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On the volatility of stock prices: an exercise in quantitative theory

RAJNISH MEHRA†

This paper examines the issues of volatility at the aggregate level. Rather than studying individual securities we focus on volatility utilizing aggregate stock market values and aggregate after-tax net cash flow as a ratio of national income. Our approach is in the tradition of the infinitely-lived classical growth model of Solow, where the behaviour of capital, consumption and investment are studied as shares of output. For the period 1946–1993 both the cash flows to equity and consumption as a share of national income were fairly constant. Yet there was significant movement in the value of the stock market as a share of national income. Our analysis suggests that these large movements cannot be rationalized within the context of the decentralized stochastic growth paradigm.

1. Introduction

The neoclassical growth model and its stochastic variants are a central construct in contemporary finance, public finance and business cycle theory. It has been used extensively by, among others, Abel *et al.* (1989), Auerbach and Kotlikoff (1987), Barro and Becker (1988), Brock (1979), Cox *et al.* (1985), Donaldson and Mehra (1984), Kydland and Prescott (1982), Lucas (1988), and Merton (1971). In fact, much of our economic intuition is derived from this model class.

The model has had some remarkable successes when confronted with empirical data, particularly in the stream of macro-economic research referred to as Real Business Cycle Theory, where researchers have found that it broadly replicates the essential macro economic features of the business cycle. See, in particular, Kydland and Prescott (1982). Unfortunately, when confronted with financial market data on stock returns, tests of these models have led, without exception, to their rejection. Perhaps the most striking of these rejections is contained in the paper by Mehra and Prescott (1985). These authors show that for reasonable values of the discount factor and the coefficient of relative risk aversion, the implied equity premium is too low when the model is calibrated to reflect historically observed aggregate consumption growth rates.

A related stream of research has focused on stock price volatility. The majority of studies to date in this area have been micro-studies (a notable exception is Grossman and Shiller 1981). This line of research has its origins in the important early work of Shiller (1981) and LeRoy and Porter (1981), which found evidence of excessive volatility of stock prices relative to the underlying dividend/earnings process. Using data for 100 years, Shiller (1981) in particular reported that, in his model, the volatility of actual stock prices exceeded the theoretical upper bound by a factor of 5.59. These studies use a constant interest rate, an assumption subsequently relaxed by Grossman and Shiller (1981) who addressed the issue of varying interest rates. They concluded that, although this reduced the excess volatility, Shiller's conclusion could not be overturned for reasonable values of the coefficient of relative risk aversion.

The conclusions of the above cited studies have been challenged, most notably by Cochrane (1992), Flavin (1983), Kleidon (1986) and Marsh and Merton (1986). These challenges appear to have merit. The interested reader is referred to Cochrane (1991), Gilles and LeRoy (1990) or Shiller (1989) for a detailed overview.

This paper shifts the focus of analysis from the firm to the aggregate level and uses the decentralized version of the representative agent stochastic growth model as a point of departure. Rather than studying individual securities we choose to examine issues of volatility utilizing aggregate stock market values and aggregate after-tax net cashflows as a ratio to National Income. In this tradition, the behaviours of capital, consumption and investment are studied as shares of output, bearing

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† Department of Economics, University of California, Santa Barbara, CA 93106, USA and the Graduate School of Business, University of Chicago, Chicago, IL 60637, USA.

Figure 1. Stock market value/national income.

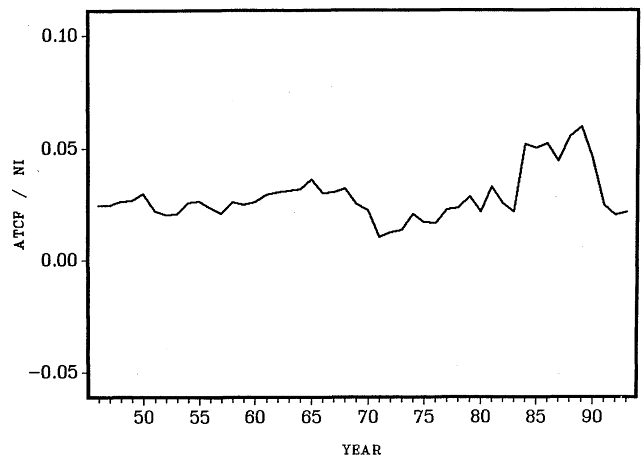

Figure 2. After tax cash flow/national income.

in mind the well-documented regularities of their ratios (Solow 1970).

