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# ON THE FINANCING AND INVESTMENT DECISIONS OF MULTINATIONAL FIRMS IN THE PRESENCE OF EXCHANGE RISK

*Rajnish Mehra**

In recent years, several papers [Mossin [13], Hamada [7], Rubinstein [14]] have addressed the normative implications of the CAPM (developed by Sharpe [15], Lintner [9] and Mossin [12]) for the capital budgeting and capital structure decisions of a value maximizing firm.[1] The model has been extended by Chen and Boness [3] to analyze the effects of uncertain inflation and by Adler and Dumas [1] to study optimal international acquisitions.

This paper, with the aid of a two-country model, attempts to investigate the effects of exchange risk on the investment and financing decisions of multinational firms. Recent work suggests that international capital markets are efficient though the evidence is far from being conclusive (see Agmon [2], Farber [6], Jacquillant and Solnik [8] and Solnik [16]). In view of this, the paper assumes that the capital markets of the two countries are not segmented. We assume that the investors in each country can freely invest in the bond and stock markets of both countries.

The paper consists of five parts I through V. Section I develops an equilibrium model for pricing securities in the presence of exchange risk. It is shown that in this case the "beta" consists of two terms; one involving the covariance of the security with the "world market portfolio" and another term to account for the exchange risk. An interesting result is that the effect of the exchange risk depends on the *net* investment position of a country (i.e., whether it is a surplus or a deficit country).

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[1]For a discussion of the appropriateness of the value-maximizing principle, see Fama and Miller [5]. For a further discussion in the context of incomplete markets, see papers by Leland and by Ekern and Wilson in the Symposium on Capital Markets [18].

Section II establishes a relationship between the interest rates in the two countries and the exchange rate. It is shown that the forward rate is a biased estimator of the future expected spot rate. Sections III and IV deal with investment and financing decisions. Criteria are developed for evaluating both domestic and foreign investments. An implication of the model is that failure to account for exchange risk can lead to a systematic bias in the investments accepted. MM propositions 1 and 2 [10], [11] are shown to hold even in the presence of exchange risk. Finally Section V discusses international mergers. In the case where investors in each country can freely invest in both countries, the irrelevance proposition is known to hold (Rubinstein [14]).

## I. Asset Pricing under Uncertain Exchange Rates

We make the following assumptions:

1. Perfect markets [see Fama [4]];
2. Homogeneous expectations;
3. Investors rational in the von Neumann-Morgenstern sense;
4. Investors with quadratic utility of terminal wealth;[2]
5. Riskless debt in both countries;
6. No stochastic inflation in both countries.[3]

Notation: There are two countries A and B. A is the domestic country and B is the foreign country. The notation for country A is presented below the notation for country B is *symmetric with* A *replaced by* B. The domestic currency is \$ and the foreign currency is £.

$S^A$: the vector of market values of the stock of *country A* firms.

$$S^A = \{S_j^A\} \qquad j = 1,2,\ldots,M^A.$$

[2]The use of quadratic utility is not particularly restrictive since any result derived under the assumption of quadratic utility can also be derived in a continuous time framework. It is well known that a quadratic utility function may result in negative marginal utility of wealth. In the analysis to follow the domain of the utility function is limited such that $EU' \geq 0$, to avoid the objectional property listed above.

[3]In this case as Adler and Dumas [1] point out "exchange rate can remain stochastic only if we assume stochastic tariffs or transportation costs . . . or if we postulate some short-term lack of adjustment of international prices to parity. That latter circumstance, which is admittedly a disequilibrium one, is very realistic since we can expect the flows of goods to react to exchange rates and prices less rapidly than flows of capital."

$Y^A$: the matrix of fractional holdings of the stock of *country A* firms by *country A* investors.

$$Y^A = [y^A_{ij}] \qquad i = 1,2,\dots,N^A \quad j = 1,2,\dots,M^A.$$

$Z^A$: the matrix of fractional holdings of the stock of *country B* firms by *country A* investors.

$$Z^A = [z^A_{ij}] \qquad i = 1,2,\dots,N^A \quad j = 1,2,\dots,M^B.$$

$B^A$: the vector of market value of the bonds of *country A* firms.

$$B^A = \{B^A_j\} \qquad j = 1,2,\dots,M^A.$$

$P^A$: the matrix of fractional holdings of the bonds of *country A* firms by *country A* investors.

$$P^A = [p^A_{ij}] \qquad i = 1,2,\dots,N^A \quad j = 1,2,\dots,M^A.$$

$Q^A$: the matrix of fractional holdings of the bonds of *country B* firms by *country A* investors.

$$Q^A = [q^A_{ij}] \qquad i = 1,2,\dots,N^A \quad j = 1,2,\dots,M^B.$$

$\tilde{R}^A$: the vector of the return on the stocks of *country A* firms.

$$\tilde{R}^A = \{\tilde{R}^A_j\} \qquad j = 1,2,\dots,M^A.$$

$\tilde{R}_e$: the return on a position in foreign exchange (for an investor in *country A*).

$$\tilde{R}_e = \frac{\tilde{s}_1 - s_o}{s_o}$$

$s_o$: is the current exchange rate \$/£

$\tilde{s}_1$: is the exchange rate at the end of the period (\$/£).

