j} \ . \tag{49}$$

Note that "b" has been replaced by "$t-j$". Since ξ and ψ do not depend on g_L, the only impact of the change in g_L to the individual savings plan of agents may come from changes in the wage rate, w_t. Nevertheless, equation (12) implies that even w_t is not affected by the change in g_L. This may seem counterintuitive for a closed-economy model. Nevertheless, in the present model the abundance of international capital keeps both interest rates and wage rates unaffected by demographic changes. Yet, aggregate domestic capital, K_t, will be affected by the savings contributions of each demographic cohort, as equation (25) implies. In what follows, we focus on characterizing analytically the impact of the demographic change on K_t and $\bar{L}_t$, in order to understand the impact of the exogenous demographic intervention on the FDI/GDP ratio through equation (48). Proposition 3.6.1 summarizes our analytical characterization.

Proposition 3.6.1 The FDI/GDP ratio transition dynamics after $\hat{t}$ are given by,

$$\frac{FDI_{\hat{t}+\ell}}{\bar{Y}_{\hat{t}+\ell}} = \frac{\alpha}{r^*+\delta} - \frac{1-\alpha}{1+r^*}\frac{1-\xi^{T_R}}{1-\xi}\lambda(\ell)\sum_{j=0}^{T-1}\xi^{-j}\Lambda(j,\ell)\,\phi(j)\ , \tag{50}$$

where,

$$\Lambda(j,\ell) \equiv \begin{cases} e^{-\left(g_{L,2}-g_{L,1}\right)\ell - g_{L,1}j} & , \quad j \geq \ell \\ e^{-g_{L,2}j} & , \quad j < \ell \end{cases} , \tag{51}$$

$$\lambda(\ell) = \left[\frac{1-e^{-g_{L,2}\ell}}{1-e^{-g_{L,2}}} + e^{-g_{L,2}\ell}\frac{1-e^{-g_{L,1}(T-\ell)}}{1-e^{-g_{L,1}}}\right]^{-1} , \tag{52}$$

and $\phi(j)$ is given by equation (42), for all $\ell \in \{1,...,T\}$.

Proof See Online Appendix E.

In Proposition 3.6.1, it is notable that, after setting $\ell = 0$, and $\ell = T$ in (50), the FDI/GDP ratio is equal to its steady-state value given by equation (34) of Proposition 3.4.2. Specifically, after setting $\ell = 0$ in (50), one steady-state FDI/GDP ratio corresponds to population growth rate $g_{\bar{L}} = g_{L,1}$ in equation (34), while setting $\ell = T$ in (50), another steady-state FDI/GDP ratio corresponds to population growth rate $g_{\bar{L}} = g_{L,2}$. We numerically study this transition in the following section.

3.7 Calibration and Simulations

While both China and India experienced major development transitions after 1982, we assume that these two countries underwent similar structural transformations, except one: the exogenous demographic intervention was effective only in China. Within the context of the present model we therefore argue that demographics alone are able to explain the different FDI/GDP-ratio dynamics in the two countries.

We focus on explaining the ratio of the FDI/GDP-ratios between China and India in the data. We further assume that India is in a demographic steady state, with a constant FDI/GDP ratio, with China following a demographic transition solely driven by its one-child policy. The main simulation question we tackle is: can the introduction of the one-child policy alone explain the behavior of the relative FDI/GDP-ratios of China and India found in the data?

There is, however, a timing issue regarding the period in which our model mechanics may reasonably be applied to the data: before 1995, both China and India had FDI restrictions; prior to 1995, FDI/GDP ratios in both China and India are, typically, below 1%, with India's FDI/GDP ratio being around 0.02%. With the theory of this paper based on competitive markets, we therefore fit the model to post-1995 available data.

A second, purely technical issue remains. China and an even smaller value of 0.017%

for India). Starting with such small values, the model-induced transition dynamics quickly pushes the generated absolute FDI/GDP value for both China and India into negative territory. Accordingly, these initial conditions force the transition-dynamics trajectory to be sensitive to the chosen calibrated parameters and initial states.

In order to address this calibration-sensitivity problem, our numerical strategy is to:

1. Employ larger starting values of the FDI/GDP ratio for China and India. Year 1995 fits this requirement (1990's values being, again, very small); China's FDI/GDP ratio was 2.90% and India's 0.192% in 1995.

2. In order to isolate the quantitative effects of the one-child policy, we assume that the Indian transition dynamics of the FDI/GDP ratio are stable for an extended period, and that they are positive. We also keep the population growth rate, $g_{\bar{L}}$, of India stable and positive, which is empirically the case: starting from 1995, India's working population growth has been essentially constant to slightly increasing.

3. The model-implied slope of the Chinese transition dynamics of the FDI/GDP ratio can be, in general, flat, or exhibit a sharp decline. Nevertheless, it must be much sharper than the Indian one. The derived Chinese transition path should intersect the X-axis around 2015; in this way, the path of transition dynamics will be positive before 2015. For the population growth, $g_{\bar{L}}$, in China before and after the implementation of the one-child policy, we consider values of $g_{L,1}$ within the range of $[1\%, 2\%]$, and values of $g_{L,2}$ within the range of $[-1\%, 0\%]$ that fit the data well.

Regarding our calibration parameter values, we fix $r^* = 10.75\%$. While the world r^* in the literature is between 4% and 3% during the examined period (see, for example, Holston et al. 2017), developing countries like China and India in the 1980s and the 1990s had a risk premium that must be added to r^*, which is the value observed in industrial economies with well-developed financial markets. The remaining calibration parameter values are summarized by Table 2:

Table 2 Calibration parameters (annual values, % rates).

	$g_{\bar{A}}$	α	β	δ	η
China	3.38	46.74	99.14	17.14	57.78
India	2.52	20.05	98.00	16.90	35.77

The difference in the physical capital intensity in production (parameter α) between China and India is justified by the rapid industrialization of China, in contrast to India's greater specialization in agriculture and services. The high depreciation rate in both countries (and especially in China), is justified by the high failure rate of firms during their rapid industrialization phase.

The transition begins in year 1985, but we only seek to match the transition path with respect to data for the period 1995-2015, since our model assumes free FDI markets without capital controls. Therefore, instead of anchoring our model using observed FDI/Y ratios in China and India, we have calibrated it using the 1985 savings rates of China and India. As mentioned above, international capital markets were not free in 1985. Therefore, the observed 1985 FDI/GDP ratios are not appropriate as calibration inputs. Hence, we choose to use the savings rate as the anchoring value. While we are unable to match the savings rates exactly, model implied savings rates are close to their respective data points, as shown in Table 3:

Table 3 Initial calibration targets (%).

	Savings rate 1985		FDI/GDP ratio 1995	
	China	India	China	India
Data	33.75	14.88	2.90	0.19
Model	29.32	13.15	2.90	0.19

Finally, we set our assumed economic lifespan to $T = 50$, and the retirement time at $T_R = 45$ for both China and India. Assuming that the economic lifetime starts at age 20, the expected life-span is 70 years and retirement occurs at 65 years. Taking 1995 as a benchmark, these life expectancy and retirement ages are realistic. The population growth rates in China are $g_{L,1} = 1.44\%$ and $g_{L,2} = -0.58\%$, whereas in India we assume $g_{L,1} = g_{L,2} = 1.59\%$. Our results are shown by Figure 4. The right-hand side panel in Figure 4 shows only model results (the level of FDI/GDP ratios), whereas the left-hand side panel shows the goodness-of fit of the model to the relative FDI/GDP-ratio data.

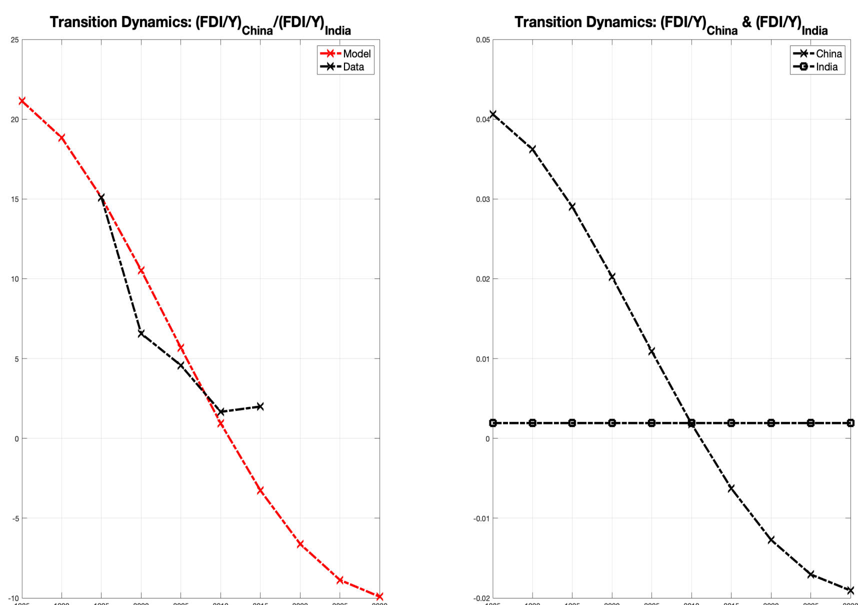

Figure 4

In Backus et al. (2014) and especially in Cooley and Henriksen (2018), there is much analysis on how changes in life expectancy and in retirement age may affect international capital flows. In order to address these concerns, we provide a sensitivity analysis of our benchmark calibration in Online Appendix F. Specifically, we analyze two cases: (i) we shorten the economic lifespan from its benchmark value $T = 50$ to $T = 45$, and the retirement time from its benchmark value $T_R = 45$, to $T_R = 40$,[20] and (ii) we increase the economic lifespan from its benchmark value $T = 50$ to $T = 55$, while keeping the retirement time to its benchmark value $T_R = 45$. In both cases, we find calibrating parameters similar to the benchmark calibration that bring simulations close to the calibration targets in both Table 3 and Figure 4.

4. Conclusion

This paper is a contribution to the nascent literature focusing on the role of demographic changes in determining FDI flows. We examine the effects of cross-country heterogeneity in population growth rates on relative FDI flows, a topic not previously addressed in the literature. In particular, we study the effects on FDI of an endogenous increases in a society's capital/labor ratio resulting from a population decline. Our empirical setting is the mandatory one-child policy in China contrasted with India's comparatively laissez faire approach to population control. This policy difference creates a natural experiment. We explore the resulting empirical evidence in the context of a neoclassical model of FDI dynamics. The evidence and our analysis support the hypothesis that population dynamics may have a major impact on relative FDI flows. More generally, our results illustrate how demographic

[20]Here we assume that working life started at 15 years old in the early agrarian regime years, with $T = 50$ and $T_R = 40$ implying 65 years of life-expectancy and retirement at 55.

policies can have substantial effects on a country's economic activity.