This paradigm has several advantages. The partial equilibrium micro-studies cited earlier ignore the interaction of consumption growth and interest rates, implicitly assuming their independence. In contrast, the neoclassical growth model explicitly captures their interaction. Dividend and stock price series are non-stationary; however, they appear to be cointegrated with National Income. Examining these aggregate values relative to Net Income induces stationarity and is natural in this theoretical setting.

The principal results of our study cover the US economy for the postwar years 1946–1993. During this period we observe that the value of equity in the US, as a ratio of National Income, has moved by a factor of about three—from a low of 0.48 of National Income in 1948 to a high of 1.33 of National Income in 1968, dropping down to 0.53 of National Income in 1974. Furthermore, there is a fair amount of persistence in the plot of the ratio of market value of equity to National Income versus time (see figure 1). During the same period, the share of claims to equity has been relatively stable (approximately 2.5%), ranging from 2.67% of National Income in 1948 to 2.91% in 1968 and 2.02% in 1974 (see figure 2). Furthermore, the share of equity to output is positively correlated with the growth rate of output. Periods where the ratio of equity to National Income was high were also periods where the growth rate of output was high.

In this paper, we analyse the behaviour of equity as a ratio of National Income. We address the question as to whether this behaviour is consistent with the decentralized stochastic growth paradigm.

The study consists of two main parts. To build some intuition, we first address these issues in a deterministic steady-state context. Next we extend our analysis to stochastic models with low frequency movements to gauge if this implies large movements in the ratio of Equity (e) to National Income (y). We also discuss the implied relationship between growth rates and e/y. The paper is organized as follows: section 2 summarizes the US historical experience for the period 1946–1993. Section 3 describes the economy studied. Section 4 presents the results and concludes the paper.

2. Data

The data used in this paper consist of a set of series for the period 1946–1993. These are individually described below.

(i) Series y: National Income data; obtained from *The Economic Report of the President*.

(ii) Series ry: Real per Capita National Income. This is series y divided by the population and the GNP deflator from *The Economic Report of the President*.

(iii) Series e: Market Value of Equity; obtained from the Board of Governors publication, *Flow of Funds Accounts Financial Assets and Liabilities Year-End Values*. (Barro and Becker 1987, provide a justification for the infinitely lived family construct in their formulation of a dynastic utility function.) The ratio of e/y is plotted in figure 1.

(iv) Series xe: Extended Market Value of Equity: For the period 1945–1993 the values were taken from the Board of Governors publication: *Flow of Funds Accounts Financial Assets & Liabilities Year-End Values* and are identical to series e. For the period 1929–1944 the values of e were taken from Holland and Myers (1984) after an adjustment discussed below. Holland and Myers report equity values from 1929–1981 for non-financial corporations. Since there is overlapping data from the period 1945–1981 (36 years), we

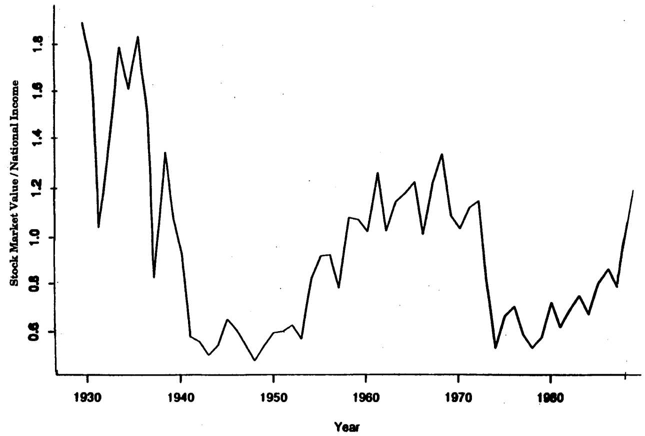

Figure 3. Stock market value/national income.

calculated the mean value of the ratio of the Holland–Myers data to the Flow of Funds year end data. The value is 0.644, i.e. the Holland–Myers data are systematically biased downward with mean 0.644 and variance 0.003 77. We used this value to adjust the Holland–Myers data from 1929–1944. The ratio xe/y is plotted in figure 3.

(v) Series d: Dividend Data; obtained from *The Economic Report of the President.*

(vi) Series *ne*: Net New Equity Issues; obtained from the Board of Governors publication, *Flow of Funds.*

(vii) Series x: After-Tax Cash Flow to Equity; computed as $x = d - ne$. The ratio of x/y is plotted in figure 2.