[N.B. without loss of generality, in the analysis that follows, we let $s_o = 1$.]

$r_A$: risk-free rate in *country A*.

$W^A$: the vector of initial wealth of investors in *country A*.

$$W^A = \{W_i^A\} \qquad i = 1,2,\dots,N^A.$$

The budget constraint for investor i of *country A* is

$$W_i^A = \sum_j y_{ij}^A S_j^A + \sum_j z_{ij}^A S_j^B + \sum_j p_{ij}^A B_j^A + \sum_j q_{ij}^A B_j^B. \tag{1}$$

The first and third terms represent the investor's holding of stocks and bonds in *country A* while the second and fourth terms represent his investment in the foreign country. [Note: We assume that the current exchange rate is 1.]

The investor's end-of-period wealth[4] is

$$\tilde{Z}_i^A = \sum_j y_{ij}^A S_j^A (1 + \tilde{R}_j^A) + \sum_j z_{ij}^A S_j^B (1 + \tilde{R}_j^B + \tilde{R}_e) + \sum_j p_{ij}^A B_j^A (1 + r_A) + \sum_j q^A_{ij} B_j^B (1 + r_B + \tilde{R}_e). \tag{2}$$

The budget constraint for an investor in *country B* is

$$W_i^B = \sum_j y_{ij}^B S_j^B + \sum_j z_{ij}^B S_j^A + \sum_j p_{ij}^B B_j^B + \sum_j q_{ij}^B B_j^A \tag{3}$$

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[4] If $1 is invested in security j of *country B*, the return is

$$\frac{1}{s_o} (1 + \tilde{R}_j^B)\tilde{s}_1 = (1 + \tilde{R}_j^B)(1 + \tilde{R}_e) \tilde{=} 1 + \tilde{R}_j^B + \tilde{R}_e.$$

Note that $\tilde{R}_e > 0$ implies the currency of *Country A* ($) has *depreciated.*

Although we do not have an explicit forward market, a forward transaction is equivalent to borrowing in one country and lending in the second. The model allows for this type of transaction.

and his end-of-period wealth[5] is

$$(4) \qquad \tilde{Z}_i^B = \sum_j y_{ij}^B S_j^B (1 + \tilde{R}_j^B) + \sum z_{ij}^B S_j^A (1 + \tilde{R}_j^A - \tilde{R}_e)$$

$$+ \sum_j p_{ij}^B B_j^A (1 + r_B) + \sum q_{ij}^B B_j^A (1 + r_A - \tilde{R}_e).$$

Since the individuals are assumed to obey the von Neumann-Morgenstern axioms, the problem facing a typical *country A* investor is

$$(5) \qquad \max_{y_{ij}^A, z_{ij}^A, p_{ij}^A, q_{ij}^A} \{EU_i(\tilde{Z}_i^A)\}$$

subject to equation (1).

The first-order conditions are

$$(6) \qquad E[U_i'(\tilde{Z}_i^A) \cdot (1 + \tilde{R}_j^A) \cdot S_j^A] = -\lambda S_j^A$$

$$(7) \qquad E[U_i'(\tilde{Z}_i^A) \cdot (1 + \tilde{R}_j^B + \tilde{R}_e) \cdot S_j^B] = -\lambda S_j^B$$

$$(8) \qquad E[U_i'(\tilde{Z}_i^A) \cdot (1 + r_A) \cdot B_j^A] = -\lambda B_j^A$$

$$(9) \qquad E[U_i'(\tilde{Z}_i^A) \cdot (1 + r_B + \tilde{R}_e) B_j^B] = -\lambda B_j^B$$

($\lambda$ is a Lagrange multiplier.)

From (6) and (8) on simplification we get

$$(10) \qquad E[U_i'(\tilde{Z}_i^A)(\tilde{R}_j^A - r_A)] = 0$$

and from (7) and (9) we get

$$(11) \qquad E[U_i'(\tilde{Z}_i^A)(\tilde{R}_j^B - r_B)] = 0.$$

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[5] If £1 is invested in security j of *country A*, the return is

$$\frac{s_o}{\tilde{s}_1}(1 + \tilde{R}_j^A) = \frac{1 + \tilde{R}_j^A}{1 + \tilde{R}_e} \simeq 1 + \tilde{R}_j^A - \tilde{R}_e.$$

(10) implies

(12) $$E(U_i'(\tilde{Z}_i^A)) \cdot (\bar{R}_j^A - r_A) + \text{Cov}(U_i', \tilde{R}_j) = 0$$

where $\bar{R}_j^A = E(\tilde{R}_j^A)$.