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Online Appendix for

Demographics and FDI: Lessons from China's One-Child Policy

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Online Appendix A – Proof of production aggregation

We omit time subscripts for simplicity. From equations (1), (2), and (3), we obtain,

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} K_i^{\alpha_i} L_i^{1-\alpha_i} \left[1 + \left(\frac{FDI_i}{K_i} \right)^{\alpha_i} \left(\frac{L_i^r}{L_i} \right)^{1-\alpha_i} \right] \ . \tag{A.1}$$

Assuming frictionless cross-country capital flows, condition (7) implies the equilibrium condition:

$$r^* + \delta = MPK_{i,t} = MPK_{i,t}^r \ . \tag{A.2}$$

Combining equations (A.2), (2), and (3), we obtain,

$$FDI_i \cdot L_i = K_i \cdot L_i^r \ . \tag{A.3}$$

Equation (A.1), combined with (A.3) and (4) becomes,

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} K_i^{\alpha_i} L_i^{-\alpha_i} \bar{L}_i \ . \tag{A.4}$$

Adding the term $K_i \cdot L_i$ to both sides of equation (A.3) leads to $(K_i + FDI_i) \cdot L_i = K_i \cdot (L_i + L_i^r)$, which implies,

$$\frac{\bar{K}_i}{\bar{L}_i} = \frac{K_i}{L_i} \ , \tag{A.5}$$

given (4), and given that $\bar{K}_i = K_i + FDI_i$. Combining (A.4) with (A.5) we obtain

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} \left(\frac{\bar{K}_i}{\bar{L}_i} \right)^{\alpha_i} \bar{L}_i \ ,$$

which coincides with equation (6), proving the aggregation result. Q.E.D.

Online Appendix B – Proof of Proposition (3.3.1)

Proof of Proposition 3.3.1

We start by solving equation (14) as a difference equation, using the simplified form,

$$x_{j+1} = \theta x_j + \zeta_j \ , \quad j = 1, ..., T \ , \tag{B.1}$$

where,

$$x_j \equiv a_{b,j} \ , \tag{B.2}$$

$$\theta \equiv 1 + r^* \ , \tag{B.3}$$

$$\zeta_j \equiv w_{b+j-1} - c_{b,j} \ . \tag{B.4}$$

Combining (16) with (B.1), provides us with the initial condition for the differential equation given by (B.1), which is given by,

$$x_1 = 0 \ . \tag{B.5}$$

Using successive substitutions,

$$\left.\begin{array}{r} \left.\begin{array}{l} (B.1) \overset{j=1}{\Longrightarrow} x_2 = \theta x_1 + \zeta_1 \\ (B.1) \overset{j=2}{\Longrightarrow} x_3 = \theta x_2 + \zeta_2 \end{array}\right\} \Rightarrow x_3 = \theta\left(\theta x_1 + \zeta_1\right) + \zeta_2 \\ (B.1) \overset{j=3}{\Longrightarrow} x_4 = \theta x_3 + \zeta_3 \qquad\qquad\qquad\qquad \end{array}\right\} \Rightarrow x_4 = \theta\left[\theta\left(\theta x_1 + \zeta_1\right) + \zeta_2\right] + \zeta_3 \ ,$$

which can be rewritten as,

$$x_4 = \theta^3 x_1 + \sum_{\ell=1}^{3} \theta^{3-\ell} \zeta_\ell \ . \tag{B.6}$$

Generalizing (B.6), we obtain,

$$x_j = \theta^{j-1} x_1 + \sum_{\ell=1}^{j-1} \theta^{j-\ell-1} \zeta_\ell \ , \quad j = 2, ..., T \ . \tag{B.7}$$

Dividing both sides of (B.7) by θ^{j-1} gives a useful form of the solution, namely,

$$\frac{x_j}{\theta^{j-1}} = x_1 + \sum_{\ell=1}^{j-1} \frac{\zeta_\ell}{\theta^\ell} \ , \quad j = 2, ..., T \ . \tag{B.8}$$

Imposing the initial condition given by (B.5) on (B.8) leads to,

$$\frac{x_j}{\theta^{j-1}} = \sum_{\ell=1}^{j-1} \frac{\zeta_\ell}{\theta^\ell} , \quad j = 2, ..., T . \tag{B.9}$$

Using (B.2), (B.3), and (B.4), (B.9) becomes,

$$\frac{a_{b,j}}{(1+r^*)^{j-1}} = \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^\ell} ,$$

or, more conveniently, after dividing both sides of the last equation by $1 + r^*$,

$$\frac{a_{b,j}}{(1+r^*)^{j-2}} = \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^{\ell-1}} , \quad j = 2, ..., T . \tag{B.10}$$

With the solution of the wealth-accumulation path at hand given by equation (B.10), we proceed in order to impose the next necessary and sufficient condition for an optimum, which is the Euler equation

$$c_{b,j+1} = [\beta (1+r^*)]^\eta c_{b,j} , \quad j = 1, ..., T-1 . \tag{B.11}$$

Solving (B.11) forward leads to equation (17). In order to solve for the optimum level of consumption $c_{b,1}$ which drives the whole optimal consumption path in (17), we can extend equation (B.10) to $j = T + 1$, and impose the terminal condition given by (15). Specifically, a necessary condition for an optimum is that (15) binds, i.e.,

$$a_{b,T+1} = 0 . \tag{B.12}$$

Extending equation (B.10) to $j = T + 1$, and imposing (B.12) gives,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} . \tag{B.13}$$

In order to calculate the right-hand side of (B.13) we substitute equation (17) to get,

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \sum_{\ell=1}^{T} \frac{[\beta (1+r^*)]^{\eta(\ell-1)}}{(1+r^*)^{\ell-1}} ,$$

which simplifies to

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \sum_{\ell=1}^{T} \psi^{\ell-1} \ ,$$

and finally,

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \frac{1-\psi^T}{1-\psi} \ . \tag{B.14}$$

To calculate the left-hand side of (B.13) we use the following change of indices:

$$t = b + \ell - 1 \ . \tag{B.15}$$

Equation (B.15) gives,

$$\ell = t - b + 1 \ , \tag{B.16}$$

which further implies,

$$\ell = 1 \overset{(B.16)}{\Longrightarrow} t = b \ , \tag{B.17}$$

and

$$\ell = T \overset{(B.16)}{\Longrightarrow} t = T - b + 1 \ . \tag{B.18}$$

Substituting (B.15), (B.16), (B.17), and (B.18) into the left-hand side of (B.13) we obtain,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \sum_{t=b}^{b+T-1} \frac{w_t}{(1+r^*)^{t-b}} \ . \tag{B.19}$$

Based on (5), equation (12) implies,

$$w_t = e^{g_{\bar{A}}(t-b)} w_b \ . \tag{B.20}$$

Before we substitute (B.20) into (B.19), notice that

$$w_t = 0 \ , \quad \text{for all } t \in \{T_R + 1, ..., T\} \ . \tag{B.21}$$

Substituting (B.20) and (B.21) into (B.19) gives,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = w_b \sum_{t=b}^{b+T_R-1} \left(\frac{e^{g_{\bar{A}}}}{1+r^*} \right)^{t-b} \ ,$$

or,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = w_b \sum_{j=0}^{T_R-1} \xi^j \ ,$$

which simplifies to,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \frac{1-\xi^{T_R}}{1-\xi} w_b \ . \tag{B.22}$$

Substituting (B.22) and (B.14) into (B.13) gives,

$$c_{b,1} = \frac{1-\xi^{T_R}}{1-\xi} \frac{1-\psi}{1-\psi^T} w_b \ , \tag{B.23}$$

which proves equation (20).

We proceed with deriving the optimal path of wealth for the representative household in cohort b.

$$\left.\begin{array}{l} (B.10) \Rightarrow a_{b,j} = (1+r^*)^{j-2} \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^{\ell-1}} \\ \qquad\qquad\qquad \ell = 1 \Rightarrow i = 0 \\ \underbrace{i = \ell - 1}_{\Downarrow \atop \ell = i+1} \ \nearrow \searrow \\ \qquad\qquad\qquad \ell = j-1 \Rightarrow i = j-2 \end{array}\right\} \Rightarrow a_{b,j} = (1+r^*)^{j-2} \sum_{i=0}^{j-2} \frac{w_{b+i} - c_{b,i+1}}{(1+r^*)^i} \ , \tag{B.24}$$

which holds for $j = 2, ..., T$. Based on (5), equation (12) implies,

$$w_{b+i} = e^{g_{\bar{A}} i} w_b \ , \quad i = 0, ..., T_R - 1 \ . \tag{B.25}$$

Equation (17) gives,

$$c_{b,i+1} = \left[\beta^\eta (1+r^*)^\eta\right]^i c_{b,1} \ , \quad i = 0, ..., T-1 \ . \tag{B.26}$$

After substituting (B.25) and (B.26) into (B.24) we obtain,

$$a_{b,j} = (1+r^*)^{j-2} \left(w_b \sum_{i=0}^{j-2} \xi^i - c_{b,1} \sum_{i=0}^{j-2} \psi^i \right) \ ,$$

which simplifies to,

$$a_{b,j} = (1+r^*)^{j-2}\left(\frac{1-\xi^{j-1}}{1-\xi}w_b - \frac{1-\psi^{j-1}}{1-\psi}c_{b,1}\right) \ , \quad j=1,...,T_R+1 \ . \qquad \text{(B.27)}$$

Substituting (B.23) into equation (B.27) gives,

$$a_{b,j} = (1+r^*)^{j-2}\left(\frac{1-\xi^{j-1}}{1-\xi} - \frac{1-\xi^{T_R}}{1-\xi}\frac{1-\psi^{j-1}}{1-\psi^T}\right) w_b \ , \quad j=1,...,T_R+1 \ ,$$

which can be written as,

$$a_{b,j} = (1+r^*)^{j-2}\,\frac{1-\xi^{T_R}}{1-\xi}\left(\frac{1-\xi^{j-1}}{1-\xi^{T_R}} - \frac{1-\psi^{j-1}}{1-\psi^T}\right) w_b \ , \quad j=1,...,T_R+1 \ . \qquad \text{(B.28)}$$