(viii) Series c: Consumption of Nondurables and Services; obtained from *The Economic Report of the President.*

The study commences with information from 1946 since reliable data for the series '*ne*' and '*e*' are unavailable prior to that year.

3. The economy and asset valuation

The economy we consider has a single representative 'stand-in' household. This unit orders its preferences over random consumption paths by

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u(c_t)\right), \qquad 0<\beta<1 \tag{1}$$

where c_t is the per capita consumption, β is the subjective time discount factor, $E_t\{\bullet\}$ is the expectation operator conditional upon information available at time zero (which denotes the present time) and u: $R_+ \to R$ is the increasing, continuously differentiable concave utility function. We further restrict the utility function to be of the constant relative risk aversion class

$$u(c,\alpha) = \frac{c^{1-\alpha}}{1-\alpha}, \qquad 0<\alpha<\infty \tag{2}$$

where the parameter α measures the curvature of the utility function. When $\alpha = 1$, the utility function is defined to be logarithmic, which is the limit of the above representation.

We assume there is a productive unit which produces output y_t in period t. Let x_t be the period dividend and e_t the price of an asset with a claim to a stochastic process $\{x_t\}$. In our initial formulation we do not explicitly model technology. Instead, we assume that $\{c, x, y\}$ is the joint equilibrium process generated by an economy with preferences specified above and a technology that incorporates capital accumulation. If we cannot account for the variation in e/y without imposing technological restrictions then the addition of further restrictions will not change our conclusions (see Appendix). If $\{c, x, y\}$ is the equilibrium stochastic process of consumption, cash-flow and output for a homogeneous consumer economy with specified preferences and technology, then we can determine an equilibrium process $\{c/y, x/y, y'/y, e/y\}$ where y' denotes the next period's output.

Since we assume maximizing behaviour on the part of the representative agent, the price of any asset (e_t) with a stochastic process $\{x_t\}$ as its claim satisfies the Euler equation

$$e_t = \beta E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}[e_{t+1} + x_{t+1}]\right\}. \tag{3}$$

Consequently, the equilibrium process $\{c/y, x/y, y'/y$ and $e/y\}$ will satisfy

$$\frac{e_t}{y_t} = \beta E_t\left\{\left(\frac{c_{t+1}}{y_{t+1}}\bigg|\frac{c_t}{y_t}\right)^{-\alpha}\left(\frac{y_{t+1}}{y_t}\right)^{1-\alpha}\left[\frac{e_{t+1}}{y_{t+1}} + \frac{x_{t+1}}{y_{t+1}}\right]\right\} \tag{4}$$

in addition to satisfying other restrictions imposed by technology.

The state of this economy $\{i, j, k\}$ follows an independent Markov process. Let g_{ijk}, x_{ijk} and c_{ijk} be the values of y'/y, x/y and c/y, respectively, in state $\{i, j, k\}$. To capture the correlations between these variables let

$$g_{ijk} = \lambda_j \tag{5}$$

$$c_{ijk} = a_1\lambda_j + \theta_i \tag{6}$$

$$x_{ijk} = a_2\lambda_j + a_3\theta_i + \gamma_k \tag{7}$$

where λ_j, θ_i and γ_k follow a Markov process and are iid. For notational simplicity let $z = \{i, j, k\}$ be the current state and z' be the next period's state. Using this notation, equation (4) can be rewritten as

$$e_z = \beta\sum_{z'}\left\{\pi_{z',z}\left(\frac{c_{z'}}{c_z}\right)^{-\alpha}(g_{z'})^{1-\alpha}[e_{z'} + x_{z'}]\right\}. \tag{8}$$

What observed quantities best correspond to the theoretical valuation expressions developed here? To build some intuition, let us first consider a deterministic setting.

At the firm level, the value of a stock is frequently represented as the discounted present value of future dividends. This representation has been used in the work of Shiller (1981) and other cited earlier. However, the value of the equity of a firm is *not* equal to the present value of all future dividends, i.e.

$$e_0 \neq \sum_{t=1}^{\infty} \frac{d_t}{(1+r)^t} \tag{9}$$

where e is the current value of the equity of the firm and d_t is the value of the aggregate dividend paid out at time t.