Since we have assumed quadratic utility of the form

$$U(\tilde{Z}) = \tilde{Z} - c\tilde{Z}^2$$

we have

(13) $$U_i'(\tilde{Z}_i^A) = 1 - 2c_i^A\tilde{Z}_i^A.$$

Therefore

(14) $$E(U_i') \cdot (\bar{R}_j^A - r_A) - 2c_i^A \text{ Cov}(\tilde{Z}_i^A, \tilde{R}_j^A) = 0$$

or

(15) $$(\bar{R}_j^A - r_A)\left[\frac{E(U_i')}{2c_i^A}\right] = \text{Cov}(\tilde{Z}_i^A, \tilde{R}_j^A) \qquad \begin{array}{l} i = 1,2,\ldots,N^A \\ j = 1,2,\ldots,M^A. \end{array}$$

Similarly from (11) one can get

(16) $$(\bar{R}_j^B - r_B)\left[\frac{E(U_i')}{2c_i^A}\right] = \text{Cov}(\tilde{Z}_i^A, \tilde{R}_j^B) \qquad \begin{array}{l} i = 1,2,\ldots,N^A \\ j = 1,2,\ldots,M^B. \end{array}$$

Solving the optimization problem for investor i of *country B*, we can get conditions similar to (15) and (16). These are

(17) $$(R^{-A}_j - r_A)\left[\frac{E(U_i')}{2c_i^B}\right] = \text{Cov}(\tilde{Z}_i^B, \tilde{R}_j^A) \qquad \begin{array}{l} i = 1,2,\ldots,N^B \\ j = 1,2,\ldots,M^A. \end{array}$$

(18) $$(\bar{R}_j^B - r_B)\left[\frac{E(U_i')}{2c_i^B}\right] = \text{Cov}(\tilde{Z}_i^B, \tilde{R}_j^B) \qquad \begin{array}{l} i = 1,2,\ldots,N^B \\ j = 1,2,\ldots,M^B. \end{array}$$

We can sum equations (15)-(18) over all individuals to get

(19) $$(\bar{R}_j^A - r_A) = \lambda^A \operatorname{Cov}(\tilde{Z}^A, \tilde{R}_j^A)$$

(20) $$(\bar{R}_j^B - r_B) = \lambda^A \operatorname{Cov}(\tilde{Z}^A, \tilde{R}_j^B)$$

(21) $$(\bar{R}_j^A - r_A) = \lambda^B \operatorname{Cov}(\tilde{Z}^B, \tilde{R}_j^A)$$

(22) $$(\bar{R}_j^B - r_B) = \lambda^B \operatorname{Cov}(\tilde{Z}^B, \tilde{R}_j^B)$$

(23)[6] $$1/\lambda^A = \sum_{i=1}^{N^A} \left[\frac{EU_i'}{2c_i^A}\right] \qquad 1/\lambda^B = \sum_{i=1}^{N^B} \left[\frac{EU_i'}{2c_i^B}\right]$$

where $$\tilde{Z}^A = \sum_i \tilde{Z}_i^A, \text{ etc.}$$

Equation (19) can be rewritten as

(24) $$(\bar{R}_j^A - r_A)/\lambda^A = \operatorname{Cov}(\sum_k y_k^A S_k^A \tilde{R}_k^A, \tilde{R}_j^A) + \operatorname{Cov}(\sum_k z_k^A S_k^B \tilde{R}_k^B, \tilde{R}_j^A)$$
$$+ \operatorname{Cov}(\sum_k z_k^A S_k^B \tilde{R}_e, \tilde{R}_j^A) + \operatorname{Cov}(\sum_k q_k^A B_k^B \tilde{R}_e, \tilde{R}_j^A)$$

and (21) as

(25) $$(\bar{R}_j^A - r_A)/\lambda^B = \operatorname{Cov}(\sum_k y_k^B S_k^B \tilde{R}_k^B, \tilde{R}_j^A) + \operatorname{Cov}(\sum_k z_k^B S_k^A \tilde{R}_k^A, \tilde{R}_j^A)$$
$$- \operatorname{Cov}(\sum_k z_k^B S_k^A \tilde{R}_e, \tilde{R}_j^A) - \operatorname{Cov}(\sum_k q_k^B B_k^A \tilde{R}_e, \tilde{R}_j^A)$$

where

(26) $$y_k^A = \sum_i y_{ik}^A, \quad q_k^A = \sum_i q_{ik}^A, \quad z_k^A = \sum_i z_{ik}^A, \text{ etc.}$$

Since *all* securities must be held by investors of *countries A* and *B*, we have the following closure conditions.