The reason why (B.28) holds for j only up to period T_R+1 is that, after period T_R the wage earnings are zero, i.e,

$$w_{b+j-1} = 0 \ , \quad j = T_R+1,...,T. \qquad \text{(B.29)}$$

We can now solve for the optimal wealth path after period T_R+1, taking the wealth in period T_R+1, i.e., a_{b,T_R+1} as given. Specifically, after setting $j=T_R+1$, (B.28) gives,

$$a_{b,T_R+1} = (1+r^*)^{T_R-1}\,\frac{1-\xi^{T_R}}{1-\xi}\left(1 - \frac{1-\psi^{T_R}}{1-\psi^T}\right) w_b \ . \qquad \text{(B.30)}$$

Starting equation (14) from $j=T_R+1$ and on, and after taking into account (B.29),

$$\left.\begin{array}{c}\left.\begin{array}{l}(14)\,,(B.29) \overset{j=T_R+1}{\Longrightarrow} a_{b,T_R+2} = \theta a_{b,T_R+1} - c_{b,T_R+1} \\ (14)\,,(B.29) \overset{j=T_R+2}{\Longrightarrow} a_{b,T_R+3} = \theta a_{b,T_R+2} - c_{b,T_R+2}\end{array}\right\} \Rightarrow a_{b,T_R+3} = \theta\left(\theta a_{b,T_R+1} - c_{b,T_R+1}\right) - c_{b,T_R+2} \\ (14)\,,(B.29) \overset{j=T_R+3}{\Longrightarrow} a_{b,T_R+4} = \theta a_{b,T_R+3} - c_{b,T_R+3}\end{array}\right\} \Rightarrow$$

$$\Rightarrow a_{b,T_R+4} = \theta\left[\theta\left(\theta a_{b,T_R+1} - c_{b,T_R+1}\right) - c_{b,T_R+2}\right] - c_{b,T_R+3}$$

which can be written in a condensed form as,

$$a_{b,T_R+4} = \theta^3 a_{b,T_R+1} - \sum_{\ell=1}^{3}\theta^{3-\ell}c_{b,T_R+\ell} \ ,$$

and can be generalized to,

$$a_{b,T_R+j} = \theta^{j-1} a_{b,T_R+1} - \sum_{\ell=1}^{j-1} \theta^{j-\ell-1} c_{b,T_R+\ell} \ ,$$

and rewritten as,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \sum_{\ell=1}^{j-1} \frac{c_{b,T_R+\ell}}{\theta^{\ell}} \right) \ , \quad j = 2, ..., T - T_R \ . \tag{B.31}$$

From (17) we obtain,

$$c_{b,T_R+\ell} = (\theta\psi)^{\ell-1} c_{b,T_R+1} \ , \quad j = 1, ..., T - T_R \ . \tag{B.32}$$

Combining (B.32) with (B.31) leads to,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - c_{b,T_R+1} \sum_{\ell=1}^{j-1} \frac{(\theta\psi)^{\ell-1}}{\theta^{\ell}} \right) \ ,$$

which is

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \theta^{-1} c_{b,T_R+1} \sum_{\ell=1}^{j-1} \psi^{\ell} \right) \ ,$$

and simplifies to,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \theta^{-1} \frac{1-\psi^{j-1}}{1-\psi} c_{b,T_R+1} \right) \ , \quad j = 2, ..., T - T_R \ . \tag{B.33}$$

Using again (17),

$$c_{b,T_R+1} = (\theta\psi)^{T_R} c_{b,1} \ . \tag{B.34}$$

Substituting (B.30), (B.23) and (B.34) into (B.33) we obtain,

$$a_{b,T_R+j} = \theta^{j-1} \left[\theta^{T_R-1} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{T_R}}{1-\psi^{T}} \right) - \theta^{T_R-1} \frac{1-\psi^{j-1}}{1-\psi} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-\psi}{1-\psi^{T}} \right] w_b \ ,$$

which simplifies, after some algebra, to,

$$a_{b,T_R+j} = \theta^{T_R+j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{T_R+j-1}}{1-\psi} \right) w_b \ , \quad j = 2, ..., T - T_R \ . \tag{B.35}$$

It remains to adjust the indices in (B.35), setting

$$\ell = T_R + j \ , \tag{B.36}$$

which implies,

$$j = \ell - T_R \ . \tag{B.37}$$

Substituting (B.36) and (B.37) into (B.35), after using (B.3) gives,

$$a_{b,\ell} = (1+r^*)^{j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{\ell-1}}{1-\psi}\right) w_b \ , \quad \ell = T_R + 2, ..., T \ . \tag{B.38}$$

Cpmbining (B.38) with (B.28) proves (21), completing the proof of the Proposition. Q.E.D.

Online Appendix C – Proof of Proposition (3.4.1)

Proof of Proposition 3.4.1 We start from equation (25), dropping the country index i, namely

$$K_t = \sum_{j=0}^{T-1} a_{t-j,j+1} L_{t-j,j+1} \ , \tag{C.1}$$

where, according to equation (21),

$$a_{t-j,j+1} = (1+r^*)^{j-1} \frac{1-\xi^{T_R}}{1-\xi} \phi(j) \, w_{t-j} \ , \tag{C.2}$$

$$\phi(j) \equiv \begin{cases} \frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T} & , \quad j = 0, ..., T_R \\ 1 - \frac{1-\psi^j}{1-\psi^T} & , \quad j = T_R+1, ..., T-1 \end{cases} \ , \tag{C.3}$$

in which the indices change must be noticed. Combining (C.1) and (C.2) we obtain,

$$K_t = \frac{1-\xi^{T_R}}{1-\xi} \sum_{j=0}^{T-1} (1+r^*)^{j-1} \phi(j) \, w_{t-j} L_{t-j,j+1} \ . \tag{C.4}$$

Given (5), and given that the size of a cohort does not change during the cohort's lifetime (deterministic death time) notice that,

$$L_{t-j,j+1} = L_{t-j} = e^{-g_{\bar{L}} j} L_t \ , \tag{C.5}$$

and that, based on (5) again, equation (12) implies,

$$w_{t-j} = e^{-g_{\bar{A}} j} w_t \ . \tag{C.6}$$

We can relate L_{t-j} in equation (C.5) to $\bar{L}_t$ for all $j \in \{0, ..., T-1\}$, since,

$$\bar{L}_t = \sum_{j=0}^{T-1} L_{t-j} \ . \tag{C.7}$$

Specifically, combining (C.5) with (C.7), we obtain,

$$\bar{L}_t = L_t \sum_{j=0}^{T-1} e^{-g_{\bar{L}} j} \ ,$$

which simplifies to,

$$\bar{L}_t = \frac{1 - e^{-g_{\bar{L}} T}}{1 - e^{-g_{\bar{L}}}} L_t \ . \tag{C.8}$$

Combining (C.8) with (C.5) gives,

$$L_{t-j,j+1} = L_{t-j} = e^{-g_{\bar{L}} j} \frac{1 - e^{-g_{\bar{L}}}}{1 - e^{-g_{\bar{L}} T}} \bar{L}_t \ . \tag{C.9}$$

With (C.9), (C.6) and (12) at hand, we return to (C.4), obtaining,

$$K_t = (1-\alpha) \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \frac{1 - \xi^{T_R}}{1 - \xi} \frac{1 - e^{-g_{\bar{L}}}}{1 - e^{-g_{\bar{L}} T}} \bar{A}_t \bar{L}_t \sum_{j=0}^{T-1} (1 + r^*)^{j-1} \phi(j) \, e^{-(g_{\bar{A}} + g_{\bar{L}}) j} \ ,$$

which can be re-written more concisely as,

$$K_t = \frac{1-\alpha}{1+r^*} \frac{1 - \xi^{T_R}}{1 - \xi} \frac{1 - e^{-g_{\bar{L}}}}{1 - e^{-g_{\bar{L}} T}} \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \bar{L}_t \sum_{j=0}^{T-1} \chi^j \phi(j) \ . \tag{C.10}$$

Recalling that $\xi = e^{g_{\bar{A}}} / (1 + r^*)$ and $\chi \equiv (1 + r^*) / e^{g_{\bar{A}} + g_{\bar{L}}}$, notice that,

$$\chi \xi = e^{-g_{\bar{L}}} \ . \tag{C.11}$$

Using (C.11) we can simplify (C.10) into,

$$K_t = \nu \cdot \left(\frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \bar{L}_t \ . \tag{C.12}$$

where

$$\nu \equiv \frac{1-\alpha}{1+r^*} \frac{1 - \xi^{T_R}}{1 - \xi} \frac{1 - \chi\xi}{1 - (\chi\xi)^T} \sum_{j=0}^{T-1} \chi^j \phi(j) \ . \tag{C.13}$$

The constant ν in (C.13) corresponds to the constant ν in (35). To prove that (C.13) and (35) are equivalent, observe that, based on (C.3),

$$\sum_{j=0}^{T-1} \chi^j \phi(j) = \sum_{j=0}^{T_R} \chi^j \left(\frac{1 - \xi^j}{1 - \xi^{T_R}} - \frac{1 - \psi^j}{1 - \psi^T} \right) + \sum_{j=T_R+1}^{T-1} \chi^j \left(1 - \frac{1 - \psi^j}{1 - \psi^T} \right) \ . \tag{C.14}$$

For calculating the first summation of the right-hand side of (C.14),

$$\sum_{j=0}^{T_R} \chi^j \left(\frac{1 - \xi^j}{1 - \xi^{T_R}} - \frac{1 - \psi^j}{1 - \psi^T} \right) = \frac{1}{1 - \xi^{T_R}} \left[\sum_{j=0}^{T_R} \chi^j - \sum_{j=0}^{T_R} (\chi\xi)^j \right] - \frac{1}{1 - \psi^T} \left[\sum_{j=0}^{T_R} \chi^j - \sum_{j=0}^{T_R} (\chi\psi)^j \right] \ ,$$

which simplifies to,

$$\sum_{j=0}^{T_R}\chi^j\left(\frac{1-\xi^j}{1-\xi^{T_R}}-\frac{1-\psi^j}{1-\psi^T}\right)=\frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi}\right]-$$
$$-\frac{1}{1-\psi^T}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\psi)^{T_R+1}}{1-\chi\psi}\right]\ . \tag{C.15}$$