The correct expression is (for a comprehensive discussion see Miller and Modigliani 1961)

$$e_0 = \sum_{t=1}^{\infty} \frac{d_t - ne_t}{(1+r)^t} \tag{10}$$

where ne_t is the net new equity financing between time $t-1$ and t. Only in the special case when a firm finances uisng only retained earnings, and neither issues nor repurchases shares, does (9) hold with equality (at the aggregate level, this implies no *net* stock issue or repurchase for the firms in the economy).

Since data on stock issues and repurchases are available from 1946, we calculate the net cash flow to equity holders in the economy as

$$x = d - ne$$

where we now interpret d as the aggregate dividend and ne the value of the net new equity issues. Hence the aggregate value of equity in this economy satisfies

$$e_t = \sum_{s=1}^{\infty} \frac{x_{t+s}}{(1+r)^s} \tag{11}$$

3.1. *Some preliminary analysis*

We begin our analysis by attempting to explain the behavior of e/y qualitatively. In the neoclassical growth economy, in steady state along a balanced growth path, capital will grow at a constant rate η. Let capital k be divided into two parts: corporate capital, k_c, and all other capital, k^*. If the ratio k^*/k is relatively constant, then k_c will grow at a constant rate η. Corporate capital k_c can be further divided into debt capital b and equity capital e so that $k_c = b + e$. If the debt/equity ratio is specified, then equity and hence the claims to equity will be growing at a constant rate η, i.e. $x_{t+1} = x_t(1+\eta)$. Substituting in equation (11)

$$e_t = \frac{x_t(1+\eta)}{r-\eta} \tag{12}$$

or

$$e/y = \frac{(x/y)(1+\eta)}{r-\eta} \tag{13}$$

Can the large changes in e/y be accounted for within this standard neoclassical growth model? In this model, by varying parameters that are exogenous to the model, we can have different values for e/k, k/y and r. Let us examine the effects of each of these on e/y as a possible explanation.

(*a*) In steady state along a balanced growth path, if the debt/equity ratio (b/e) is low with corporate capital k_c fixed, then e/y is high. Hence, if there was a change in capital structure with corporations buying back debt by issuing equity, we would see an increase in e/y, the ratio of equity to output. (Figure 4 illustrates the effect of a change in b/e ratios on e/y, for an economy in steady state.)

Historically for the US, the debt/equity ratio (b/e) has steadily increased since 1950. Taggart (1985) reports that while in 1945 the debt/equity ratio (b/e) was ≈10%, in 1980 it was ≈40% (see figure 5).* Taggart (1985) reports, '...the use of debt financing has increased considerably in the post war period.... This trend emerges regardless of the method of measurement employed....' Does historical evidence support the steady state result that e/y and b/e move inversely?

Debt/equity ratios in the 1980s were comparable to those in the late 1920s, whereas the ratios of market-value of equity to National Income (e/y)

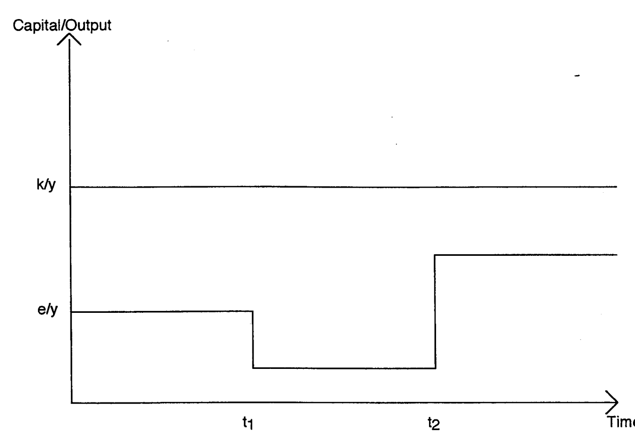

Figure 4. The effect of an increase in the b/e ratio at time t_1 and a decrease at time t_2. The economy is in a steady state.