(27) $$y_k^A + z_k^B = 1 \qquad k = 1,2,\ldots,M^A$$

(28) $$y_k^B + z_k^A = 1 \qquad k = 1,2,\ldots,M^B.$$

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[6] $1/\lambda^A$ can be interpreted as $\sum_i E(\frac{1}{R_A^i})$ where $R_A^i = -\frac{U''}{U'}$ is the Arrow-Pratt coefficient of absolute risk aversion for individual i.

Adding (24) and (26) and making use of the closure conditions (27) and (28), we get

$$(29) \qquad (\bar{R}_j^A - r_A)(1/\lambda^A + 1/\lambda^B) = \mathrm{Cov}(\sum_k S_k^A \tilde{R}_k^A, \tilde{R}_j^A) + \mathrm{Cov}(\sum_k S_k^B \tilde{R}_k^B, \tilde{R}_j^A)$$

$$+ (\sum_k z_k^A S_k^B - \sum_k z_k^B S_k^A)\mathrm{Cov}(\tilde{R}_e, \tilde{R}_j^A) + (\sum_k q_k^A B_k^B - \sum_k q_k^B B_k^A)\mathrm{Cov}(\tilde{R}_e, \tilde{R}_j^A).$$

Define

$$R_m^A = \frac{\sum_k S_k^A \tilde{R}_k^A}{\sum_k S_k^A}$$

as the return on the market in *country A*.

Note that $\sum_k z_k^A S_k^B$ is the total stock of *country B* held by investors of *country A*.

Therefore

$$\sum_k z_k^A S_k^B - \sum_k z_k^B S_k^A$$

is the *net* investment *by country A in country B's* stocks.

Let

$$\Delta S^A = \sum_k z_k^A S_k^B - \sum_k z_k^B S_k^A \qquad (= -\Delta S^B).$$

Similarly let

$$\Delta B^A = \sum_k q_k^A B_k^B - \sum_k q_k^B B_k^A \qquad (= -\Delta B^B)$$

be the *net* investment by *country A* in *country B's* bonds. Therefore we get

$$(30) \qquad (\bar{R}_j^A - r_A)(1/\lambda^A + 1/\lambda^B) = S^A \ \mathrm{Cov}(\tilde{R}_j^A, R_m^A) + S^B \ \mathrm{Cov}(\tilde{R}_j^A, R_m^B)$$

$$+ (\Delta S^A + \Delta B^A) \ \mathrm{Cov}(\tilde{R}_j^A, \tilde{R}_e).$$

Define

$$\tilde{R}_m^{AB} = \frac{\tilde{R}_m^A S^A + \tilde{R}_m^B S^B}{S^A + S^B}$$

as the return on the "world market"

$$1/\lambda^{AB} = 1/\lambda^{A} + 1/\lambda^{B} \qquad S^{AB} = S^{A} + S^{B}$$

$$\Delta SB^{A} = \Delta S^{A} + \Delta B^{A} = - \Delta SB^{B}$$

as the *net* investment *by country A* in *country B* (in both stocks and bonds). Therefore

$$(31)^{7} \qquad \bar{R}_j^A = r_A + \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{R}_j^A,\tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{R}_j^A,\tilde{R}_e)].$$

The risk-return relationship is

$$(32) \qquad \bar{R}_j^A = r_A + \lambda^{AB}\cdot\beta_j^{*A}$$

where

$$(33) \qquad \beta_j^{*A} = S^{AB}\ \mathrm{Cov}\ (\tilde{R}_j^A,\tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{R}_j^A,\tilde{R}_e).$$

Equation (33) shows that the risk of security j can be decomposed into two parts: a) the covariance of the security with the "world market portfolio," and b) the covariance of the security with the return on a position in foreign exchange. It is interesting to note that, if the net foreign investment is zero (i.e., $\Delta SB^A = 0$), then the exchange risk vanishes.

From equation (30) it is clear that if the value of one particular stock market is much larger than the other (e.g., $S^A >> S^B$), then the security will be priced on the larger market. So if the value of the NYSE >> than that of the other exchanges, it is reasonable to assume that the securities are priced on the NYSE. (This lends some support to the studies testing the CAPM on the NYSE.)