Regarding the second summation of the right-hand side of (C.14), observe that

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}+\chi^{T_R+2}+...+\chi^{T-1},$$

which simplifies to,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}\left(1+\chi+\chi^2+...+\chi^{T-T_R-2}\right)\ ,$$

or,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}\frac{1-\chi^{T-T_R-1}}{1-\chi}\ ,$$

i.e.,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\frac{\chi^{T_R+1}-\chi^T}{1-\chi}\ . \tag{C.16}$$

Based on (C.16), the second summation of the right-hand side of (C.14) simplifies to,

$$\sum_{j=T_R+1}^{T-1}\chi^j\left(1-\frac{1-\psi^j}{1-\psi^T}\right)=\frac{\chi^{T_R+1}-\chi^T}{1-\chi}-\frac{1}{1-\psi^T}\left[\frac{\chi^{T_R+1}-\chi^T}{1-\chi}-\frac{(\chi\psi)^{T_R+1}-(\chi\psi)^T}{1-\chi\psi}\right]\ . \tag{C.17}$$

Substituting (C.15) and (C.17) into (C.14) we obtain, after some algebra,

$$\sum_{j=0}^{T-1}\chi^j\phi(j)=\frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi}\right]+$$
$$+\frac{1}{1-\psi^T}\left[\frac{1-(\chi\psi)^T}{1-\chi\psi}-\frac{1-\chi^T}{1-\chi}\right]+\frac{\chi^{T_R+1}-\chi^T}{1-\chi}\ . \tag{C.18}$$

Finally, combining (C.18) with (C.13), we obtain the expression in (35) for ν.

It remains to obtain the expressions for $FDI_{i,t}/\bar{Y}_{i,t}$ and $K_{i,t}/\bar{Y}_{i,t}$ given by (34) and (37). Equation (32) can be rewritten as,

$$S_t = K_{t+1} - (1-\delta)\, K_t \ . \tag{C.19}$$

Observe that equation (C.12) holds for all t, namely,

$$\frac{K_{t+1}}{\bar{A}_{t+1}\bar{L}_{t+1}} = \frac{K_t}{\bar{A}_t\bar{L}_t} = \nu \cdot \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \ ,$$

which means that,

$$\frac{K_{t+1}}{K_t} = \frac{\bar{A}_{t+1}\bar{L}_{t+1}}{\bar{A}_t\bar{L}_t} \ ,$$

and based on (5), it is,

$$K_{t+1} = e^{g_{\bar{A}}+g_{\bar{L}}} K_t \ . \tag{C.20}$$

Combining (C.20) with (C.19) we arrive at,

$$S_t = \left(e^{g_{\bar{A}}+g_{\bar{L}}} - 1 + \delta\right) K_t \ . \tag{C.21}$$

Recall from equation (6) that,

$$r^* + \delta = \alpha \frac{\bar{Y}_t}{\bar{K}_t} \ ,$$

which can be rewritten as,

$$\frac{\bar{K}_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ ,$$

or,

$$\frac{K_t}{\bar{Y}_t} + \frac{FDI_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ . \tag{C.22}$$

From equation (6) we obtain,

$$\bar{Y}_t = \left(\frac{\bar{K}_t}{\bar{A}_t\bar{L}_t}\right)^{\alpha} \bar{A}_t\bar{L}_t \ ,$$

and based on (11) it is,

$$\bar{Y}_t = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t\bar{L}_t \ . \tag{C.23}$$

Combining (C.23) with (C.12), we obtain,

$$\frac{K_t}{\bar{Y}_t} = \nu \ . \tag{C.24}$$

From (C.24) and (C.22) we prove equation (34). After dividing both sides of (C.21) by $\bar{Y}_t$ and substituting (C.24), we arrive at equation (37), proving the Proposition. Q.E.D.

Online Appendix D – Proof of Propositions (3.4.2) and (3.4.3)

Proof of Proposition 3.4.2 Our main goal is to characterize what happens to the FDI/GDP ratio as $g_{\bar{L}}$ decreases. From equation (34) we can see that

$$\frac{\partial\left(\frac{FDI_t}{\bar{Y}_t}\right)}{\partial g_{\bar{L}}} > 0 \Leftrightarrow \frac{\partial \nu}{\partial g_{\bar{L}}} < 0 \ . \tag{D.1}$$

According to equation (21),

$$a_{b,j+1} = (1+r^*)^{j-1} \frac{1-\xi^{T_R}}{1-\xi} \phi(j)\, w_b \ , \tag{D.2}$$

where,

$$\phi(j) \equiv \begin{cases} \frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T} & , \quad j = 0, ..., T_R \\ 1 - \frac{1-\psi^j}{1-\psi^T} & , \quad j = T_R+1, ..., T-1 \end{cases} . \tag{D.3}$$

It is straightforward to show that,

$$\frac{1-\xi^{T_R}}{1-\xi} > 0 \text{ , for all } \xi > 0,\ \xi \neq 1 \ . \tag{D.4}$$

To see that (D.4) is true, notice that the signs of the numerator and the denominator of $\left(1-\xi^{T_R}\right)/(1-\xi)$ will be the same, no matter if $0 < \xi < 1$ or $\xi > 1$. Therefore, according to (D.2) and (D.4), the only way to guarantee that accumulated wealth, $a_{b,j+1}$, along the lifecycle of a cohort (leaving out $a_{b,1} = a_{b,T+1} = 0$) are positive, is to pick calibrating parameters r^*, $g_{\bar{A}}$, $g_{\bar{L}}$, β and η, so that ξ and ψ in (D.3) guarantee that,

$$\phi(j) > 0 \text{ , for all } j \in \{1, ..., T-1\} \ . \tag{D.5}$$

Returning now to (D.1), combining (C.11) and (C.13), ν can be re-written as,

$$\nu \equiv \frac{1-\alpha}{1+r^*} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} \sum_{j=0}^{T-1} \chi^j \phi(j) \ . \tag{D.6}$$

Using a similar argument to this for proving (D.4), we can see that

$$\frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} > 0 \ , \quad \text{for all } g_{\bar{L}} \neq 0 \ . \tag{D.7}$$

In order to find the sign of $\partial\nu/\partial g_{\bar{L}}$ from (D.6), notice that, among constants ξ, ψ, and χ, only $\chi = (1+r^*)/e^{g_{\bar{A}}+g_{\bar{L}}}$ depends on $g_{\bar{L}}$. Therefore, let's express χ as,

$$\chi = e^{-g}\kappa \ ,$$

where $\kappa \equiv (1+r^*)/e^{g_{\bar{A}}}$, and we express $g_{\bar{L}}$ as "g", for notational simplicity. Therefore,

$$\nu = \zeta \underbrace{\frac{1-e^{-g}}{1-e^{-gT}}}_{f(g)} \cdot \underbrace{\sum_{j=0}^{T-1} \left(e^{-g}\kappa\right)^j \phi(j)}_{h(g)} \ , \tag{D.8}$$

where

$$\zeta = \frac{1-\alpha}{1+r^*}\frac{1-\xi^{T_R}}{1-\xi} > 0 \ .$$

Based on (D.8),

$$\frac{\partial\nu}{\partial g} = \zeta\left[f'(g)\,h(g) + f(g)\,h'(g)\right] \ . \tag{D.9}$$

Notice that,

$$f(g)\,h(g) > 0 \ , \text{ and } h'(g) < 0 \ . \tag{D.10}$$

Since $\zeta > 0$, equations (D.9) and (D.10) imply that,

$$\frac{\partial\nu}{\partial g} < 0 \Longleftrightarrow \frac{f'(g)}{f(g)}\frac{h(g)}{-h'(g)} < 1 \ . \tag{D.11}$$

From the definition of $h(g)$ in (D.8) we see that

$$\frac{h(g)}{-h'(g)} = \frac{\sum_{j=0}^{T-1}\left(e^{-g}\kappa\right)^j\phi(j)}{\sum_{j=0}^{T-1} j\left(e^{-g}\kappa\right)^j\phi(j)} \ , \tag{D.12}$$

i.e.,

$$\frac{h(g)}{-h'(g)} = \frac{\overbrace{\phi(0)}^{\overset{0}{\shortparallel}} + e^{-g}\kappa\ \phi(1) + ... + (e^{-g}\kappa\ )^{T-1}\phi(T-1)}{0\cdot\phi(0) + e^{-g}\kappa\ \phi(1) + ... + (T-1)(e^{-g}\kappa\ )^{T-1}\phi(T-1)}\ . \tag{D.13}$$

From (D.12) and (D.13) we can see that

$$\frac{h(g)}{-h'(g)} = 1\ ,\ \text{if}\ T = 2\ , \tag{D.14}$$

and

$$\frac{h(g)}{-h'(g)} < 1\ ,\ \text{if}\ T > 2\ . \tag{D.15}$$

Coming now to $f(g)$,

$$f'(g) = \frac{e^{-g}\left(1 - e^{-gT}\right) - Te^{-gT}\left(1 - e^{-g}\right)}{\left(1 - e^{-gT}\right)^2}\ . \tag{D.16}$$

After some algebra, from the definition of $f(g)$ in (D.8) we see that,

$$\frac{f'(g)}{f(g)} = \frac{1}{e^g - 1} - \frac{T}{e^{gT} - 1}\ . \tag{D.17}$$

Based on (D.17), we can see that,

$$\frac{f'(g)}{f(g)} < 1 \Longleftrightarrow \frac{T}{e^{gT} - 1} - \frac{1}{e^g - 1} + 1 > 0\ , \tag{D.18}$$

which implies,

$$\frac{f'(g)|_{T=2}}{f(g)|_{T=2}} < 1 \Longleftrightarrow \frac{2}{e^{2g} - 1} - \frac{1}{e^g - 1} + 1 > 0 \Longleftrightarrow \frac{e^g\left(e^g - 1\right)}{e^{2g} - 1} > 0\ , \tag{D.19}$$

which is a true statement for all $g \neq 0$ (i.e., for all $g_{\bar{L}} \neq 0$). Combining (D.19) with (D.14) proves the part of the proposition that refers to $T = 2$.

For $T > 2$, inequality (D.18) is not guaranteed to be true, therefore, combining (D.17), (D.12), (D.11) and (D.1), proves inequality (43) of the proposition, completing the proof. Q.E.D.