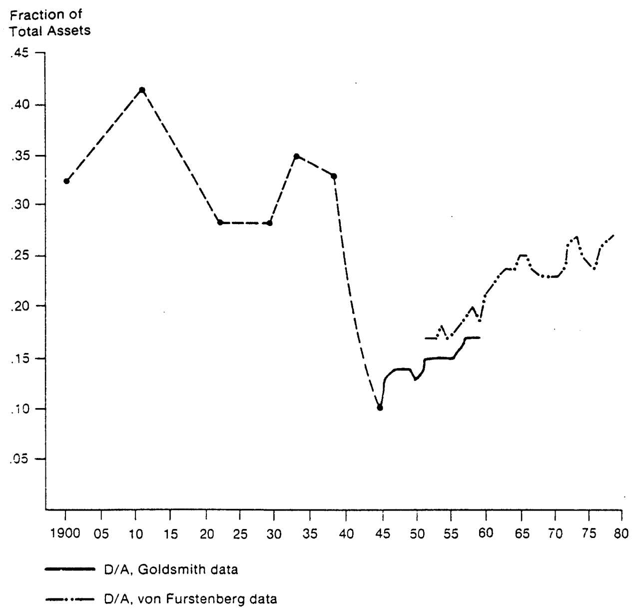

Figure 5. Ratios of market value of debt to replacement cost of assets. Source Taggart (1985).

were significantly different (see figure 3). During the period 1950–1970, when b/e was monotonically increasing, e/y was persistently high—in direct contradiction to our theoretical expectation.

Some caveats are in order.

(i) We are implicitly assuming a Miller (1977) model of capital structure.

(ii) Inflation tends to lower the value of equity since assets are depreciated on the basis of historical cost; on the other hand, the real value of a firm's long-term debt obligations declines, thereby raising the value of equity. We implicitly assume that these effects offset each other.

(*b*) In steady state, if the capital/output ratio (k/y) is large, then e/y is large (holding b/e and k^*/k fixed). If b/e is fixed, then

$$\frac{b}{e}+1=\frac{b+e}{e}=\frac{k_c}{e}$$

is fixed, which implies e/y and k/y are positively correlated.

Historically, the capital/output ratio (k/y) is trendless and constant and cannot, therefore, account for the movement in e/y.

* In a private communication, Robert Taggart suggested that Debt/Assets was a reasonable approximation for Debt/(Debt & Equity), from which the debt/equity ratio can be easily calculated. Note that, in this study, we use the qualitative fact that this ratio has been increasing in the postwar period.

(*c*) A third implication of the deterministic neoclassical growth model concerns the interaction between real interest rates and consumption growth rates. Along a balanced growth path, $r=\rho+\eta\alpha$ implying that r is high when the growth rate of consumption is high (given $\alpha>0$). (This result can be found in Arrow and Kurz 1970, Solow 1970 or Dixit 1976.) Hence, a high growth rate (η) implies a low e/y (given x/y and b/e are the same). This is not substantiated by our data. During the 1960s we observe a high η as well as record high values of e/y.

To summarize, our preliminary analysis suggests that historical movements in e/y cannot be systematically accounted for, even qualitatively, within the deterministic neoclassical growth model. In an effort to achieve greater congruence between theory and empirical data, we next analyse the behaviour of e/y in the stochastic economy outlined earlier.

4. Results

The parameters defining preferences are α and β; the parameters defining technology are the constants a_1, a_2, a_3 and elements of $[p_{ij}]$ and $[\lambda_i]$, $[q_{ij}]$ and $[\theta_i]$ and $[r_{ij}]$ and $[\gamma_i]$. Our approach is to assume two states for each Markov chain (on λ, θ and γ) and to restrict each process as follows. For example, for the process on λ

$$\lambda_1=\lambda+\sigma(\lambda),\quad \lambda_2=\lambda-\sigma(\lambda)$$

$$p_{11}=p_{22}=p,\quad p_{12}=p_{21}=(1-p)$$

This parametrization allows us to change independently the mean values λ, θ and γ by changing $\bar{\lambda}$, $\bar{\theta}$ and $\bar{\gamma}$, to change their variability by altering $\sigma(\lambda)$, $\sigma(\theta)$ and $\sigma(\gamma)$ and to vary their serial correlation by adjusting p, q and r.

The parameters were selected so that the average values of g_z, c_z and x_z, their standard deviations and their cross-correlations with respect to the model's stationary distribution matched the sample values for the US economy between 1946–1993.

The sample statistics for the US economy are presented in table 1. Table 2 summarizes the calibrated parameters.