The contribution of the exchange risk depends on the net investment position of the country and the covariance of the firm's return with $\tilde{R}_e$. If for country A, $\Delta SB^A$ is positive, the Sharpe-Lintner-Mossin (SLM) CAPM will overstate (understate) the risk of a firm if its return is negatively (positively) correlated with $\tilde{R}_e$. Since a negative correlation corresponds to the case where a

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[7]For a firm in Country B we have
$\bar{R}_j^B = r_B + \lambda^{AB}[S^{AB}\mathrm{Cov}(\tilde{R}_j^B,\tilde{R}_m^{AB}) + \Delta SB^A\mathrm{Cov}(\tilde{R}_j^B,\tilde{R}_e)].$

firm's return is expected to increase due to a revaluation by Country A (devaluation by Country B), the SLM-CAPM, for instance overstates the risk of a firm if its return is expected to increase due to a revaluation by Country A.[8] (Assuming $\Delta SB^A > 0$.)

The tables below show the cases in which the SLM-CAPM understates [-] or overstates [+] the risk of Country A and B firms.

COUNTRY A FIRMS

| | Firms whose returns are likely to increase (decrease) due to a devaluation (revaluation) by country A, i.e., $COV(\tilde{R}_j^A, \tilde{R}_e) > 0$ | Firms whose returns are likely to decrease (increase) due to a devaluation (revaluation) by country A, i.e., $COV(\tilde{R}_j^A, \tilde{R}_e) < 0$ |
|---|---|---|
| Country A surplus, i.e., $\Delta SB^A > 0$ | - | + |
| Country A deficit, i.e., $\Delta SB^A < 0$ | + | - |

[8] It should be emphasized that in carrying out the comparison the same "market" is used in both models. The SLM-CAPM in this case would be

$$\bar{R}_j^A = r_A + \lambda^{AB}[S^{AB}\text{Cov}\ (\tilde{R}_j^A, \tilde{R}_m^{AB})]$$

and not the one in the single country case, i.e.,

$$\bar{R}_j^A = r_A + \lambda^{A}[S^{A}\text{Cov}\ (\tilde{R}_j^A, \tilde{R}_m^{A})].$$

COUNTRY B FIRMS

| | Firms whose returns are likely to decrease (increase) due to a devaluation (revaluation) by country B, i.e., $COV(\tilde{R}_j^B, \tilde{R}_e) > 0$ | Firms whose returns are likely to increase (decrease) due to a devaluation (revaluation) by country B, i.e., $COV(\tilde{R}_j^B, \tilde{R}_e) < 0$ |
|---|---|---|
| Country B surplus i.e., $\Delta SB^B > 0$ | + | - |
| Country B deficit i.e., $\Delta SB^B < 0$ | - | + |

## II. Equilibrium in the Bond Market

From equations (8) and (9) we get

$$E[U_i'(\tilde{Z}_i^A)(r_A - r_B - \tilde{R}_e)] = 0. \tag{34}$$

After some straightforward simplification and summing over all investors in country A, we get

$$(-r_A + r_B + \bar{R}_e)/\lambda^A = Cov(\tilde{Z}^A, \tilde{R}_e). \tag{35}$$

Similarly one can get an equation for investors in country B,

$$(r_B - r_A + \bar{R}_e)/\lambda^B = Cov(\tilde{Z}^B, \tilde{R}_e). \tag{36}$$

Therefore

$$(r_B - r_A + \bar{R}_e)(1/\lambda^A + 1/\lambda^B) = Cov(\tilde{Z}^A + \tilde{Z}^B, \tilde{R}_e) \tag{37}$$

which on simplification gives

$$\bar{R}_e = r_A - r_B + \lambda^{AB}[S^{AB}\, Cov(\tilde{R}_e, R_m^{AB}) + \Delta SB^A \sigma^2(\tilde{R}_e)]. \tag{38}$$

From the interest rate parity theorem, we know that

$$(39) \qquad \frac{F - s_o}{s_o} \cong r_A - r_B$$

where F is the forward rate at the beginning of the period ($/£).

Since

$$\bar{R}_e = \frac{\bar{s}_1 - s_o}{s_o},$$

we get

$$(40) \qquad \bar{s}_1 = F + s_o \lambda^{AB} [S^{AB} \operatorname{Cov}(\tilde{R}_e, \tilde{R}_m^{AM}) + \Delta SB^A \sigma^2(\tilde{R}_e)]$$

which shows that the forward rate is a biased estimator of the expected future spot rate, a fact observed by Solnik [16]. Equation (38) gives an equilibrium relationship between $R_e$, $r_A$, and $r_B$. It should, however, be clear that though intuitively appealing $\bar{R}_e$ *cannot* be replaced by $r_A - r_B$.