Proof of Proposition 3.4.3

Based on equation (34), the inequality given by (44) holds if,

$$\frac{\partial \nu}{\partial r^*} > 0 \ . \tag{D.20}$$

Therefore, we focus on providing sufficient conditions for (D.20). Based on (35),

$$\nu = f\left(r^*\right) g\left(r^*\right) h\left(r^*\right) \underbrace{\left[j\left(r^*\right) + k\left(r^*\right) + \ell\left(r^*\right)\right]}_{\substack{\shortparallel \\ m\left(r^*\right)}} \ , \tag{D.21}$$

where,

$$f\left(r^*\right) = \frac{1-\alpha}{1+r^*} \ , \tag{D.22}$$

$$g\left(r^*\right) = \frac{1-\chi\xi}{1-\left(\chi\xi\right)^T} \ , \tag{D.23}$$

$$h\left(r^*\right) = \frac{1-\xi^{T_R}}{1-\xi^T} \ , \tag{D.24}$$

$$j\left(r^*\right) = \frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi} - \frac{1-\left(\chi\xi\right)^{T_R+1}}{1-\chi\xi}\right] \ , \tag{D.25}$$

$$k\left(r^*\right) = \frac{1}{1-\psi^T}\left[\frac{1-\left(\chi\psi\right)^T}{1-\chi\psi} - \frac{1-\chi^T}{1-\chi}\right] \ , \tag{D.26}$$

$$\ell\left(r^*\right) = \frac{\chi^{T_R+1}-\chi^T}{1-\chi} \ . \tag{D.27}$$

Therefore, based on the notation given by (D.21),

$$\begin{aligned} \frac{\partial \nu}{\partial r^*} &= f'\left(r^*\right) g\left(r^*\right) h\left(r^*\right) m\left(r^*\right) \\ &+ f\left(r^*\right) g'\left(r^*\right) h\left(r^*\right) m\left(r^*\right) \\ &+ f\left(r^*\right) g\left(r^*\right) h'\left(r^*\right) m\left(r^*\right) \\ &+ f\left(r^*\right) g\left(r^*\right) h\left(r^*\right) \underbrace{m'\left(r^*\right)}_{\substack{\shortparallel \\ j'\left(r^*\right)+k'\left(r^*\right)+\ell'\left(r^*\right)}} \end{aligned} \tag{D.28}$$

Therefore, we need to investigate the signs of $f(r^*)$, $g(r^*)$, $h(r^*)$, $jk\ell(r^*)$ as well as the signs of $f'(r^*)$, $g'(r^*)$, $h'(r^*)$, $jk\ell'(r^*)$.

From (D.22), we can immediately see that,

$$f(r^*) > 0 \text{ and } f'(r^*) < 0 \text{, as long as } r^* > -1 . \tag{D.29}$$

Similarly, since $\chi = (1+r^*)/e^{g_{\bar{A}}+g_{\bar{L}}}$ and $\xi = e^{g_{\bar{A}}}/(1+r^*)$, (D.23) can be rewritten as,

$$g(r^*) = \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} ,$$

which implies,

$$g(r^*) > 0 \text{ and } g'(r^*) = 0 \text{, for all } r^* \in \mathbb{R} . \tag{D.30}$$

Regarding the signs of $h(r^*)$ and $h'(r^*)$, let's re-define $h(r^*)$ as

$$h(r^*) \equiv n(\xi(r^*)), \quad \text{where } n(\xi) \equiv \frac{1-\xi^{T_R}}{1-\xi^T} \text{, and } \xi(r^*) \equiv \frac{e^{g_{\bar{A}}}}{1+r^*} . \tag{D.31}$$

Notice from (D.31) that

$$h(r^*) > 0 \quad \text{for all } \xi \neq 1 , \tag{D.32}$$

and,

$$h'(r^*) \equiv n'(\xi(r^*))\,\xi'(r^*) . \tag{D.33}$$

Equation (D.31) implies,

$$\xi'(r^*) < 0 \text{, for all } r^* > -1 , \tag{D.34}$$

and, after some algebra,

$$n'(\xi) = \xi^{T+T_R-1}\frac{T\left(\frac{1}{\xi^{T_R}}-1\right)-T_R\left(\frac{1}{\xi^T}-1\right)}{\left(1-\xi^T\right)^2} . \tag{D.35}$$

Therefore, (D.33), (D.34), and (D.35) imply,

$$h'(r^*) < 0 \Leftrightarrow T\left(\frac{1}{\xi^{T_R}}-1\right) > T_R\left(\frac{1}{\xi^T}-1\right) . \tag{D.36}$$

At this point, given (D.30), equation (D.28) implies,

$$\frac{\partial \nu}{\partial r^*} = f' \cdot g \cdot h \cdot m + f \cdot g \cdot h' \cdot m + f \cdot g \cdot h \cdot m' \ ,$$

i.e.,

$$\frac{\partial \nu}{\partial r^*} = f \cdot g \cdot h \cdot m \cdot \left(\frac{f'}{f} + \frac{h'}{h} + \frac{m'}{m} \right) \ ,$$

and based on (D.21),

$$\frac{\partial \nu}{\partial r^*} = \nu \cdot \left(\frac{f'}{f} + \frac{h'}{h} + \frac{m'}{m} \right) \ . \tag{D.37}$$

Given that $f'/f = d\left[\ln\left(f\right)\right]/dr^*$, equation (D.22) implies,

$$\frac{f'\left(r^*\right)}{f\left(r^*\right)} = \frac{d\left[\ln\left(\frac{1-\alpha}{1+r^*}\right)\right]}{dr^*} = \frac{-1}{1+r^*} \ . \tag{D.38}$$

Similarly, equation (D.24) combined with (D.31) implies,

$$\frac{h'\left(r^*\right)}{h\left(r^*\right)} = \frac{d\left[\ln\left(\frac{1-\xi^{T_R}}{1-\xi^{T}}\right)\right]}{dr^*} = \frac{\xi}{1+r^*} \cdot \left(\frac{T_R \xi^{T_R - 1}}{1-\xi^{T_R}} - \frac{T\xi^{T-1}}{1-\xi^{T}} \right) \ . \tag{D.39}$$

Combining (D.37) with (D.38) and (D.39) proves (44). Q.E.D.

Online Appendix E – Proof of Proposition (3.6.1)

Proof of Proposition 3.6.1 The proof relies on combining equations (48) and (49). Based on (48),

$$\frac{FDI_{\hat{t}+\ell}}{\bar{Y}_{\hat{t}+\ell}} = \frac{\alpha}{r^* + \delta} - \left(\frac{\alpha}{r^* + \delta}\right)^{-\frac{\alpha}{1-\alpha}} \frac{K_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} \ . \tag{E.1}$$

We focus on relating the dynamics of $\bar{L}_{\hat{t}+\ell}$ with the dynamics of $\bar{K}_{\hat{t}+\ell}$ in (E.1). During the transition, each cohort grows at rate $g_L(t)$, given by,

$$g_L(t) = \begin{cases} g_{L,1} & , \quad t \leq \hat{t} \\ g_{L,2} & , \quad t > \hat{t} \end{cases} \ . \tag{E.2}$$

Moreover,

$$\bar{L}_{\hat{t}+\ell} = \sum_{j=0}^{T-1} L_{\hat{t}+\ell-j} \ , \ \ \ell = 1, ..., T \ . \tag{E.3}$$

From (E.2), we can see that,

$$L_{\hat{t}+\ell-j} = \begin{cases} e^{-g_{L,2}\ell - g_{L,1}(j-\ell)} L_{\hat{t}+\ell} & , \quad j \geq \ell \\ e^{-g_{L,2}j} L_{\hat{t}+\ell} & , \quad j < \ell \end{cases} \ , \ \ \ell = 1, ..., T \ , \ \ j = 0, ..., T-1 \ ,$$

which can be summarized as,

$$L_{\hat{t}+\ell-j} = \Lambda(j,\ell)\, L_{\hat{t}+\ell} \ , \tag{E.4}$$

where $\Lambda(j,\ell)$ is given by (51). Combining (E.3) and (E.4) we obtain,

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell} \sum_{j=0}^{T-1} \Lambda(j,\ell) \ \ ,$$

and based on the definition of $\Lambda(j,\ell)$ from (51),

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell} \left[\sum_{j=0}^{\ell-1} e^{-g_{L,2}j} + \sum_{j=\ell}^{T-1} e^{-\left(g_{L,2} - g_{L,1}\right)\ell - g_{L,1}j} \right] \ , \tag{E.5}$$

where the convention

$$\sum_{i=a}^{b} x_i = 0 \ , \text{ if } \ a > b \ , \tag{E.6}$$

applies. After some algebra, (E.5) implies,

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell}\left[\frac{1-e^{-g_{L,2}\ell}}{1-e^{-g_{L,2}}} + e^{-g_{L,2}\ell}\frac{1-e^{-g_{L,1}(T-\ell)}}{1-e^{-g_{L,1}}}\right] \ . \tag{E.7}$$

Combining (E.4) and (E.7) gives,

$$L_{\hat{t}+\ell-j} = \Lambda\left(j,\ell\right)\lambda\left(\ell\right)\bar{L}_{\hat{t}+\ell} \ , \tag{E.8}$$

where $\lambda(\ell)$ is given by (52).

With (E.8) at hand, we can now relate the dynamics of $\bar{L}_{\hat{t}+\ell}$ with the dynamics of $\bar{K}_{\hat{t}+\ell}$. The definition of $\bar{K}_{\hat{t}+\ell}$ is,

$$\bar{K}_{\hat{t}+\ell} = \sum_{j=0}^{T-1} a_{\hat{t}+\ell-j,j+1} L_{\hat{t}+\ell-j} \ . \tag{E.9}$$

From (49) we obtain,

$$a_{\hat{t}+\ell-j,j+1} = (1+r^*)^{j-1}\frac{1-\xi^{T_R}}{1-\xi}\phi\left(j\right)w_{\hat{t}+\ell-j} \ , \tag{E.10}$$

where $\phi(j)$ is given by equation (42). From (12),

$$w_{\hat{t}+\ell} = (1-\alpha)\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}\bar{A}_{\hat{t}+\ell} \ , \tag{E.11}$$

therefore,

$$w_{\hat{t}+\ell-j} = w_{\hat{t}+\ell}e^{-g_{\bar{A}}j} \ . \tag{E.12}$$

Combining (E.9) with (E.8), (E.11), and (E.12), gives

$$\frac{\bar{K}_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} = (1-\alpha)\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}\frac{1-\xi^{T_R}}{1-\xi}\lambda\left(\ell\right)\sum_{j=0}^{T-1}(1+r^*)^{j-1}\Lambda\left(j,\ell\right)\phi\left(j\right)e^{-g_{\bar{A}}j} \ . \tag{E.13}$$

Keeping in mind that $\xi \equiv e^{-g_{\bar{A}}}/(1+r^*)$, (E.13) becomes,

$$\frac{\bar{K}_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} = \frac{1-\alpha}{1+r^*}\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}\frac{1-\xi^{T_R}}{1-\xi}\lambda\left(\ell\right)\sum_{j=0}^{T-1}\xi^{-j}\Lambda\left(j,\ell\right)\phi\left(j\right) \ . \tag{E.14}$$

Combining (E.14) with (E.1), leads to (50), proving the proposition. Q.E.D.