All the variables in this economy are in real per capita terms. However, since we are interested in the ratios of e/y, c/y and x/y we can use nominal aggregate values in both the numerator and denominator without affecting the results. The only exception is y'/y where we must use the values of real per capita National Income (series ry). Hence the mean and standard deviations of g_z are for real per capita National Income. Established economic theory typically uses low values for the coefficient of risk aversion α. In this study we do not challenge this vast literature, but, based upon it, we upper bound α by a

Table 1. Sample moments for the US economy: 1946–1993

Variable	g_z	c_z	x_z	e_z
Mean	1.0159	67.69%	2.82%	79.88%
Standard Deviation	3.305%	2.965%	1.096%	23.55%

$\text{Cov}(g_z, c_z) = 6.79 \times 10^{-5}$ $\quad$ $\text{Cov}(g_z, g_{z'}) = 4.15 \times 10^{-5}$
$\text{Cov}(g_z, x_z) = 8.42 \times 10^{-5}$ $\quad$ $\text{Cov}(c_z, c_{z'}) = 7.93 \times 10^{-4}$
$\text{Cov}(c_z, x_z) = 9.97 \times 10^{-5}$ $\quad$ $\text{Cov}(x_z, x_{z'}) = 9.32 \times 10^{-5}$

Table 2. Parameter calibrated to the US economy: 1946–1993

$a_1 = 0.062$	$a_2 = 0.077$	$a_3 = 0.11$
$\bar{\theta} = 0.614$	$\bar{\lambda} = 1.016$	$\bar{\gamma} = -0.12$
$\sigma(\theta) = 2.96 \times 10^{-2}$	$\sigma(\lambda) = 3.31 \times 10^{-2}$	$\sigma(\gamma) = 1.015 \times 10^{-2}$
$\theta_1 = 0.643$	$\lambda_1 = 1.049$	$\gamma_1 = -0.11$
$\theta_2 = 0.584$	$\lambda_2 = 0.983$	$\gamma_2 = -0.13$
$q = 0.95$	$p = 0.52$	$r = 0.90$

Table 3. Statistics for (e/y) generated by the model economy: $\beta = 0.96$

$\alpha =$	1	2	4	10	US 1946–93
Mean (e/y)	0.615	0.447	0.300	0.187	0.80
$\sigma(e/y)$	0.052	0.055	0.054	0.057	0.235
Range of (e/y)	0.54–0.69	0.37–0.53	0.23–0.38	0.11–0.27	0.48–1.33

Table 4. Statistics for (e/y) generated by the model economy: $\beta = 0.99$

$\alpha =$	1	2	4	10	US 1946–93
Mean (e/y)	2.537	1.016	0.490	0.249	0.80
$\sigma(e/y)$	0.134	0.102	0.084	0.077	0.235
Range of (e/y)	2.36–2.71	0.88–1.15	0.38–0.61	0.15–0.36	0.48–1.33

value of 10. We refer the reader to Mehra and Prescott (1985) for a detailed discussion.

Once the economy has been calibrated, the state contingent values of e/y (e_{ijk}) can be calculated from the following set of equations

$$e_{ijk} = \beta \sum_{i=1}^{2} \sum_{m=1}^{2} \sum_{n=1}^{2} {}_{ijk}\phi_{lmn} g_{lmm}^{1-\alpha} \left(\frac{c_{lmm}}{c_{ijk}}\right)^{-\alpha} [e_{lmn} + x_{lmn}] \tag{14}$$

where ${}_{ijk}\phi_{lmn} = q_{il} \cdot p_{jm} \cdot r_{kn}$.

Note that equation (14) is linear in e_{ijk}. Since we have assumed two states for θ, λ and γ, there are eight such equations. These can be solved for the eight values of e_{ijk}.

The average value of the share of equity to output (e_z) over the period 1946–1993 was 0.80, with a standard deviation of 0.24. During the same period e_z varied from 0.48 to 1.33, i.e. by a factor of 3.

Given the estimated processes on the growth rate of output (g_z), consumption as a share of output (c_z) and cashflows as a share of output (x_z), we calculate the values of e_z which are consistent with the model. These values were obtained by varying x between zero and 10 and β between zero and 1. We report values for $\beta = 0.96$ and 0.99 and $\sigma = 1$, 2, 4 and 10 in tables 3 and 4. (For lower values of β, the standard deviation of e_z and its range rapidly shrink, rendering it uninteresting from our perspective.)

As illustrated in tables 3 and 4, the standard deviation of e_z for the calibrated economy was, in all cases, significantly lower than that observed in the sample period (0.235). For $\beta = 0.96$, for all values of α, the sample standard deviation exceeded the calibrated value by a factor of 4, while for $\beta = 0.99$, this factor was between 2 and 3. In addition, the level of e_z moved in a narrower range, in contrast to the wide range [0.48–1.33] observed for the sample period. In fact, for the period 1929–1993, the range for e/y was [0.45–1.90], with almost a fourfold increase between the lower and upper values. This is in spite of the fact that the variation in x_z in the calibrated economy matched that for the US economy in the postwar period.