## III. Asset Expansion

Rubinstein [14] has shown that a firm should accept a project if

$$E(R^O) > r + \lambda \operatorname{Cov}(R^O, R_m)$$

where $R^O$ is the rate of return of the project. In an international setting this is easily translated to the condition that a project in country A should be accepted if and only if

$$E(\tilde{R}^{OA}) > r_A + \lambda^{AB}[S^{AB} \operatorname{Cov}(\tilde{R}^{OA}, \tilde{R}_m^{AB}) + \Delta SB^A \operatorname{Cov}(\tilde{R}^{OA}, \tilde{R}_e)]$$

or

$$(41)^9 \qquad \frac{E(\tilde{R}^{OA}) - r_A}{S^{AB} \operatorname{Cov}(\tilde{R}^{OA}, \tilde{R}_m^{AB}) + \Delta SB^A \operatorname{Cov}(\tilde{R}^{OA}, \tilde{R}_e)} > \lambda^{AB}.$$

So in accepting a project, the relevant factors are its covariance with the world market portfolio and its covariance with the exchange rate. The effect of the latter depends on the net investment position of the country (i.e., $\Delta SB^A$ > or < 0). If $\Delta SB^A$ is > 0, a project whose returns decrease due to a devaluation (by A) would be "preferred" in the sense that it would have a lower

[9] Assuming the denominator is positive.

cost of capital. For a country like Canada which has $\Delta SB^B < 0$, a "preferred" project would be one whose returns increase due to a domestic devaluation. So in a deficit country, even if two projects have the same covariance with the market, the one that is favored by a devaluation will have a lower cost of capital.

The above analysis suggests that, if the effects of exchange risk are not considered explicitly in capital budgeting decisions, a systematic bias will develop. In the case of a deficit country the project acceptance criteria would be biased against devaluation favored projects, resulting in an economy-wide under investment in projects aided by a devaluation.

Foreign Investments

Consider the case where a firm in country A is evaluating an investment in country B whose rate of return in *country B* is $\tilde{R}^{OB}$. However, viewed from country A, the return on this investment is $\tilde{R}^{OB} + \tilde{R}_e$. The project should, therefore, be accepted if

$$\bar{R}^{OB} + \bar{R}_e > r_A + \lambda^{AB}[S^{AB}\,\mathrm{Cov}(\tilde{R}^{OB},\tilde{R}_m^{AB}) + \Delta SB^A\,\mathrm{Cov}(\tilde{R}^{OB},\tilde{R}_e)]$$

$$+ \lambda^{AB}[S^{AB}\,\mathrm{Cov}(\tilde{R}_e,\tilde{R}_m^{AB}) + \Delta SB^A\,\sigma^2(\tilde{R}_e)].$$

This can be simplified using equation (38) to give

$$\bar{R}^{OB} > r_B + \lambda^{AB}[S^{AB}\,\mathrm{Cov}(\tilde{R}^{OB},\tilde{R}_m^{AB}) + \Delta SB^A\,\mathrm{Cov}(\tilde{R}^{OB},\tilde{R}_e)]$$

which is the acceptance criteria a *country B* firm would have used. In other words, a country A firm evaluating a project in country B should use the same acceptance criteria as a country B firm. (See also Section V.)

## IV. Financing Decision

In this section we examine the effect of exchange rate fluctuation on the financing decision of firms. [It should be clear that except for the exchange rate risk the form of the International CAPM (ICAPM) we have derived is similar to the SLM-CAPM. Indeed if we ignore the term containing $\tilde{R}_e$, we get

$$E(R_j^A) = r_A + \lambda^{AB}S^{AB}\,\mathrm{Cov}(\tilde{R}_j^A,\tilde{R}_m^{AB})$$

which is similar to the asset pricing model derived by Solnik [16] in continuous time and to the SLM-CAPM.]

Let $\tilde{X}_j^A$ = the value of the net operating income of the $j^{th}$ firm in country A.

$$\tilde{R}_j^A = \frac{\tilde{X}_j^A - r_A B_j^A}{S_j^A}.$$

Let u stand for an unlevered firm, i.e.,

$$\tilde{R}_j^{Au} = \frac{X_j^{Au}}{S_j^{Au}}, \qquad V_j^{Au} = S_j^{Au}.$$

Substituting for $\tilde{R}_j^A$ in equation (31) we get

$$E(\tilde{X}_j^A) = r_A V_j^A + \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{X}_j^A, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{X}_j^A, \tilde{R}_e)]. \tag{42}$$

For the pure equity firm we get

$$E(\tilde{X}_j^{Au}) = r_A V_j^{Au} + \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{X}_j^{Au}, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{X}_j^{Au}, R_e)]. \tag{43}$$

Therefore if $\tilde{X}_j^A = \tilde{X}_j^{Au}$ it follows that

$$V_j^A = V_j^{Au}.$$

As a firm alters its capital structure even in the presence of exchange risk, its value does not change. So the MM [10] theorem on capital structure holds even in the presence of exchange risk.

Following Chen and Boness [3] we can derive MM's second theorem using the ICAPM.