Online Appendix F – Sensitivity Analysis

Case 1: shortening the economic lifespan from its benchmark value $T = 50$ to $T = 45$, and the retirement time from its benchmark value $T_R = 45$, to $T_R = 40$

Here, the interest rate is $r^* = 10.67\%$ and all calibration parameters appear in Table F.1.

Table F.1 Calibration parameters (annual values, % rates).

	$g_{\bar{A}}$	α	β	δ	η
China	3.08	46.76	99.15	17.17	57.81
India	2.52	20.07	98.01	16.94	35.78

The goodness of fit to key calibration targets (as in Table 3 in the main body of the paper), is given by Table F.2.

Table F.2 Initial calibration targets (%).

	Savings rate 1985		FDI/GDP ratio 1995	
	China	India	China	India
Data	33.75	14.88	2.90	0.19
Model	28.69	13.23	2.90	0.19

The goodness of fit of the transition dynamics in this case appear in Figure F.1.

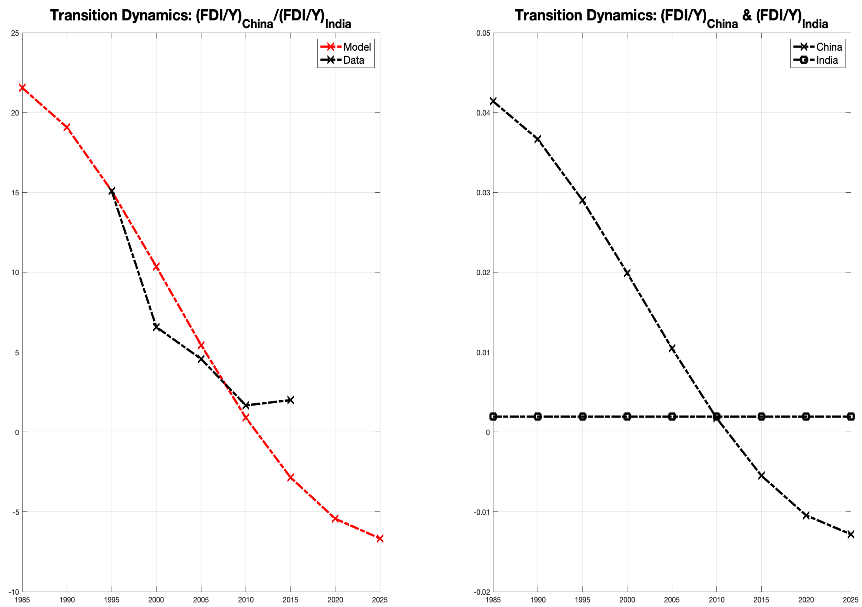

Figure F.1

In brief, the goodness of fit in this case of shorter life expectancy and shorter retirement age (that may fit to China's agrarian regime some decades ago), reveals that the model-implied effects of an exogenous demographic intervention such as the one-child policy are robust to shortening life expectancy and to having an earlier retirement age.

Case 2: expanding the economic lifespan from its benchmark value $T = 50$ **to** $T = 55$**, while keeping the retirement time to its benchmark value** $T_R = 45$

Here, the analysis refers to the recent improvements in healthcare in China that have led to more longevity (75 years). Because a higher life expectancy motivates more savings, here the interest rate is set to a slighlty lower value, with $r^* = 8.95\%$. Tables F.3 gives the calibration parameters in this case

Table F.3 Calibration parameters (annual values, % rates).

	$g_{\bar{A}}$	α	β	δ	η
China	3.11	46.74	99.14	17.14	57.77
India	2.52	20.05	98.01	16.91	35.77

Table F.2 shows the goodness of fit to key calibration targets.

Table F.4 Initial calibration targets (%).

	Savings rate 1985		FDI/GDP ratio 1995	
	China	India	China	India
Data	33.75	14.88	2.90	0.19
Model	32.30	14.68	2.90	0.19

In this case we can see from that the increase in life expectancy helps the model to better match the savings calibration targets. This is intuitive, because households that live longer must save more in order to finance a longer post-retirement period. The goodness of fit of the transition dynamics in this case of more longevity appear in Figure F.2.

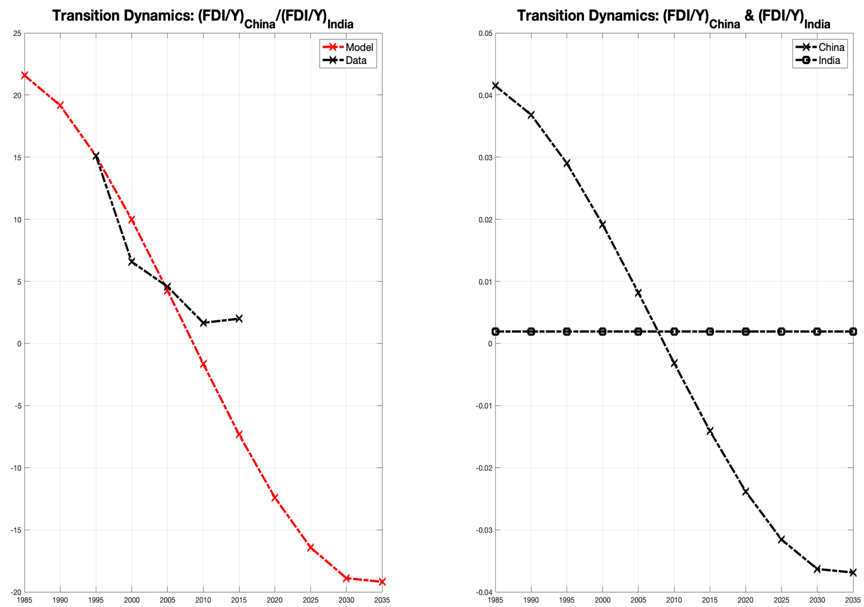

Figure F.2

In this case we can see again that the model's implied effects of China's one-child policy on the FDI/GDP ratio are robust to life-expectancy changes.

Online Appendix G – Literature review

Literature related to China's high savings rate

Curtis et al. (2015) and Choukhmane (2020) hypothesize that reduced fertility implies fewer children to support parents in their old age, thereby inducing parents to increase their own savings. Wei and Zhang (2011) explain the increased savings rate as a competitive response to the policy-induced sex ratio imbalance: families save more to increase the wealth of their sons in order to enhance their position in the competition for increasingly scarce spouses. Imrohoroglu and Zhao (2018) emphasize the long-term care insurance traditionally provided by families, and how the one-child policy has decreased the ability of families to provide it. Parents are thus forced to self-insure and do so by saving more. Other related work includes Chamon and Prasad (2010) and Yang et al. (2013). Finally, Zhang (2017) provides a comprehensive overview of the socio-economic effects of the one-child policy in China.

Another likely reason behind the documented increase in China's the savings rate is the remarkable improvement in life expectancy in China (to compare the progression of life expectancy indices between China and India, see https://data.worldbank.org/). Accordingly, the associated health care and medical costs have increased tremendously, all of which encourage Chinese households to save more. Moreover, in the past decades, the geographical mobility of young Chinese cohorts is much higher than the previous generation due to the drastic relaxation of the residential registration system (Hukou system). Hence, the monetary cost for supporting elder parents has also increased due to mobility-induced spatial separations, which also compels elderly parents to save more for retirement.

Regarding the extent of the change in the Chinese savings rate since 1980, there is some disagreement in the literature. Using the gross domestic savings to GDP ratio as a measure according to the World Bank, the Chinese savings rate increased from 33.4% in 1982 to 47.5% in 2014, a 14.1 percentage points increase. Choukhmane et al. (2020) used the Chinese Urban Household Survey (CEIC data) and showed an increase of 20 percentage points from 10% in 1980 to approximately 30% in 2015. Imrohoroglu and Zhao (2018) document the savings rate in China as increasing from 20% to 40%, an extreme view in the literature that we adopt for illustrative purposes.

Literature related to China's high capital returns

Bai et al. (2006) were the first to document the high capital returns in China (exceeding 20% post 1993) carefully. They conclude that China's high investment rate is consistent with the observed high returns. Nevertheless, mapping the documented high returns reported by Bai et al. (2006) to the aggregative concept of MPK under perfect foresight that we employ in this paper is not a straightforward task. Cochrane, in the discussion of Bai et al. (2006, p. 99), notes that the comparatively high return in China should be adjusted for differences in risk. Nordhaus and Cooper's discussion of Bai et al. (2006) emphasizes that a sudden conversion of land from agricultural to residential use is a process that can increase capital returns (capital gains) in ways that are not captured in standard equilibrium capital theory analysis. The discussion appears on pages 93-98, following Bai et al. (2006). Bai et al. (2006, Table 1 and Figure 2, pp. 72-75) also report a nearly 60% decline in capital returns in China from 1993-2001. This dramatic decline cannot be fully attributed to a TFP-growth decline, possibly validating the comments by Nordhaus and Cooper in Bai et al. (2006, pp. 93-98). Part of this decline can be explained, however, by the anticipated rapid decline in

China's population growth rate, as reported in Figures 1 and 2 of the present paper. Song et al. (2011) explore the seeming contradiction implicit in China's simultaneous high capital returns and high capital outflows. Their model rests on the internal reallocation of capital out of low growth firms that are large, externally financed, and whose capital needs are low. In contrast, high growth, high productivity firms are small and subject to capital constraints. They thus finance their rapidly increasing investments out of internally generated funds alone. As a result, the surplus capital from low growth firms migrates abroad, while the relative growth in the high productivity firms allows the high overall capital returns to be observed. A more recent study also reporting high capital returns in China and focusing on the link between these returns and the housing boom in China, is Chen and Wen (2017). Nothing in the present model depends on the precise level of capital returns.