The empirically observed values for the mean, standard deviation and range of e_z are clearly inconsistent with those predicted by the model.

Some recurring trends are evident in our results. In all the simulations, in states where consumption is low relative to output (i.e. c_z is low), e_z is correspondingly low, *ceteris paribus*. A similar congruence is observed between x_z and e_z. For every scenario, in states where the cash flow to equity holders was low relative to output, e_z was also low. Both these observations conform with our intuition. More significantly, e_z and g_z were positively correlated; irrespective of the level of α, a high growth rate resulted in a high value for e/y. This is consistent with observations for the US economy.

4.1. *Robustness of results*

In an attempt to reconcile the discrepancy between theory and observation, we tested the sensitivity of our results to model misspecification. We report the findings for two polar cases of interest. First, we consider the

Table 5. $\beta = 0.96, p = q = r = 0.99$

$\alpha =$	1	2	4	10	US 1946–93
Mean (e/y)	0.616	0.568	Equilibrium does not exist		0.80
$\sigma(e/y)$	0.175	0.272			0.235
Range of (e/y)	0.36–0.88	0.11–1.08			0.48–1.33

Table 6. $\beta = 0.96, p = q = r = 0.50$

$\alpha =$	1	2	4	10	US 1946–93
Mean (e/y)	0.615	0.446	0.300	0.196	0.80
$\sigma(e/y)$	0.027	0.039	0.052	0.081	0.235
Range of (e/y)	0.59–0.64	0.41–0.49	0.25–0.36	0.11–0.28	0.48–1.33

case where p, q and $r = 0.99$, implying that g_z, c_z and x_z almost follow a random walk. The results are summarized in table 5.

Since c_z and x_z already display considerable persistence in the data ($q = 0.95$ and $r = 0.90$), it is the additional increase in persistence in the growth rate of output g_z (from $p = 0.52$ to $p = 0.99$) that is responsible for both the increase in standard deviation and the range. If, in fact, the growth rate of output showed persistence, the model would match observations on volatility for α between 1 and 2.

However, in this case where $\alpha > 1$, in states where the growth rate was low, the ratio e/y was high. This negative correlation between g_z and e_z is inconsistent with empirical observations. For the US data, the range and standard deviation of e_z was large and e_z was positively correlated with the growth rate g_z.

Intuitively, with high levels of persistence the economy behaves like a deterministic one, switching between two growth rates η_1 (high) and η_2 (low). In a deterministic economy along a balanced growth path

$$e/y = \frac{(x/y)(1+\eta)}{\rho + (\alpha - 1)\eta}$$

where we have used equation (13) and the fact that $r = \rho + \alpha\eta$.

We see that $\partial(e/y)/\partial\eta < 0$ if $\alpha > 1 + \rho \approx 1$. Therefore, the e/y ratio will be low when η is high, just as we observe in our simulations.

In the second case, when $p = q$ and $r = 0.5$, successive changes in g_z, c_z and x_z are independent. Table 6 summarizes the statistics for e_z.

It is remarkably similar to table 3. Since g_z displays almost no persistence in the data ($p = 0.52$), the decrease in persistence in c_z and x_z appears to have almost no impact on the behaviour of e_z.

Tables 5 and 6 clearly demonstrate that introducing low frequency movements in the growth rate of output greatly increases both the variability and range of e_z and underscores the importance of these movements to any discussion of volatility. To capture the implications of low frequency movements consider the following thought experiment.

We retain all the parameters of the calibrated economy except that we consider the case where output t grows at two rates, 3% and 1%, for an expected period of 10 years. (Of course, when the process on g_z changes, the process on consumption c_z and cash flows x_z will also change) What will be the implications for our model?