By definition

$$\tilde{R}_j^A = \tilde{R}_j^{Au}(1 + k_j^A) - r_A k_j^A \tag{44}$$

where $k_j^A = B_j^A/S_j^A$ is the debt equity ratio or

$$\bar{R}_j^A = \bar{R}_j^{Au}(1 + k_j^A) - r_A k_j^A. \tag{45}$$

Therefore

$$\bar{R}_j^A = r_A + (1 + k_j^A)\lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{R}_j^{Au}, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{R}_j^{Au}, \tilde{R}_e)] \tag{46}$$

or

(47) $$\bar{R}_j^A = r_A + \theta + \phi$$

$$\theta = \lambda^{AB}\sigma(\tilde{R}_j^{Au})[S^{AB}\rho_{jm}\sigma(\tilde{R}_m^{AB}) + \Delta SB^A\rho_{je}\sigma(\tilde{R}_e)]$$

$$\phi = \lambda^{AB}\sigma(\tilde{R}_j^{Au})[S^{AB}\rho_{jm}\sigma(\tilde{R}_m^{AB}) + \Delta SB^A\rho_{je}\sigma(\tilde{R}_e)]k_j^A.$$

$\rho_{jm}$ is the correlation coefficient between $\tilde{R}_j^{Au}$ and $\tilde{R}_m^{AB}$. $\rho_{je}$ is similarly defined. $\theta$ is the premium for the business risk of the firm and $\phi$ is the premium for the financial risk (including exchange risk) of the firm.

Financing Decision with Corporate Taxes

The after-tax rate of return on the equity of an unlevered firm is

$$\tilde{R}_j^{u\tau A} = \frac{\tilde{X}_j^{uA}(1-\tau)}{V_j^{uA}}$$

and on the levered firm it is

$$\tilde{R}_j^{\tau A} = \frac{(\tilde{X}_j^A - r_A B_j^A)(1-\tau)}{S_j^A}.$$

For a pure equity firm we have

(48) $$E(\tilde{X}_j^{uA})(1-\tau) = r_A[V_j^{uA}] + \lambda^{AB}(1-\tau)[S^{AB}\,\mathrm{Cov}(\tilde{X}_j^{Au},\tilde{R}_m^{AB}) + \Delta SB^A\,\mathrm{Cov}(\tilde{X}_j^{uA},\tilde{R}_e)]$$

and for the levered firm

(49) $$E(\tilde{X}_j^A)(1-\tau) = r_A[S_j^A + B_j^A(1-\tau)] + \lambda^{AB}(1-\tau)[S^{AB}\,\mathrm{Cov}(\tilde{X}_j^A,\tilde{R}_m^{AB}) + \Delta SB^A\,\mathrm{Cov}(\tilde{X}_j^A,\tilde{R}_e)].$$

If $\tilde{X}_j^A = \tilde{X}_j^{uA}$, then (48) and (49) imply that

$$V_j^A = V_j^{uA} + \tau B_j^A$$

which is precisely MM's [11] result with taxes.

## V. International Mergers

In this section we examine the effect on the market value of a country A firm when it takes over a country B firm.

Consider two firms, one in country A (firm a) and the other in country B (firm b). The values of the two firms (in country A) can be determined as follows.

For firm a

$$(50) \qquad 1 + \tilde{R}_a^A = \frac{\tilde{X}_a^A + S_a^A - r_A B_a^A}{S_a^A} .$$

Therefore, using equation (31)

$$(51) \qquad V_a^A = \frac{\bar{X}_a^A - \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{X}_a^A, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{X}_a^A, \tilde{R}_e)]}{r_A} .$$

The return from firm b as viewed in country A is

$$(52) \qquad 1 + \tilde{R}_b^B = \frac{\tilde{s}_1(S_b^B + \tilde{X}_b^B - r_B B_b^B)}{S_b^B} .$$

This can be written as (see footnote 4)

$$(53) \qquad 1 + \tilde{R}_b^B \cong \frac{S_b^B + \tilde{X}_b^B - r_B B_b^B + \tilde{R}_e S_b^B}{S_b^B} .$$

Using equations (31) and (38) we get

$$(54) \qquad V_b^B = \frac{\bar{X}_b^B - \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{X}_b^B, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{X}_b^B, \tilde{R}_e)]}{r_B} .$$

Therefore

$$(55) \quad V_a^A + V_b^B = \frac{\bar{X}_a^A}{r_A} + \frac{\bar{X}_b^B}{r_B} - \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\frac{\tilde{X}_a^A}{r_A} + \frac{\tilde{X}_b^B}{r_B}, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\frac{\tilde{X}_a^A}{r_A} + \frac{\tilde{X}_b^B}{r_B}, \tilde{R}_e)] .$$