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Appendix H - Data Descriptions and Sources

year	China FDI/GDPratio (%)	India FDI/GDPratio (%)	Ratio	log(ratio)
1981	0.155	0.027	5.822	0.765
1982	0.239	0.022	10.731	1.031
1983	0.430	0.002	255.079	2.407
1984	0.661	0.006	111.110	2.046
1985	0.824	0.032	25.571	1.408
1986	0.710	0.028	25.256	1.402
1987	0.613	0.043	14.218	1.153
1988	0.707	0.016	44.807	1.651
1989	0.751	0.043	17.615	1.246
1990	0.709	0.048	14.783	1.170
1991	0.800	0.022	35.780	1.554
1992	1.750	0.070	24.988	1.398
1993	3.930	0.146	26.934	1.430
1994	4.198	0.246	17.046	1.232
1995	3.832	0.476	8.044	0.905
1996	4.074	0.544	7.489	0.874
1997	4.402	0.801	5.495	0.740
1998	4.251	0.555	7.655	0.884
1999	3.450	0.431	7.997	0.903
2000	3.321	0.637	5.216	0.717
2001	3.744	0.974	3.845	0.585
2002	3.781	0.934	4.048	0.607
2003	3.033	0.561	5.407	0.733
2004	3.092	0.660	4.687	0.671
2005	4.240	0.783	5.418	0.734
2006	4.290	1.978	2.168	0.336
2007	4.404	2.070	2.127	0.328
2008	4.322	3.518	1.228	0.089
2009	3.234	2.952	1.095	0.040
2010	4.914	2.209	2.224	0.347
2011	4.811	2.597	1.853	0.268
2012	4.178	1.816	2.300	0.362
2013	4.290	1.989	2.157	0.334
2014	5.132	2.829	1.814	0.259

Table H.1 Data on FDI/GDP ratios

Foreign Direct Investment[1]

We use four different data sources to cross-verify the FDI inflows and outflows of China and India.

1. OECD: 1990-2013. Historic time series from OECD FDI statistics to end-2013 (http://www.oecd.org/daf/inv/investment-policy/fdi-statistics-according-tobmd3.htm).
2. National Accounts: 1982 – 2014. National Bureau of Statistics China (NBS-China) provides FDI outflow and inflow information (http://datE.stats.gov.cn/english/index.htm).
3. UNCTAD (United Nations Conference on Trade and Development): 1981-2013.The UNCTAD work program on FDI Statistics documents and analyzes global and regional trends in FDI.
4. DataStream: 1981-2016 (Quarterly). Thomson Reuters DataStream provides quarterly data on FDI inflows and outflows for China and India.[2]

Population Estimates and Forecasts: 1950-2100. United Nations: probabilistic population projections based on the world population prospects (the 2015 revision).[3]

GDP Series: 1990-2014, 2015-2018 (estimates). Work Bank, PPP adjusted at constant 2011 international USD.

Capital Stock -GDP ratio (K/Y ratio): PWT 9.0 (The Penn World Table).

FDI data come from four sources: (a) National Accounts, (b) OECD, (c) Datastream, and (d) UNCTAD. These sources cover different years, so we specify which we use in each context and document the correlation among these data sources. National account data for India is downloaded from the RBI website (https://rbi.org.in/Scripts/SDDSView.aspx) and it is identical to the data provided by OECD. So, we only report the OECD source.

[1] All FDI statistics from different sources use 2010 USD as the base dollar value.
[2] The quarterly data sources are composed by Oxford Economics (http://www.oxfordeconomics.com/).
[3] United Nations (2015). Probabilistic Population Projections based on the World Population Prospects: The 2015 Revision. Population Division, DESA. http://esE.un.org/unpd/ppp/.

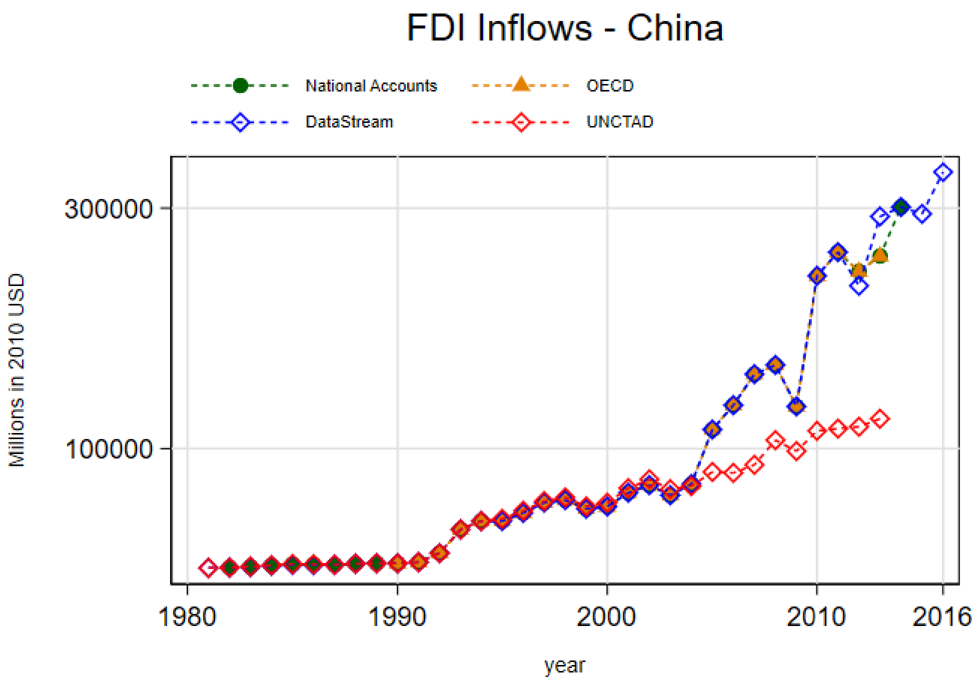

Figure H.1

The sources used in the paper are National-accounts data for the period 1982-2014 and Datastream data for years 2015-2016. National-accounts data and Datastream data overlap over the period 1982-2014 with a correlation coefficient of 99.79%.

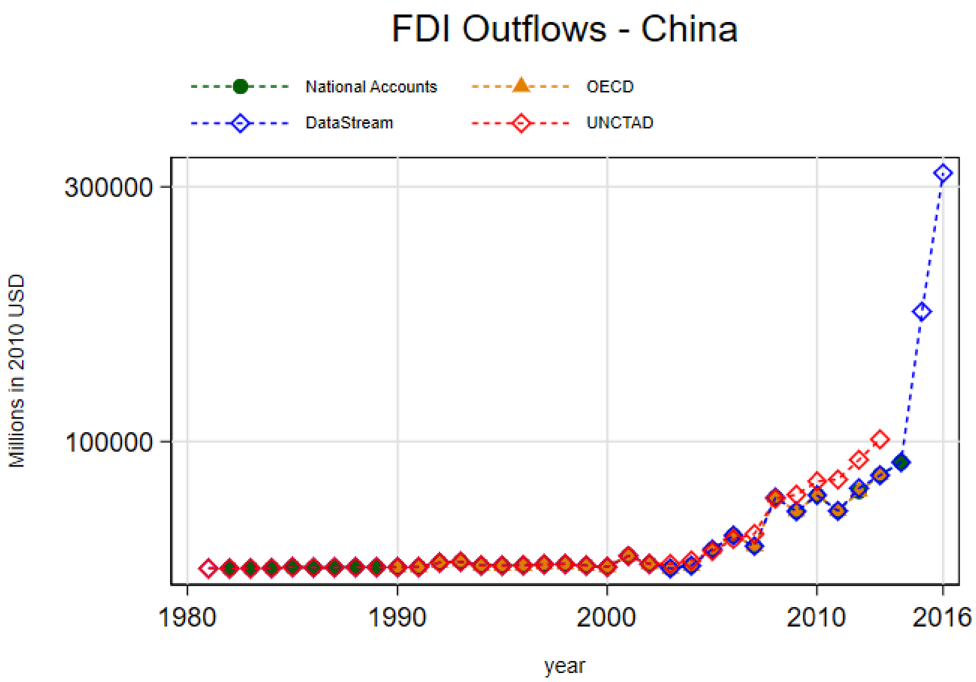

Figure H.2

The sources used in the paper are National-accounts data for the period 1982-2014 and Datastream data for years 2015-2016. National-accounts data and Datastream data overlap over the period 1982-2014 with a correlation coefficient 99.99%.

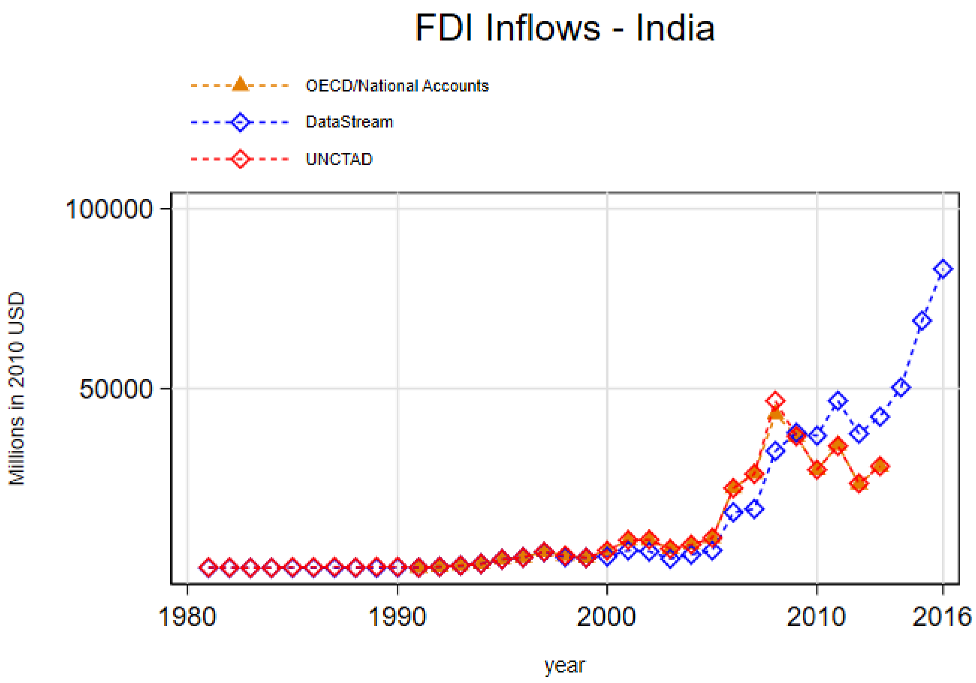

Figure H.3

The sources used in the paper are UNCTAD data for the period 1981-2013 and Datastream data for years 2014-2016. UNCTAD data and Datastream data overlap over the period 1981-2013 with a correlation coefficient of 92.56%. The reason we have chosen UNCTAD data for the period 1981-2013 is because, (a) for the period between 1981 and 1989 Datastream reports zero values (but not missing values), and (b) the two data sources overlap over the period 1991-2013 with a correlation coefficient of 99.87%.