Table 7. $\beta = 0.96, \lambda_1 = 1.03, \lambda_2 = 1.01, p = 0.90$

$\alpha =$	1	2	4	10	US 1946–93
Mean (e/y)	0.623	0.42	0.26	0.123	0.80
$\sigma(e/y)$	0.052	0.053	0.05	0.04	0.235
Range of (e/y)	0.55–0.70	0.34–0.50	0.17–0.32	0.06–0.21	0.48–1.33

In this case, $\lambda_1 = 1.03$ and $\lambda_2 = 1.01$ and p, the transition probability, satisfies the relation (see table 7)

$$\sum_{k=1}^{\infty} k(1-p)p^{k-1} = 10$$

$$p = \frac{10-1}{10} = 0.9$$

As expected, the range of e/y has increased (the effects being more pronounced for higher values of α), but, once again, high values of e/y correspond to states where the growth rate is low. Moreover, in sharp contrast to table 5, the range and standard deviation of (e/y) is quite different from that observed for the sample period (1946–1993). Clearly, extremely high levels of persistence are needed for the model to be consistent with observations on volatility.

An insight gained from this study is that while low frequency movements in the growth rate are important in determining the volatility of stock prices, persistence in growth rates reduces the equity premium. In the latter case, it is the high frequency movements that are crucial in increasing the premium. In other words, a paradigm that satisfactorily explains one of the phenomena would not necessarily resolve the other.

To summarize, the stochastic model results do not match the values for the standard deviation and range of (e/y) observed in the US sample data. Our simulations underscore the importance of low frequency movements (persistence) in the growth rate to any study of volatility. Consistent with actual observations, in periods of low persistence in growth rates, equity as a share of output will be positively correlated with growth rates. However, an increase in persistence (while increasing the range and standard deviation of e/y), reverses the correlation. Hence, in the stochastic economy studied, it was not possible to match both the standard deviation and the range for e/y and also generate a positive correlation between growth rates and e/y.

Our results are in concurrence with the conclusions of Grossman and Shiller (1981), Shiller (1989) and Gilles and LeRoy (1990) regarding excessive volatility in the US economy. For the period 1946–1993, both the cash flows to equity and consumption as a share of National Income were fairly constant. Yet there was significant movement in the value of the stock market as a share of National Income. Our analysis suggests that these large movements cannot be rationalized within the context of the decentralized stochastic growth paradigm.

It is possible that an alternative structure, incorporating market incompleteness and overlapping generations, such as Constantinides *et al.* (1997), will prove more useful in understanding why there are such large movements in the stock market relative to National Income and only small, relatively transitory, movements in earnings as a share of National Income. This is likely to be a fruitful area for future research.

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Appendix

This Appendix presents a simple proof to demonstrate that expanding the set of technologies in a pure exchange, Arrow–Debreu economy to admit capital accumulation and production does not increase the set of joint equilibrium processes on consumption and asset prices.

Let θ denote preferences, τ technologies, E the set of the exogenous processes on the aggregate consumption good, P the set of technologies with production opportunities, and $m(\theta, \tau)$ the set of equilibria for the economy (θ, τ).

Theorem:

$$\bigcup_{\tau \in E} m(\theta, \tau) \supset \bigcup_{\tau \in P} m(\theta, \tau)$$

Proof: For $\theta_0 \in \theta$ and $\tau_0 \in P$ let (a_0, c_0) be a joint equilibrium process on asset prices and consumption. A necessary condition for equilibrium is that the asset prices a_0 be consistent with c_0, the optimal consumption for the household with preferences θ_0. Thus, if (a_0, c_0) is an equilibrium then

$$a_0 = g(c_0, \theta),$$

where g is defined by the first-order necessary conditions for household maximization. This functional relation must hold for all equilibria, regardless of whether they are for a pure exchange or a production economy.

Let (a_0, c_0) be an equilibrium for some economy (θ_0, τ_0) with $\tau_0 \in P$. Consider the pure exchange economy with $\theta_1 = \theta_0$ and $\tau_1 = c_0$. Our contention is that (a_0, c_0) is a joint equilibrium process for asset prices and consumption for the pure exchange economy (θ_1, τ_1). For all pure exchange economies, the equilibrium consumption process is τ, so $c_1 = \tau_1 = c_0$, given that more is preferred to less. If c_0 is the equilibrium process, the corresponding asset price must be $g(c_0, \theta_1)$. But $\theta_1 = \theta_0$, so $g(c_0, \theta_1) = g(c_0, \theta_0) = a_0$. Hence a_0 is the equilibrium for the pure exchange economy (θ_1, τ_1), proving the theorem. Since the set of equilibria in a production company is a subset of those in an exchange economy, it follows immediately that if the variation in e/y cannot be accounted for in an exchange economy, modifying the technology to incorporate production will not alter this conclusion. □

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