Now consider a situation where firm a takes over firm b, i.e., B becomes a foreign subsidiary of firm A. Let c denote firm a after the takeover. Therefore we have

$$(56) \qquad 1 + R_c^A = \frac{S_a^A + \tilde{X}_a^A - r_A B_a^A + \tilde{s}_1(S_b^B + \tilde{X}_b^B - r_B B_b^B)}{S_a^A + S_b^B}$$

$$(57) \qquad R_c^A \cong \frac{\tilde{X}_a^A - r_A B_a^A + \tilde{X}_b^B - r_B B_b^B + S_b^B \tilde{R}_e}{S_a^A + S_b^B}$$

Once again using equations (31) and (38) one can get

$$(58) \quad V_c^A r_A + V_b^B (r_B - r_A) = \bar{X}_c^A - \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\tilde{X}_c^A, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\tilde{X}_c^A, \tilde{R}_e)]$$

where

$$\tilde{X}_a^A + \tilde{X}_b^B = \tilde{X}_c^A$$

$$S_a^A + S_b^B = S_c^A$$

$$B_a^A + B_b^B = B_c^A$$

$$S_c^A + B_c^A = V_c^A .$$

Substituting for $V_b^B$ from equation (54) in equation (58) we get

$$(59) \quad V_c^A = \frac{\bar{X}_a^A}{r_A} + \frac{\bar{X}_b^B}{r_B} - \lambda^{AB}[S^{AB}\ \mathrm{Cov}(\frac{\tilde{X}_a^A}{r_A} + \frac{\tilde{X}_b^B}{r_B}, \tilde{R}_m^{AB}) + \Delta SB^A\ \mathrm{Cov}(\frac{\tilde{X}_a^A}{r_A} + \frac{\tilde{X}_b^B}{r_B}, \tilde{R}_e)].$$

Comparing (59) and (55) we see that

$$V_c^A = V_a^A + V_b^B.$$

The result illustrates that, even in the presence of exchange risk. there is no incentive for firms to merge if individuals can freely invest in both capital markets. This is in contrast to the results of Adler and Dumas [1] where they show that for segmented markets the irrelevance proposition does not hold and the firm in general does have an optimal acquisition decision.

## VI. Conclusions

This paper, with the aid of a two-country model, focuses on the effects of exchange risk on the various decisions facing a firm. It is an attempt at synthesizing corporate financial theory in an international setting and derives several normative implications of the model developed in Section I for a value-maximizing firm.

## REFERENCES


[1] Adler, M., and B. Dumas. "Optimal International Acquisitions." *Journal of Finance* (March 1975).

[2] Agmon, T. "The Relationship among Equity Markets: A Study of Share Price Co-movements in the U.S., U.K., Germany and Japan." *Journal of Finance* (September 1972).

[3] Chen, A. H., and A. J. Boness. "Effects of Uncertain Inflation on the Investment and Financing Decisions of a Firm." *Journal of Finance* (May 1975).

[4] Fama, E. F. "Risk Return and Equilibrium: Some Clarifying Comments." *Journal of Finance* (March 1968).

[5] Fama, E. F., and M. H. Miller. *The Theory of Finance*. New York: Holt, Rinehart and Winston, Inc. (1972).

[6] Farber, A. L. "Performance of Internationally Diversified Mutual Funds." In *International Capital Markets*, edited by E. J. Elton and M. J. Gruber. North Holland (1975).

[7] Hamada, S. "Portfolio Analysis, Market Equilibrium and Corporation Finance." *Journal of Finance* (March 1969).

[8] Jacquillant, B., and B. H. Solnik. "Pricing of Multinational Firms: An Empirical Investigation of European and U.S. MNF Stock Prices." European Institute for Advanced Studies in Management (January 1976).

[9] Lintner, J. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics* (February 1965).

[10] Modigliani, F., and M. H. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review* (June 1958).

[11] __________. "Corporate Income Taxes and the Cost of Capital: A Correction." *American Economic Review* (June 1963).

[12] Mossin, J. "Equilibrium in a Capital Asset Market." *Econometrica* (October 1966).

[13] __________. "Security Pricing and Investment Criteria in Competitive Market." *American Economic Review* (December 1969).

[14] Rubinstein, M. E. "A Mean-variance Synthesis of Corporate Financial Theory." *Journal of Finance* (March 1973).

[15] Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium under Condition of Risk." *Journal of Finance* (September 1974).

[16] Solnik, B. H. "An Equilibrium Model of the International Capital Market." *Journal of Economic Theory*, vol. 8 (1974).

[17] __________. "The International Pricing of Risk: An Empirical Investigation of the World Capital Market Structure." *Journal of Finance* (May 1974).

[18] "Symposium on the Optimality of Capital Markets." *Bell Journal of Economics and Management Science* (Spring 1974).