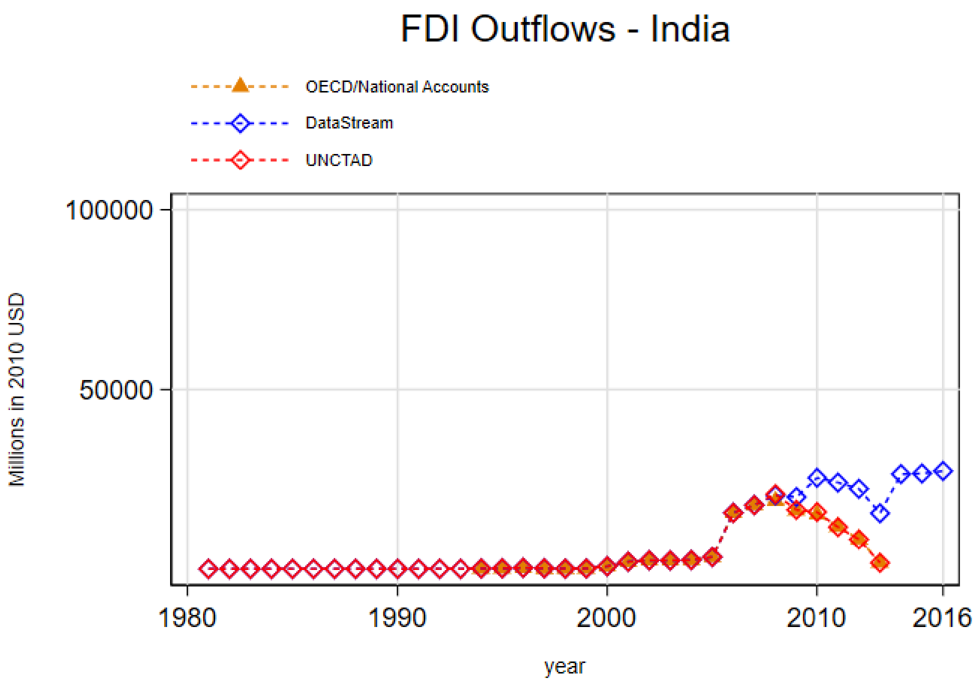

Figure H.4

The sources used in the paper are UNCTAD data for the period 1981-2013 and Datastream data for years 2014-2016. UNCTAD data and Datastream data overlap over the period 1981-2013 with a correlation coefficient of 89.32%. The reason we have chosen UNCTAD data for the period 1981-2013 is because, (a) for the period between 1981 and 1993 Datastream reports zero values (but not missing values), and (b) the two data sources overlap over the period 1994-2013 with a correlation coefficient of 99.86%.

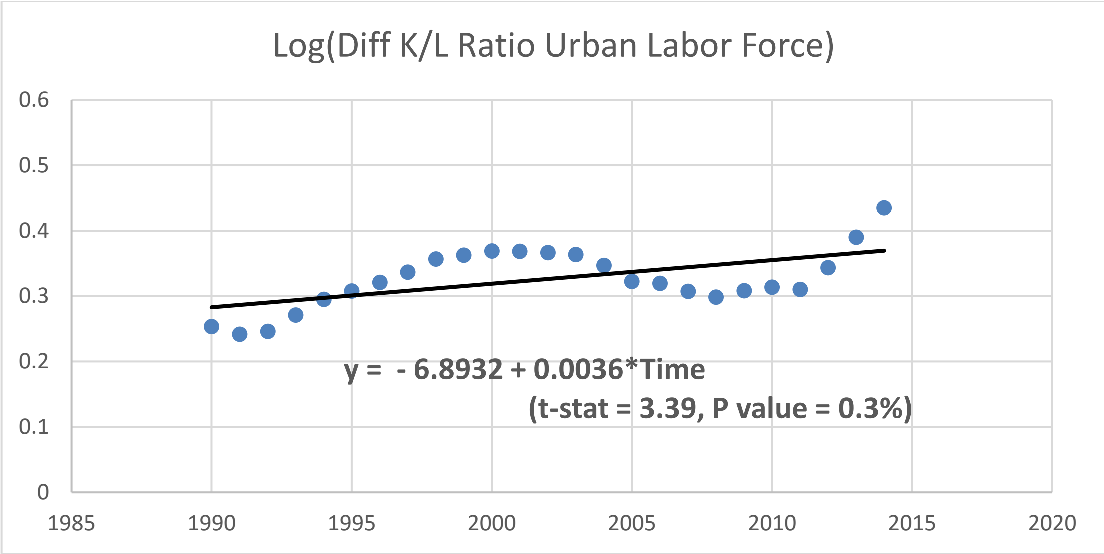

Figure H.5

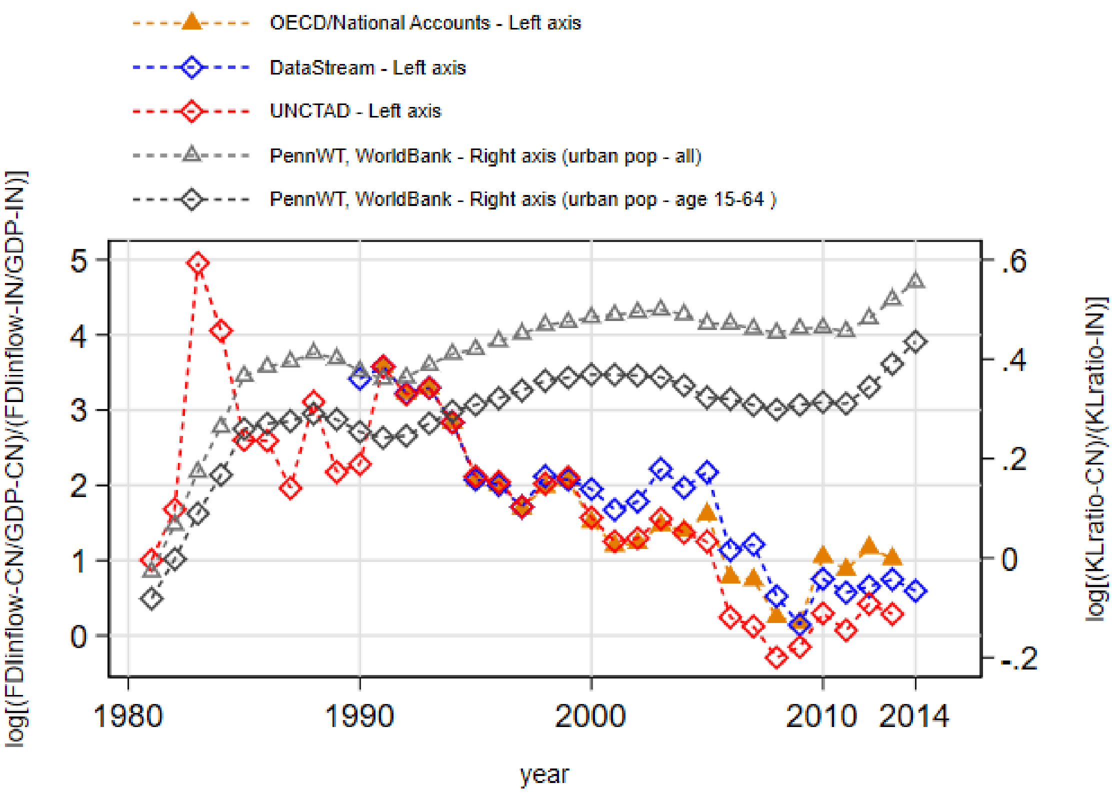

Figure H.6

To address the concern that large-scale internal migration in China would decrease the capital-labor ratio instead of increasing it, we use the urban population, restricted to ages 15-64 and perform a robustness check. Figure E.5 shows that the linear time trend coefficient (of the log K/L ratio of China over the K/L ratio of India) is positive and statistically significant (not equal to 0 with p-value at 0.3%). In Figure E.6 where we plot a similar data series as Figure 5 (in the paper) using this restricted sample, all the quantitative results remain.

The first two columns of Table E.2 provide the data appearing in Figure E.6 (without the logarithmic conversion of ratios). The last two columns of Table E.2 are the two new urban (working) population series appearing in Figure E.6.

year	Ratio_FDIY	Ratio_FullPop	Ratio_PopUrban	Ratio_PopUrbanWorking
1990	30.45	0.96	1.46	1.29
1991	35.73	0.99	1.43	1.27
1992	25.08	1.02	1.44	1.28
1993	26.94	1.07	1.47	1.31
1994	17.04	1.13	1.51	1.34
1995	7.96	1.17	1.52	1.36
1996	7.42	1.22	1.55	1.38
1997	5.54	1.27	1.57	1.40
1998	8.32	1.32	1.60	1.43
1999	7.95	1.36	1.61	1.44
2000	7.02	1.41	1.62	1.45
2001	5.33	1.50	1.63	1.45
2002	5.94	1.57	1.64	1.44
2003	9.19	1.66	1.65	1.44
2004	7.14	1.73	1.63	1.41
2005	8.79	1.72	1.60	1.38
2006	3.11	1.72	1.60	1.38
2007	3.36	1.71	1.59	1.36
2008	1.69	1.68	1.57	1.35
2009	1.16	1.70	1.59	1.36
2010	2.12	1.72	1.59	1.37
2011	1.77	1.72	1.58	1.36
2012	1.92	1.77	1.62	1.41
2013	2.11	1.87	1.68	1.48
2014	1.81	1.97	1.74	1.55

Table H.2